COMPARATIVE ADVANTAGE ACROSS GOODS AND PRODUCT QUALITY

Francisco Alcalá*

First version May 2007

This version: October 2010

Abstract: This paper explores the connection between specialization across goods and specialization within goods along the quality dimension. It introduces quality differentiation and firm heterogeneity into the Dornbusch, Fischer and Samuelson (1977) framework. Efficiency can be quality biased (i.e., more-efficient firms are relatively more efficient at producing higher quality). In industries in which this is the case, the highest quality is produced by the country that has the absolute advantage in the industry and the lowest quality is produced by the country that has the lowest wage. More importantly, for each good, average export quality increases with the exporter's revealed comparative advantage, conditional on its wage level. This prediction is shown to be consistent with the data on US imports of apparel and clothing accessories.

Keywords: International trade, Vertical specialization, Horizontal specialization, Quality margin, Extensive margin. JEL classification: F10.

---

* Universidad de Murcia and Ivie. I thank Christian Broda, Jonathan Eaton, and Tim Kehoe for comments and useful suggestions on an earlier version, as well as participants in presentations at NYU and New York Federal Reserve Bank. This paper was written while visiting the Economics Department at NYU whose hospitality is very much appreciated. Financial support was provided by the Spanish Ministry of Education and Science project ECO2008-02654/ECON, and by the Fundación Séneca de la Región de Murcia, project 05710/PHCS/07. Contact: alcala.paco@gmail.com .

1. INTRODUCTION

Specialization within goods across the quality dimension (vertical specialization) is becoming increasingly important relative to specialization across goods (horizontal specialization). Gradually, the same types of industrial goods are being exported from developed and developing economies. Meanwhile, richer countries are specializing in exporting the higher qualities (Schott 2004, Hummels and Klenow 2005, and others thereafter). Besides this characterization, our knowledge about the patterns and determinants of vertical specialization is still quite limited. This paper explores the connections between vertical and horizontal specialization.

Intuitively, one can expect absolute advantage and product quality to be positively related in agricultural products having a wide range of qualities. Consider cigars or coffee. If a country has the best land and climate to produce cigars or coffee, it is likely that it will produce high quality products and that it will also have some horizontal specialization in these type of goods. Can a similar argument be applied to industrial goods?

The key theoretical hypothesis in this paper is that, in some industries, efficiency is quality biased. This means that the relative efficiency between two firms is not constant across different levels of output quality but more-efficient firms are comparatively even more efficient at producing the higher qualities. This potential quality bias of efficiency is a common assumption in models with heterogeneous firms and quality differentiation (e.g. Baldwin and Harrigan 2007, Johnson 2007, Verhoogen 2008, Kugler and Verhoogen 2008, and Alcalá and Hernández 2010) and has received empirical support from Gervais (2009). Consider now a country having an absolute advantage in some industry (e.g., this advantage could be due to knowledge spillovers and a long tradition of the industry in its territory). In that industry, firms in from that country will on average be more efficient than firms from other countries. Therefore, they will on average produce higher quality.

As it will be shown below, the potential link between absolute advantage and quality should produce a positive correlation (conditional on wages) between revealed comparative advantage (RCA) and quality. The intuition is that horizontal specialization, as measured by RCA, can be related to the countries' relative labor unit costs (i.e., wages over efficiency) in the different industries. Thus, conditional on wages, a larger RCA in a given industry tends to reveal more efficiency (i.e., larger absolute advantage). Hence, conditional on wages, RCA will tend to be positively correlated with quality in industries in which efficiency is quality biased.

Figure 1 shows several examples of this correlation for several 5-digit goods within the clothing and accessory industry (industry 84 of the SITC). The data correspond to US imports. The vertical axes measure unit values of imports from different countries (using unit values as a proxy for average quality) and the horizontal axes measure exporter RCA in that specific good (see Section 5 for more details about the data). In all the cases, a positive relationship between exporter RCA and import unit value is apparent. This positive relationship holds regardless of whether or not the sample of exporting countries is split into two different groups according to their per capita income.

The previous discussion suggests that both the horizontal and the vertical dimensions of trade are important and are likely to be connected. Thus, ideally, trade models should be able to incorporate both dimensions in a unified framework. This paper introduces quality differentiation and firm heterogeneity in the Dornbusch-Fischer-Samuelson model. By that means, it provides a simple integrated model where both the horizontal and vertical dimensions of specialization are present so that their interactions can be investigated. The main prediction of the model is that, in industries in which efficiency is quality biased, average output quality increases with the exporter's RCA conditional on its wage level. This prediction is tested for the particular case of the apparel and clothing accessories industry using data on US imports. This industry is particularly interesting: it is the manufacturing industry with the largest number of exporters to the US including a wide range of both low- and high-income countries, it contains a wide variety of products, and products show significant quality differences as proxied by unit values. It is found that exporter RCA, conditional on exporter wages, is significantly correlated with US import unit values for more than 40% of the goods being considered (72 goods of industry 84 of the SITC at the 5-digit level, which are the goods remaining after filtering those with too few observations).

Firm heterogeneity also implies in the model's equilibrium that each good tends to be produced in both countries in many different qualities. As a result, the spectrum of qualities of each good produced by each country may overlap with the other country's spectrum. The model then predicts that, within each good, (i) the highest quality is produced by the country with the absolute advantage; (ii) the lowest quality is produced by the country with the lowest wage; and (iii) the country with the lowest labor unit cost produces the widest spectrum of qualities.

These predictions can lead to different patterns of the overlap of the qualities produced by each country. In industries in which the lower-wage country has the absolute advantage, the higher-wage country will produce a range of qualities that is a strict subset of the lower-wage country's range (for example, Spain exports carpets but the range of qualities reaches neither the highest qualities exported by Turkey nor its lowest ones). Furthermore, the model generalizes some central results of the Dornbusch-Fischer-Samuelson model to a setting with heterogeneous firms and quality differentiation. In the model's equilibrium, richer countries export a wider set of goods (the extensive margin of exports)[1] and country market shares across goods are a continuous decreasing function of relative labor unit costs.[2]

The model is very stark on the demand side. In particular, it assumes the same homothetic demand across goods and quality varieties in the two countries. To be sure, non-homotheticicies are important in shaping the patterns of trade along the quality dimension (see Hallak 2006, Choi, Hummels and Xiang 2009, and Fieler 2008). However, homotheticity proves to be a useful simplification to derive some new and interesting predictions. Moreover, there is no reason to expect that these predictions would be reversed by introducing non-homotheticicies.

Existing trade models concentrate on only one of these two dimensions of specialization. There is by now an important list of theoretical models analyzing the patterns of country specialization along the quality dimension (Flam and Helpman 1987, Falvey and Kierzkowski 1987, Grossman and Helpman 1991, Stokey 1991, and Murphy and Shleifer 1997, among others). They all predict that richer countries specialize in producing higher quality, which conforms to the general evidence cited above. However, these papers assume either only one vertically differentiated good in the economy (together with some non-differentiated good) or only one quality level per good at each point of time. As a consequence, they cannot provide a comprehensive description of the trade patterns indicated above and cannot explore the possible interactions between horizontal and vertical specialization. A second limitation of available trade models on country

[1] Even if vertical specialization is increasingly important, the relevance of the horizontal dimension of specialization and the extensive margin of exports (i.e., the increase in range of goods being exported) cannot be underestimated. Kehoe and Ruhl (2009) show that the extensive margin of exports accounts for the bulk of trade growth after trade liberalizations. Hummels and Klenow (2005) find that a 10-percent increase in per capita income brings about, on average, an 8.5-percent increase in the range of goods being exported.

[2] Recall that in the original Dornbusch-Fischer-Samuelson markets shares are discontinuous, going from zero to one at the commodity for which the two countries' labor unit costs are equalized.

specialization along the vertical dimension is that they assume homogeneous producers within each country. However, heterogeneity of firms' efficiency is a prominent phenomenon whose consideration has proven to be very fruitful in capturing important features of international trade.[3] This paper shows that accounting for firm heterogeneity is also important for describing the patterns of vertical specialization.

The paper is organized as follows. Section 2 lays out the model. Section 3 analyzes specialization across goods. Section 4 analyzes quality specialization within goods. Section 5 explores some empirical evidence using data for US imports of clothing and accessories. Section 6 concludes.

2. THE MODEL

Consider a two-country economy. Home and foreign countries are denoted H and F, respectively. Subscript W indicates world aggregates. There is a measure-one continuum of goods indexed by j. Each good defines an *industry*. Every good can be produced along a continuum of qualities. For each good, there is an infinite set of efficiency-heterogeneous potential producers in each country. In equilibrium, only a finite measure of firms will be active. Each firm produces only one good and chooses which quality and how many units to produce taking as given the other firms' quality and quantity choices (*Cournot* equilibrium). Firm k from country i in industry j produces $x_{ki}(j)$ units of good j with quality $q_{ki}(j)$. Firms choosing zero output are said to be inactive. There are no transportation costs. Hence the las of on e price in both countries holds for any good and quality variety.

2.1. Demand

Let $c_{ki}^{h}(j)$ be country-h representative agent's consumption of firm $k_i(j)$'s output. Consumers from both countries maximize the same utility function

[3] See Bernard and Jensen (1995) for pioneering empirical work; Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) for path breaking general equilibrium trade models; and Eaton, Kortum, and Kramarz (2005) for how the heterogeneous-firm framework coupled with Cournot equilibrium fits regularities on the distribution of firms and market shares across output destinations. Tybout (2003) and Bernard et al. (2007) review the literature. Bernard, Redding and Schott (2007) analyze firm heterogeneity and comparative advantage in a two-country two-factor two-industry model with no quality differentiation.

(1) $$\int_0^1 \ln\left(\sum_{i=H,F}\sum_k q_{ki}(j)c_{ki}^h(j)\right)dj$$

with respect to $c_{ki}^h(j)$, for every $k_i(j)$, subject to the budget constraint $Y^h = \int_0^1\left[\sum_{i=H,F}\sum_k P_{ki}(j)c_{ki}^h(j)\right]dj$; where Y^h is country-*h* representative consumer's income. $\sum_{i=H,F}\sum_k q_{ki}(j)c_{ki}^h(j)$ is referred to as the number of *quality units* of good *j* consumed by *h*. Maximization of (1) leads to the following first-order conditions:

(2) $$\frac{P_{ki}(j)}{P_{k'i'}(j)} = \frac{q_{ki}(j)}{q_{k'i'}(j)};$$

(3) $$\sum_{i=H,F}\sum_k P_{ki}(j)c_{ki}^h(j) = Y^h.$$

Condition (2) states that the relative price between any two varieties of the same good *j* produced by firms $k_i(j)$ and $k'_{i'}(j)$ is given by their relative quality (i.e., the products of two firms producing the same quality variety of the same good are perfect substitutes and the marginal rate of substitution between any two quality varieties of the same good is constant). Condition (3) states that expenditure is the same across all goods, as with any symmetric Cobb-Douglas utility function. Denote by $P(j)$ the price of a unit of good *j* with quality equal to 1. Then, from expression (2) we have:

$$P_{ki}(j) = P(j)\cdot q_{ki}(j),$$

I will refer to $P(j)$ as the *price level in industry j*. Using this expression to substitute in (3) and assuming market clearing for each firm's output (e.g., $x_{ki}(j) = \sum_{h=H,F} c_{ki}^h(j)$), yields the price level in industry *j* as a function of firms' output and quality choices:

(4) $$P(j) = \frac{Y_H + Y_F}{\sum_{i=H,F}\sum_k q_{ki}(j)x_{ki}(j)}.$$

This is the inverse demand function to be used in solving for the Cournot equilibrium of each industry.

The general equilibrium of an economy with this demand setting yields a determinate composition for each firm's output in terms of both quantity and quality, and therefore a determinate composition of world consumption. However, if no further considerations are made, in equilibrium, each representative consumer is indifferent between consuming any quality variety of each good, as long as relative prices between quality varieties satisfy (2). In other words, the composition of each individual's consumption basket in terms of quality varieties within each good is indeterminate (even if the composition of aggregate world consumption is fully determinate). Since a characterization of exports requires a determinate composition of each country representative agent's consumption basket, I will informally consider a common slight variation of utility function (1) that renders this composition fully determinate. I will assume an infinitesimal preference for variety over the varieties produced by the set of firms. As a result, each individual's consumption basket is a scaled down version of the world's consumption basket. Moreover, the composition of each country's exports is exactly the same as the composition of its production. Formally, if υ_i is country i's share in world income ($\upsilon_H = Y_H/(Y_H+Y_F) = 1-\upsilon_F$), then country i consumes a portion υ_i of every firm's output, and exports a portion $1-\upsilon_i$ of every domestic firm's output. Under this assumption, the vertical and horizontal characterization of each country's output in Sections 3 and 4 should also be interpreted as a characterization of its exports.

2.2. Technology

Labor is the only production factor. Increasing output quality comes at the cost of lower output per worker. Efficiency of firm k from country i in industry j is given by the product of three positive parameters: T_i, $a_i(j)$, and z_k; where T_i is a country-specific aggregate efficiency parameter, $a_i(j)$ is a country-industry-specific efficiency parameter, and z_k is a firm-specific efficiency parameter. Firm $k_i(j)$'s production function is given by:

$$(5) \qquad x_{ki}(j) = \left[T_i a_i(j) z_k\right]^{1-\sigma} \frac{l_{ki}(j)}{e^{q_{ki}(j)/[T_i a_i(j) z_k]^{\sigma}}}, \quad 0 \le \sigma \le 1;$$

where $l_{ki}(j)$ is its input of labor. The parameter σ measures the extent to which more-efficient firms have a relative advantage in producing higher quality goods. If $\sigma>0$ then efficiency is *quality biased*, whereas if $\sigma=0$ then efficiency is neutral with respect to producing higher quality. Parameter T_i captures differences across countries in general sources of productivity (e.g., good institutions, social capital or public infrastructures). Parameter $a_i(j)$ captures country-industry asymmetries, which may be due to differences in industry-specific knowledge, skills, and natural resource endowments. z_k captures firm-specific components such as the entrepreneur's skills.

Units of goods are normalized so that $T_F=a_F(j)=1$ for all j. Thus, we can drop subscripts for home technology parameters; i.e., $T_H=T$ and $a_H(j)=a(j)$. The function $a(j)$:[0,1]→R_{++} is assumed to be continuous, differentiable, and strictly decreasing. There are an infinite number of potential firms in each country and industry, indexed $k=0,...,\infty$, which are ordered according to efficiency. Firm 0 is the most efficient one and its efficiency is normalized $z_0=1$. In equilibrium, only a finite number of firms will be active. The distribution of firm efficiencies z_k is the same in all industries and countries.[4]

Country i's wage is denoted by w_i. The foreign wage is used as the *numeraire*: $w_F=1$. Let $c_{ki}(q, j)$ be the (constant) marginal cost of firm k from country i producing good j with quality q:

$$c_{ki}(q,j)=\frac{w_i}{[T_i a_i(j)z_k]^{1-\sigma}}e^{q/[T_i a_i(j)z_k]^{\sigma}}. \tag{6}$$

The use of the terms absolute and comparative advantage may lead to some confusion when both horizontal and vertical specialization are considered and become intertwined. I reserve these terms for comparison of advantage across goods and avoid their use in the analysis of trade along the quality dimension. Country H (respectively, F) is said to have an *absolute advantage* over country F (resp. H) in industry j if $Ta(j)>1$ (resp. $Ta(j)<1$). This implies that for each k, efficiency of firm k from country H, $Ta(j)z_k$, is higher than efficiency of firm k from country F. At any rate, even if the home country has an absolute advantage in the industry, some foreign firms may be more efficient than some home-country firms in that industry. Additionally, I will refer to the ratio $[w_i/T_i a_i(j)]/[w_h/T_h a_h(j)]$ as country i's *comparative cost* in industry j with respect to country h. Note that, due to normalizations, $w_F/T_F a_F(j)=1$ for all j. Hence, home country's comparative cost in industry j is simply $w/Ta(j)$. Country H (respectively, F) is said to have a *cost advantage* over country F (resp. H) in industry j if $w/Ta(j)<1$ (resp. $w/Ta(j)>1$).

2.3. Equilibrium

[4] The assumption that the distribution of firm-specific parameters z_k is the same in every industry and country is not necessary for the results but it greatly simplifies the exposition. The same results could be obtained by assuming the following first-order stochastic dominance of firm productivities across countries: if $T_i a_i(j)>T_h a_h(j)$ then $T_i a_i(j)z_{ki}(j)>T_h a_h(j)z_{kh}(j)$; where $z_{ki}(j)$ and $z_{kh}(j)$ are efficiency of the k^{th} most efficient firm in industry j in countries i and h, respectively.

Each firm maximizes profits $\pi_{ki}(j)=x_{ki}(j)[q_{ki}(j)P(j)-c_{ki}(q,j)]$ with respect to its output $x_{ki}(j)$ and quality $q_{ki}(j)$, taking as given the inverse industry demand function (4) and other firms' output and quality choices (Cournot equilibrium). From each firm's first order conditions of maximization we have:

$$(7) \qquad s_{ki}(j)=\begin{cases} 1-\dfrac{e}{P(j)}\dfrac{w_i}{T_i a_i(j) z_k} & \text{if} \quad 1-\dfrac{e}{P(j)}\dfrac{w_i}{T_i a_i(j) z_k}\geq 0; \\ 0 & \text{otherwise.} \end{cases}$$

$$(8) \qquad q_{ki}(j)=\left[T_i a_i(j) z_k\right]^{\sigma};$$

where $s_{ki}(j)$ is firm $k_i(j)$'s world market share in value terms, $s_{ki}(j)\equiv q_{ki}x_{ki}(j)/\sum_{h=H,F}\sum_g q_{gh}(j)x_{gh}(j)$.[5] We can define firm $k_i(j)$'s cost of producing one unit of quality as $c_{ki}(q,j)/q_{ki}(j)$. Substituting with (8) in (6) yields:

$$(9) \qquad \frac{c_{ki}(q,j)}{q_{ki}(j)}=\frac{w_i}{T_i a_i(j) z_k}$$

The least-efficient active firm from i in industry j will be denoted by $\bar{k}_i(j)$. Firm $\bar{k}_i(j)$ satisfies:

$$(10) \qquad z_{\bar{k}_i(j)}=\frac{e}{P(j)}\frac{w_i}{T_i a_i(j)}.$$

Interestingly, the parameter σ does not play any role when comparing these unit costs across firms and countries.

Expressions (7) and (8) imply that market share and output quality are increasing in the firm's efficiency. The positive link between efficiency and market shares is common to *Cournot* models with heterogeneous firms, whereas the positive link between efficiency and quality is

[5] See Appendix for details. This approach brings about exactly the same result than a Cournot equilibrium where each firm first chooses how many *quality units* of the good to produce (i.e., it chooses the product $x_{ki}(j)\cdot q_{ki}(j)$), given the other firms' production of quality units; and second, it chooses which combination of quantity and quality minimizes the cost of producing this optimal number of quality units.

conditional on the relative advantage of more efficient firms in producing higher quality (i.e., σ>0).

Some discussion of this second link will clarify some differences and similarities between this and other trade models with quality differentiation. Most models of trade with quality-differentiated goods contain some connection between efficiency and quality. Notwithstanding, models differ on how this connection works: it may be work at the aggregate, at the industry, or at the firm level, depending on the source of efficiency differences. General equilibrium models of international specialization with only one quality differentiated good such as Flam and Helpman (1987) only consider efficiency differences at the country level. As a consequence, the only link between efficiency and quality occurs at the aggregate level: richer countries specialize in producing the higher quality goods. Heterogeneous firm models of international trade with quality differentiation tend to consider efficiency differences only at the firm level. Therefore, the equilibrium is not characterized in terms of country or industry characteristics. In addition to country and firm specific efficiency components, the model in this paper also considers an industry-country component of firm efficiency (as in the Dornbusch-Fischer-Samuelson model). This component is what will bring about a link between a country's specialization in a particular industry and average output quality in that industry: a country's advantage in a given industry implies that its firms will tend to be more efficient relative to the world and therefore will tend to produce higher quality.

In addition to the equilibrium conditions (7)-(8) obtained from firm maximization, industry equilibrium requires that market shares add up to 1. Let ψ_{ij} be the sum of country-*i* firms' market shares in industry *j*: $\psi_{ij} \equiv \sum_{k=0}^{\bar{k}_i(j)} s_{ki}(j)$. Note substituting with (7) that ψ_{ij} is a function of the ratio $P(j)T_i a_i(j)/w_i$. Industry *j* equilibrium condition is:

$$\text{(11)} \qquad \psi_{Fj}\left(P(j)\right)+\psi_{Dj}\left(P(j)\frac{Ta(j)}{w}\right)=1.$$

For any wage and industry technology parameters –more specifically, for any ratio $Ta(j)/w$– there is always an equilibrium industry price level $P(j)^*$ such that (11) is satisfied. Intuitively, given technology and wages, all firm market shares would go to zero for an industry price level $P(j)$ sufficiently low (see (7). Note that even producing output of zero quality is costly). On the other hand, for an industry price level high enough we would have

$\psi_{Fj}\left(P(j)\right)+\psi_{Dj}\left(P(j)Ta(j)/w\right)>1$. Continuity of market shares on $P(j)$ ensures the existence of an industry equilibrium price level.

> **Proposition 2.1:** *For any $T>0$ and $w>0$, there exists a price level $P_j^*>0$ such that industry j is in equilibrium (i.e., expression (10) is satisfied). Moreover, P_j^* is a continuous and decreasing function of the ratio $Ta(j)/w$, $P_j^{*}= P^*(Ta(j)/w)$.*
>
> Proof: See Appendix.

In addition to equilibrium in every industry, the general equilibrium of the world economy requires that labor demand matches aggregate labor supply in each country. Labor supply is assumed to be equal to one in each country. Using expressions (5), (7), and (8) we get the following labor market equilibrium conditions, one for each country:

$$\text{(12)} \qquad \int_0^1 l_i(j)dj = \frac{1}{w_i} Y_W \int_0^1 \left(\sum_{k=0}^{\infty} s_{ki}(j)\left[1-s_{ki}(j)\right]\right) dj = 1\,; \quad i = H, F.$$

These two equilibrium conditions determine the relative wage w and the scale of world income Y_W. The relative wage w may be seen as determining how the sum of industry market shares is distributed between the two countries in a way that is consistent with their relative labor supplies (and productivities). In turn, world income Y_W adjusts to the scale that is consistent with the absolute size of world labor supply.[6]

> **Proposition 2.2:** *For any $T>0$, there exists a wage w^* and a world income Y_W^* satisfying all the equilibrium conditions. Moreover, w^* is continuous and strictly increasing in T.*
>
> Proof: See Appendix.

Given w^* and Y_W^*, we can solve for the rest of variables. Given w^*, the function $P_j^*=P^*(Ta(j)/w)$ determines prices. Then, expressions $\psi_{ij}(P(j)T_i a_i(j)/w_i)$ determine country market shares in each industry. In turn, production levels are obtained using market shares, Y_W^*, and prices.

[6] Some intuition on how the world economy equilibrium is reached is as follows. For a home wage w low enough, home country market shares would be equal to one in all industries. For w high enough the opposite is true: foreign-country firms would get all the market in all industries. Since industry equilibrium prices and country market shares are continuous in w, there is an intermediate w^* such that the distribution of market shares across countries is consistent with their relative labor supplies. Then, the scale of the world economy, Y_W, adjusts to the absolute size of the labor supply.

3. SPECIALIZATION ACROSS GOODS

This section characterizes country specialization across industries. The main goal is to extend the analysis in Dornbusch, Fischer and Samuelson (1977) to a setting with vertical differentiation and firm heterogeneity. In this way, horizontal and vertical specialization can be analyzed within the same model and its interactions explored. The first result highlighted in this section (Proposition 3.2: richer countries export a wider set of goods) is the same than can be obtained within the original DFS model. The second one (Proposition 3.3: market shares across goods are a continuous function of comparative costs) contrast with the unrealistic prediction in DFS where each country has either zero or 100% of world exports of any given good.

Implications of the model are more easily derived by letting the number of firms vary in a continuous way. In what follows I assume that there is a continuous number of potential firms in each industry and country. In every industry and country, the firm-specific efficiency parameters z_k are given by a continuous and differentiable function $g(k)$, g:$[0,\infty)\rightarrow(0,1]$, such that $g(0)=1$ and $dg/dk<0$. The analysis in the previous section carries over exactly the same by substituting sums across the set of firms with integrals. In equilibrium, only a finite measure $\bar{k}_i(j)$ of firms from each country is active in each industry.[7]

3.1. Aggregate efficiency, wages, and income

In this subsection I obtain some intermediate results that will be useful in the following. An increase in a country's aggregate efficiency lowers its firms' costs and raises their incentives to hire more labor and increase production. This increases wages and lowers the prices in industries where the country is a producer. However, the wage increase and the price reductions do not completely offset the positive impact of the aggregate efficiency increase on firms'

[7] The use of the continuum in the Cournot setting is not infrequent in the literature (in particular, in the comparative static analysis of the Cournot equilibrium with respect to the number of firms). It is of great mathematical convenience even if somewhat counterintuitive. Note that when each firm takes as given the distribution of competitors' output and qualities, it is irrelevant whether the set of competitors is measured in discrete or in continuous units. Then, given competitors' choices, the firm's optimal output and quality is computed as if it were a *full measure-one firm* (thus, having a non-negligible impact on the industry equilibrium). This implies the same first order conditions as in the model with a discrete number of firms (i.e., expressions (A.1) and (A.2) in the Appendix). Finally, each firm's output computed in this way is integrated with the rest of firms' output to constitute industry output as in any other model using the convention of a continuum of agents.

profits (except in industries where the country is the unique producer). The following proposition summarizes these and other related facts.

> **Proposition 3.1:** *An increase in the home-country aggregate efficiency T brings about: (i) a less than proportional increase in the home wage w; (ii) a less than proportional reduction in the price level* $P^*(j)$ *and an increase in the* $P^*(j)T/w$ *ratio in those industries where both countries are producers; (iii) an increase in the home-country relative income* υ*.*
>
> Proof: See Appendix.

Results (*i*) and (*iii*) imply that country aggregate efficiency, wage, and relative income are positively connected. Throughout the rest of the paper I use the expressions *country with higher aggregate efficiency*, *country with the higher wage*, and *richer country* as roughly equivalent.[8]

3.2. The Extensive Margin of Exports

How does horizontal specialization relate to income and comparative cost? The model's basic implications on horizontal specialization can be presented graphically. Consider Figure 3 which is drawn for a given set of technology parameters and the corresponding relative equilibrium wage w^*. Define country *i*'s *marginal firm* in industry *j* as the firm that would just be in the margin of being active should firms from the other country have zero share in market *j* (this could be the consequence of the other country having very low productivity in this industry). Subscript *M* will denote marginal firm variables. Figure 3 draws firm costs per unit of quality $w_i/T_i a_i(j) z_k$ across industries, for four types of firms: home-country most efficient firms (k=0), home-country marginal firms (k=M), foreign-country most efficient firms, and foreign-country marginal firms. Solid lines correspond to home-country firms whereas dotted lines correspond to foreign-country firms. For each country, the lower line corresponds to the most efficient firms' costs (recall that, due to normalizations, $w_F/T_F a_F(j) z_0=1$). Upper lines correspond to

---

[8] The equivalence may not be exact if the share of profits in national income is not the same in both countries. A sufficient condition for this equivalence to be precise is that the schedule $a(j)$ is symmetric; where symmetry is defined as $a(j)=1/a(1-j)$ for every j. To see this, consider an economy where this condition holds and assume T=1. It can then be shown that both countries would have the same wage and income (moreover, $P(1-j)=a(j)P(j)$). Now, consider the case $T\neq1$. Since according to Proposition 3.1 we have $dw^*/dT>0$ and $d\upsilon/dT>0$, we conclude that $w^*>1$ and $\upsilon>0.5$ if and only if $T>1$. However, even if the schedule $a(j)$ is not symmetric, the richer country will also have a higher wage as long as the difference between the countries' aggregate efficiencies is large enough.

marginal firms' costs. Since $a(j)$ is decreasing, domestic costs are increasing as we move towards higher j.

Expression (7) implies that two firms $k_H(j)$ and $k_F(j)$ in a given industry, one from each country, have the same market share if they have the same cost ratio. That is, if $w/Ta_j z(k_H(j)) = z(k_F(j))$. Now note that for industry $j=\bar{j}_H$, the home-country most efficient firm has the same cost ratio as the foreign marginal firm. Hence both firms have zero market share since, by definition, marginal firms have zero market share. Then, for $j>\bar{j}_H$ the most efficient home firm has higher costs than the marginal foreign firm. Hence home-country output for $j\geq\bar{j}_H$ is zero. Symmetrically, for $j\leq\bar{j}_F$ foreign output is zero. Now, any increase in T shifts downwards the domestic schedules since the ensuing increase in the equilibrium wage w^* is less than proportional (Proposition 3.1). It therefore moves both cutoffs $\bar{j}_H$ and $\bar{j}_F$ to the right. Therefore,

> **Proposition 3.2:** *An increase in a country's aggregate efficiency expands the range of industries where the country is a producer.*

As long as higher income is linked to aggregate efficiency, this proposition implies that richer countries export a wider set of goods. As pointed out in the Introduction, Hummels and Klenow (2005) have shown that this richer countries' extensive margin of exports has a large quantitative importance.

The formal argument for this result is as follows. Denote by $\bar{P}$ the value of $P(j)$ that solves $\psi_{Fj}(P(j))=1$.[9] This is the equilibrium price at the cutoff industry $\bar{j}_H$. Moreover, the most efficient domestic firm in industry $\bar{j}_H$ must be exactly on the edge of being active. Hence, $\bar{j}_H(T)$ is the industry satisfying $s_{0H}(\bar{j}_H) = 1 - ew^*(T)/\left[\bar{P}Ta(\bar{j}_H)z_0\right] = 0$. That is,

$$(13) \quad a(\bar{j}_H) = e\frac{w^*(T)}{\bar{P}T}.$$

Differentiating with respect to T and using Proposition 3.1, yields the result in Proposition 3.2:

$$\frac{d\bar{j}_H}{dT} = \frac{1}{\partial a(\bar{j}_H)/\partial \bar{j}_H}\frac{a(\bar{j}_H)}{T}\left[\frac{dw^*}{dT}\frac{T}{w^*} - 1\right] > 0.$$

3.3. Comparative Costs and Country Market Shares

---

[9] $\bar{P}$ is the same for all j; see the proof of Proposition 2.1 for details. It can also be shown that marginal firms' efficiency is given by $z_M = e/\bar{P}$.

The cutoff between industries where the home country has a cost advantage and those where the foreign country does corresponds to the crossing of the two upper lines in Figure 3. This cutoff is denoted by $\bar{j}^{CA}$ and satisfies

(14) $$Ta(\bar{j}_H^{CA})/w^*(T)=1.$$

This is the single cutoff determining country specialization in the DFS model without transportation costs. In this model, country H is the only producer and exporter for $j<\bar{j}^{CA}$, and F is the only producer and exporter for $j>\bar{j}^{CA}$. In the model in this paper, continuity of firms' market shares on wages and efficiency parameters leads to the following

> **Proposition 3.3:** *A county's market share in a given industry j is a continuous and decreasing function of its comparative cost $w_i/T_ia_i(j)$ in that industry.*

Figure 4 illustrates the pattern of market shares implied by this proposition. It is straightforward to check this proposition taking into account that country *i*'s market share is the sum of its firms' markets shares. Expression (7) implies that each domestic firm's market share is decreasing in $w/Ta(j)$. Moreover, expression (10) for $\bar{k}_H(j)$ implies that the set of domestic active firms is decreasing in $w/Ta(j)$. Thus, lower comparative cost implies larger home-country market share both because the number of its active firms is larger and because each firm has larger market share. Note that lower ratio $w/Ta(j)$ also implies a lower industry price level $P(j)^*$ (Proposition 2.1), which reduces foreign output and markets shares.

4. Countries' Output Quality Within Each Industry

This section characterizes countries' specialization within each industry along the quality dimension. Subsection 4.1 investigates which countries produce each quality within each industry, and how this relates to wages and absolute advantage. In turn, Subsection 4.2 focuses on average quality.

4.1. Who Produces Which Qualities Within each Industry?

For obvious reasons, the analysis in this section focuses on industries in the interval $(\bar{j}_F, \bar{j}_H)$ where both countries have positive production. Consider an industry in this interval. Active firms from country *i* span the interval of efficiencies $(T_ia_i(j)z_{\bar{k}i(j)}, T_ia_i(j)]$ which determines the interval of qualities produced by the country. From expression (8), it is clear that the country with the highest value of $T_i a_i(j)$ (i.e., the country with the absolute advantage in this industry) produces the highest quality. This expression also implies that the least efficient active firm in each country produces the lowest quality in that country. Now, who does produce the lowest quality in the world market? Consider the least efficient firms from H and F: $\bar{k}_H(j)$, $\bar{k}_F(j)$. From (8) and (10), we have:

$$1 = \frac{e\, w_H}{P(j)\left[q_{\bar{k}H}(j)\right]^{1/\sigma}} = \frac{e\, w_F}{P(j)\left[q_{\bar{k}F}(j)\right]^{1/\sigma}}.$$

This yields

$$\text{(15)} \qquad \frac{q_{\bar{k}H}(j)}{q_{\bar{k}F}(j)} = \left(\frac{w_H}{w_F}\right)^{\sigma}.$$

Therefore, the lowest quality is produced in the country with the lowest wage. The reason is that lower wages allow lower-efficiency firms to be competitive. In turn, lower efficiency implies lower quality. Now, consider the range of quality varieties produced by country *i*. The span of this range is measured by the ratio between the highest and the lowest quality so that it is independent of units:

$$\text{(16)} \qquad \frac{q_{0i}(j)}{q_{\bar{k}i}(j)} = \left(\frac{T_i a_i(j)}{T_i a_i(j) z_{\bar{k}i(j)}}\right)^{\sigma} = \frac{1}{\left(z_{\bar{k}i(j)}\right)^{\sigma}}.$$

Since for any $P(j)$, $z_{\bar{k}i(j)}$ is lower for the country with lower cost $w_i/T_i a_i(j)$ (see expression (10)), we conclude that the country with the cost advantage in industry j produces a wider spectrum of qualities. The reason is that the range of active firms' efficiencies is wider in the county with the cost advantage.[10] Summarizing,

> **Proposition 4.1:** *Consider an industry where both countries have positive market shares.*
>
> *(i) The highest quality is produced in the country with the absolute advantage.*

[10] See Bernard, Redding and Schott (2007) for a similar result in a heterogeneous firm model with no quality differentiation. Also note that this last result implies an *extensive margin of exports within the quality dimension*: an increase in a country's income brings about an increase in the range of exported qualities within each industry. The reason is that an increase in a country's aggregate efficiency raises its comparative cost advantage in all industries where it is an exporter (since, according to Proposition 3.1, the ensuing wage increase is less than proportional).

(ii) The lowest quality is produced in the country with the lowest wage.

(iii) The country with the cost advantage produces a wider spectrum of qualities.

Figures 5a and 5b illustrate this proposition assuming that the home country has higher wage level. Figure 5a considers an industry where the home country has an absolute advantage. Hence it is the unique producer of the highest qualities $q\in(q_{0F}(j), q_{0H}(j)]$. On the other hand, the foreign country is the unique producer of the lowest qualities $q\in(q_{\bar{k}F}(j), q_{\bar{k}H}(j))$ since it has lower wage. Figure 5b considers an industry where the foreign country has an absolute advantage. In this case, the foreign country is the unique producer of the highest qualities $q\in(q_{0H}(j), q_{0F}(j)]$ as well as the lowest qualities. The richer (home) country only produces some intermediate qualities.

Proposition 4.1 highlights some of the connections between vertical and horizontal specializations. Comparative costs $w/Ta(j)$ determine market shares across goods as well as the relative width of the range of qualities produced by each country. Furthermore, each of the two components of this ratio plays a specific role: a high denominator (high absolute advantage) involves producing the high qualities; whereas a low numerator (low wage) involves producing the low qualities because it allows less-efficient firms to be active. Of course, both circumstances are simultaneously possible as in Figure 5b.

Figure 6 provides an overall picture of the countries' horizontal and vertical specializations. In comparison to Figure 3, it includes a new dotted line that depicts the inverse of country H's efficiency across industries; i.e., $1/Ta(j)$. As in the rest of figures, this new line is drawn assuming that country H has a higher wage than F (i.e., $w>1$), so that it is below the $w/Ta(j)$ line. Industry $\bar{j}^{AA}$ is the cutoff between industries where the home country has an absolute advantage (to the left of $\bar{j}^{AA}$) and industries where the foreign country has it (to the right). According to Proposition 4.1, country H produces the highest-quality varieties to the left of $\bar{j}^{AA}$ and country F does so to the right. Moreover, to the left of $\bar{j}^{CA}$ country H also has a cost advantage and produces the widest spectrum of quality varieties (symmetrically for F to the right of this cutoff). As explained for Figure 3, both countries have positive production for industries between $\bar{j}_F$ and $\bar{j}_H$. Between $\bar{j}^{AA}$ and $\bar{j}_H$, the poorer country is the only producer of the highest quality varieties in spite of the richer country also being a producer of these goods. To the right of $\bar{j}_H$ the poorer country is also the unique producer of the highest qualities, but in a trivial sense since the richer country does not produce these goods at all. The lowest qualities of all the goods are produced in country F except to the left of $\bar{j}_F$, which corresponds to the interval of goods not produced in this poorer country.

Note that $\bar{j}^{AA}$ in Figure 5 could be to the right of $\bar{j}_H$. This would imply that the richer country does not produce some of the goods for which it has an absolute advantage.[11] Another particular case occurs when $Ta(1)>1$ (the richer H country has an absolute advantage in all goods) and $Ta(1)/w>z_M$ (it produces some varieties of every good). Country specialization in this case roughly corresponds to the one described in trade models with only one quality differentiated good: the richer country is the only producer and exporter of the higher-quality goods.

Proposition 4.1 points out that the production of the highest qualities is not directly linked to country income but to absolute advantage in each specific industry. Still, it is likely that the richer countries produce the higher qualities of most goods because those countries are richer just because they have an absolute advantage (i.e., they are more efficient) in more or more important industries. Some sources of absolute advantage may have an imperfect, low, or even null correlation with per capita income (e.g., specific natural resources or histories of specialization that created some local knowledge and other positive externalities in particular industries). However, the general sources of absolute advantage (e.g., high average education, easy and cheap access to financial resources, good institutions, or public infrastructures) tend to be positively correlated with income. Richer countries tend to produce the highest qualities for a larger set of goods than poorer countries. To be precise, in the model the richer country produces the highest qualities for a set of industries that is larger than the set of industries for which it has a cost advantage. This can be checked in Figure 6. If H is richer, then $1/Ta(\bar{j}^{CA}) < w/Ta(\bar{j}^{CA}) = 1 = 1/Ta(\bar{j}^{AA})$. Therefore, since $a(j)$ is decreasing, $\bar{j}^{CA}$ is to the left of $\bar{j}^{AA}$.

4.2. Average Quality

4.2.1. Average quality and income level

Average quality of world's and country i's output in industry j are, respectively:

$$Q_W(j) \equiv \sum_{i=H,F} \int_0^{\bar{k}_i} r_{ki}(j) q_{ki}(j) dk; \quad Q_i(j) \equiv \frac{\int_0^{\bar{k}_i} r_{ki}(j) q_{ki}(j) dk}{\int_0^{\bar{k}_i} r_{ki}(j) dk};$$

[11] The country with the absolute advantage in a given industry may gradually abandon it as wages increase. For example, a state like Florida might have the highest absolute advantage for producing oranges, but it would loose market share as its wage level rises (Proposition 3.3). The market share reduction will concentrate on the cheapest (lower-quality) varieties which are produced by the least efficient firms. Eventually, for sufficiently high wages, even the most efficient firms would be unable to survive in spite of the absolute advantage.

where $r_{ki}(j)$ is firm $k_i(j)$'s share in the world market of j, in physical units of output. Since

$$r_{ki}(j) \equiv \frac{x_{ki}(j)}{\sum_{i=H,F}\int_0^{\bar{k}_i} x_{ki}(j)dk} = s_{ki}(j)\frac{Q_W(j)}{q_{ki}},$$

we have:

$$\text{(17)} \qquad Q_i(j) = \frac{\int_0^{\bar{k}_i} s_{ki}(j)dk}{\int_0^{\bar{k}_i} s_{ki}(j)/q_{ki}(j)dk} = w_i^{\sigma}\left(\frac{T_i a_i(j)}{w_i}\right)^{\sigma}\frac{\int_0^{\bar{k}_i} s_{ki}(j)dk}{\int_0^{\bar{k}_i} s_{ki}(j)/\left(z_{ki}(j)\right)^{\sigma} dk}.$$

Note that differences across countries in market shares $s_{ki}(j)$ and in the measure of active firms $\bar{k}_i(j)$ can only arise as a consequence of differences in comparative costs $w_i/T_i a_i(j)$ (see expressions (7) and (10)). It is then immediate the following

Proposition 4.2: *Consider an industry where both countries have positive production. If more-efficient firms have a relative advantage in producing higher quality goods* (*i.e., if* σ>0) *then, conditional on comparative costs, higher wage implies higher average quality.*

The intuitive argument for this result is as follows. Two countries with the same cost level $w_i/T_i a_i(j)$ in a given industry will have the same measure of active firms; and for every k, firm k in one country will have the same market share than the corresponding firm k in the other country. However, if the wage in one of the countries is higher, then it must be the case that this country has higher absolute efficiency in the industry. Thus, for every pair of firms with the same cost and market share, one from each country, the firm from the country with higher wage is more efficient and produces higher quality.

This proposition suggests that regressions between export average quality and country per capita income should be run conditional on some measure of comparative costs. This motivates the empirical approach in Section 5. Still, unconditional regressions between average quality of exports and country per capita income have delivered significant positive coefficients for a large set of industrial goods, as well as at the aggregate level (see Schott 2004, and Hummels and Klenow 2005). Proposition 4.2 is consistent with these empirical findings as long as, for a large set of industries, cross-country differences in costs are moderate or are not positively correlated with country per capita income (since, as Proposition 4.3 states below, larger comparative costs bring about lower quality).

4.2.2. Average quality and comparative advantage

Consider now the average quality consequences of differences in comparative costs. Given wages, lower comparative costs in an industry imply higher absolute advantage. In an economy with homogeneous firms and the same wages, average output quality would unquestionably be higher in the country with higher absolute advantage in the industry. However, the relationship may be uncertain if firms are heterogeneous. The reason is that higher absolute advantage involves a larger set of active firms in the country and a reallocation of market shares across firms producing different qualities. Market shares of less-efficient firms (which produce lower quality) could increase relative to the market shares of more-efficient firms when the country's efficiency increases. In fact, it is straightforward from (7) that this will be the case. For some distribution of firm efficiencies, this could give rise to a negative composition effect such that higher efficiency involves lower average output quality. Nevertheless, reasonable assumptions on the distribution of efficiency across firms may rule out this possibility.

Let us consider the following distribution pattern of firms' efficiency, which satisfies the general characteristics assumed on $g(k)$ at the beginning of Section 4:

(18) $$z_k = e^{-\theta k}\ , \quad \theta > 0\ , \quad k \in [0, \infty).$$

Larger θ involves wider heterogeneity across firms. This distribution is flexible enough to approximate a wide array of possible industry configurations. Using (18) to substitute in (17) we have:

(19) $$Q_i(j) = \frac{\frac{1}{\theta}\left[\ln b + \frac{1}{b} - 1\right]}{\left(\frac{1}{T_i a_i(j)}\right)^{\sigma} \frac{1}{\theta}\frac{1}{\sigma}\left[\frac{1}{1+\sigma} b^{\sigma} + \frac{\sigma}{1+\sigma}\frac{1}{b} - 1\right]}; \quad \text{where } b \equiv \frac{P(j) T_i a_i(j)}{w_i e}.$$

The derivative of the log of $Q_i(j)$ with respect to country's efficiency in industry j, $T_i a_i(j)$, yields:

(20) $$\frac{\partial Q_i(j)}{\partial (T_i a_i(j))}\frac{T_i a_i(j)}{Q_i(j)} = \frac{1}{\theta}\frac{s_{0i}(j)}{\psi_i(j)}\left[1 - \frac{Q_i(j)}{q_{0i}(j)}\right] > 0.$$

Thus, higher efficiency in industry j implies higher average quality. Note that differences in $Q_i(j)$ across countries only depend on differences in the wage level and efficiency in the industry. Moreover, conditional on wages, higher efficiency in the industry implies lower comparative cost. Therefore, we have the following

Proposition 4.3: *Consider an industry where both countries have positive production. If more-efficient firms have a relative advantage in producing higher quality goods* (*i.e.*, *if* σ>0) *then, conditional on wages, lower comparative cost implies higher average output quality.*

5. IMPORT UNIT VALUES OF APPAREL AND CLOTHING ACCESSORIES

In this section we test the predictions in Propositions 4.2 and 4.3 using data on US import of apparel and clothing accessories. As already pointed out, the apparel and clothing accessories is a particularly interesting industry: it contains a wide variety of products, those products show significant quality differences as proxied by unit values, and it is the manufacturing industry with the largest set of exporters, which includes low- as well as high-income countries.

5.1. Empirical Strategy

As the main observable measure of comparative advantage I use the index of *revealed comparative advantage*, *RCA* (Balassa 1965). This index is a measure of relative export performance (or specialization) by industry and country. The index for country *i* and good *j* can be defined as $RCA_i(j) \equiv (V_i^W(j)/V_i^W)/(V_W(j)/V_W)$; where $V_i^W(j)$ is country *i*'s exports of good *j* to the world, V_i^W is country *i*'s total exports to the world, $V_W(j)$ is total international trade of good *j*, and V_W is total world trade (all variables in value terms).

The connection between the *RCA* index used in regressions and the comparative cost concept that appears in Propositions 4.2 and 4.3 can be made explicit as follows. Using the notation developed for the model, the *RCA* index can be computed as (recall that country *i* exports a fraction $(1-\upsilon_i)$ of each good):

$$(21) \qquad RCA_i(j) = \frac{(1-\upsilon_i)\psi_{ij}Y_W}{(1-\upsilon_i)Y_i} / \frac{\sum_{h=H,F}(1-\upsilon_h)\psi_{hj}Y_W}{\sum_{h=H,F}(1-\upsilon_h)Y_h} = \frac{\psi_{ij}(T_i a_i(j)/w_i)}{\upsilon_i}\cdot\zeta(j);$$

where $\zeta(j) \equiv \sum_{h=H,F}(1-\upsilon_h)\upsilon_h / \sum_{h=H,F}(1-\upsilon_h)\psi_{hj}$. Note that $\zeta(j)$ only depends on the industry and therefore enters the expression for $RCA_i(j)$ in the same way for all countries. Hence, conditional on the country's income share $\upsilon_i = Y_i/Y_W$, there is an increasing one-to-one mapping between the country's *RCA* in a given industry and its comparative cost ratio $T_i a_i(j)/w_i$. Therefore, we can use *RCA* to test Propositions 4.2 and 4.3 by including exporter's *GDP* in the estimating equation, in addition to *RCA* and *PCGDP* (this last variable being used as a proxy for the wage level). The baseline equation to be estimated using exports to the US is:

(22) $$\text{Log } unit\ value_{it}^{US}(j) = \delta_t + a_1 \text{ Log } PCGDP_{it} + a_2 \text{ Log } RCA_{it}(j) + u_{it}(j)\ ;$$

where $unit\ value_{it}^{US}(j)$ is the ratio of the value of country i's exports of good j to the US over the quantity exported to this country in period t (in number of items or in kilograms, depending on the commodity), $PCGDP_{it}$ is country i's PPP per capita GDP in period t, δ_t is a time fixed effect to control for level differences in unit values across time, and $u_{it}(j)$ is the error term. As additional controls, I consider country i's PPP GDP in period t and the distance between country i and the US.

To perform some robustness checks, I also consider two alternative measures of country comparative advantage and specialization: comparative advantage excluding exports to the US (RCA^{UE}) and quantity revealed comparative advantage (*QRCA*). The RCA^{UE} measure is defined as:

$$RCA_i^{UE}(j) \equiv ([V_i^W(j) - V_i^W(j)]/[V_i^W - V_i^W])/(V_W(j)/V_W).$$

Hence this measure excludes exports to the US. In turn, the *QRCA* has the same definition as $RCA_i(j)$ except that $V_i^W(j)$ is replaced by the physical quantity of country i's exports of good j to the world, $X_i^W(j)$. That is,

$$QRCA_i(j) \equiv (X_i^W(j)/V_i^W)/(V_W(j)/V_W).$$

Using these to alternative measures should eliminate any concern about a potential spurious correlation between unit values of exports to the US in the left-hand-side of the estimated equation and revealed comparative advantage in the right-hand-side of the equation. Below, I discuss this potential problem in detail after explaining the data.

5.2 The Data

Equation (21) is estimated using data on unit values of US imports from all countries, as reported by the US. Using a single destination country has the advantage of making unnecessary to control for destination characteristics. Moreover, using unit values of imports as reported by the US instead of unit values of exports to the US as reported by the exporting countries has the advantage of using a single reporter and, therefore, of higher statistical homogeneity and reliability.[12] However, exporter revealed comparative advantage has to be computed considering all the exporter's sales to the whole world market.[13] This requires using export data as reported by each exporter country. The fact that the data for used to compute unit values have a different reporter that the data used to compute the *RCA* will in fact turn into a statistical advantage: it will make very unlikely finding spurious correlations between the left and right hand side of the estimated equation due to common measurement errors (see the discussion below).

PCGDP and GDP data are from WDI World Bank. Trade data are from the United Nations Commodity Trade Statistics Database (UN Comtrade), SITC Rev.3. I consider the data for articles of apparel and clothing accessories (industry 84 of the SITC). Estimation pools data from 2005 and 2006 (the UN Comtrade database does not provide data on physical quantities for most commodities before 2005). These data contain a large number of measurement errors and outliers. To minimize this problem we implement several filters. First, for each commodity at the 5-digit level, we drop all observations whose import unit value is larger than 10 times or smaller than 1/10 of the median unit value of the corresponding year. Second, some countries are reported to export an extremely small amount of items to the US. Observations for these countries are likely to reflect marginal and atypical commercial activities and re-exports. To avoid this problem, for each commodity we drop all observations from countries whose exports in physical units are less than 1/10,000 of the mean export per country in the sample. Finally, we exclude the commodities having less than 40 observations. This leaves us with 72 commodities at the 5-digit level within the apparel and clothing accessories group.

As already noted, using the RCA^{EU} and *QRCA* alternative measures of revealed comparative advantage may be useful in eliminating any potential concern about spurious correlations. Nonetheless, there is no clear reason to expect that using the *RCA* measure would lead to this sort of problem. The reason is that the data used to compute unit values on left-hand-side of the equation and the data used to compute *RCA* on the right-hand-side have different sources. To see this in more detail, consider rewriting equation (21) as:

---

[12] Since unit costs are then CIF values, I include distance between the US and the exporter as well as exporter size as controls to acount for differences in transportation costs.

[13] Note that a country may have a comparative advantage in some commodity even if it does not export any unit to the US (e.g., Cuba and cigars).

$\log V_{it}^{US}(j) - \log X_{it}^{US}(j)$

$$= \delta_t + a_1 \log PCGDP_{it} + a_2 ([\log V_{it}^{W}(j) - \log V_{it}^{W}] - [\log V_{Wt}(j) - \log V_{Wt}]) + u_{ij}(j);$$

where $X_{ij}^{US}(j)$ is the physical quantity of country i's exports of good j to the US. The potential concern could be that measurement errors in $V_{it}^{W}(j)$ are carried on into $V_{it}^{US}(j)$. If this were the case, having $V_{it}^{US}(j)$ and $V_{it}^{W}(j)$ on each side of the equation would generate a positive bias in the estimate of a_2. However, $V_{it}^{US}(j)$ and $V_{it}^{W}(j)$ correspond to different variables and are based on statistics reported by different countries: $V_{it}^{US}(j)$ are US imports of good j from country i as reported by the US, whereas $V_{it}^{W}(j)$ are country i´s exports of good j to the whole world, as reported by country i. Therefore, There is no reason to expect that measurement errors in $V_{it}^{W}(j)$ are carried on into $V_{it}^{US}(j)$ since they are different data reported by different countries. Contrarily, using the *QRCA* measure may indeed create a spurious negative bias in the estimation of the coefficient on revealed comparative advantage.[14] Still, we estimate equation (21) using the RCA^{UE} and *QRCA* alternative measures of international specialization to check the robustness of the results.

5.3 Results

Equation (21) is estimated by pooling in a single regression all the commodities in the apparel and cloth accessories industry, as well as by running independent regressions for each of the 72 commodities in the industry. In the estimation that pools all the 72 commodities, we include a dummy for each of them.

Table 1 reports the results of estimating equation (21) by pooling the data of all the 72 commodities. Column 1 shows the results for the estimation using the standard *RCA* measure without controlling for exporter GDP and distance to the US. The remaining columns always control for these factors and use the three alternative measures of revealed comparative advantage. In all four regressions, both *PCGDP* and revealed comparative advantage are positive and significant at the 1-percent level. Including the controls for exporter size and distance does not have almost any effect on the results. As expected due to its negative bias,

[14] Note in this respect that we can rewrite $QRCA_i(j)$ as $QRCA_i(j)=RCA_i(j)/unit\ value_i^{W}(j)$. Then, since $RCA_i(j)$ is the appropriate measure from the theoretical model and $unit\ value_i^{W}(j)$ is likely to be positively correlated with $unit\ value_i^{US}(j)$, we may expect that using the *QRCA* measure leads to a spurious negative bias in the estimate of a_2. Therefore, the higher share of negative signs found when using the QRCA measure should be interpreted with care.

the estimated coefficient using the *QRCA* measure is much smaller, though it is still highly significant.

Table 2 shows the result of independently estimating equation (21) for each of the 72 commodities at the 5-digit level in the apparel and clothing accessories industry. Each of the four rows in the table shows the results corresponding to a different specification. In rows 2-4, the additional controls included in the estimated equation are average distance between the exporter and the US, and exporter GDP. The same four equations were independently estimated for each of the 72 commodities. The results in each row show the percentage of the 72 regressions (one for each commodity) that yield a positive estimated coefficient for the measure of revealed comparative advantage, as well as the percentage of the 72 commodities that yield a statistically significant coefficient with either sign. According to the second row of results, 81.9-percent of the commodities yield a positive coefficient on *RCA*. More importantly, 43.1-percent of the commodities (that is, 31 out of 72 commodities) yield a positive coefficient on *RCA* that was significant at the 5-percent level (and only 1.4-percent delivered a negative and significant coefficient). In turn, all the coefficients on *PCGDP* are positive and 97.2-percent of them are significant at the 5-percent level. Results using the *RCA* and the *QRCA* measures are similar. Still, results using the RCA^{UE} measure are more favorable than those using the conventional *RCA* measure. When using the *QRCA* measure, 75-percent of commodities yield a positive coefficient and 26.4-percent yield a positive coefficient that is significant at the 5-percent level. However, as pointed out, using the *QRCA* measure introduces a negative bias in the estimation.

Figure 7 provides a synthetic overview of the estimated coefficients on *RCA* corresponding to the regressions whose results are reported in the second row of Table 2. In Figure 7a, the coefficients are ordered according to the SITC-rev3 5-digit number of the corresponding commodity, whereas in Figure 7b the same coefficients are ordered according to their magnitude.

Thus, the empirical exercise in this section provides support for the model's main result. For some commodities, the factors providing an advantage in exporting a particular good also provide an advantage at producing the higher-quality varieties of that good. If more-efficient firms produce higher quality, countries with an absolute advantage in a particular commodity will export higher quality of that commodity. This does not hold for all commodities but seems to be true at least for an important group of manufactures that are exported by the largest number of countries.

6. CONCLUDING COMMENTS

Recent empirical research has documented the importance of country specialization across both the horizontal and the vertical dimensions of goods to characterize the current patterns of trade. Existing models tend to concentrate on only one of these two dimensions of specialization and neglect their possible connections. This paper provides a model where both the horizontal and the vertical dimensions of trade are present and shows that their connections are very important. It is found that richer countries export a wider set of goods (the extensive margin). Within each good or industry, (i) the highest quality is produced by the country with the absolute advantage in that good; (ii) the lowest quality is produced by the country with the lowest wage; and (iii) each country's average quality and unit value of exports of the good increase with the country's revealed comparative advantage, conditional on its wage level.

The paper also provides some preliminary empirical evidence that supports the main prediction of the model. It analyses the correlations between US unit values of merchandise imports and exporter revealed comparative advantage as well as per capita GDP for 72 commodities of apparel and clothing accessories at the 5-digit level of the SITC. Robustness is checked using several measures of RCA and controlling for exporter size and distance. Consistently with the theoretical model, it is shown that exporter per capita income and revealed comparative advantage have a positive and jointly significant correlation with exported unit values.

Some of the most important simplifications of the model relate to demand. Generalizing this element and carrying out a systematic empirical test of the theoretical results across a large number of industries are relevant directions for further research.

REFERENCES

Alcalá, F., and P. J. Hernández (2010): "Firms' Main Market, Human Capital, and Wages," *SERIEs, Journal of the Spanish Economic Association*, 1(4), 433-458.

Balassa, B. (1965): "Trade Liberalization and 'Revealed' Comparative Advantage," *Manchester School*, 33, 99-123.

Baldwin, R., and J. Harrigan (2007): "Zeros, Quality and Space: Trade Theory and Trade Evidence," NBER working paper 13214, Cambridge, MA.

Bernard, A.B., and Jensen, J.B. (1995): "Exporters, jobs, and wages in U.S. manufacturing: 1976-1987," *Brookings Papers on Economic Activity, Microeconomics*, 67-119.

Bernard, A.B., J. Eaton, J.B. Jensen, and S.S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review*, 93, 1268-1290.

Bernard, A.B, J.B. Jensen, S. J. Redding, and P. K. Schott (2007): "Firms in International Trade," *Journal of Economic Perspectives*, 21, 105-130.

Bernard, A.B., S. J. Redding and P. K. Schott (2007): "Comparative Advantage and Heterogeneous Firms," *Review of Economics Studies*, 74, 31-66.

Choi, Y. , D. Hummels, and C. Xiang (2009): "Explaining import quality: The role of the income distribution," *Journal of International Economics*, 77(2), pages 265-275.

Dornbusch, R., S. Fischer, and P.A. Samuelson (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 67, 823-839.

Eaton, J., S. Kortum and F. Kramarz (2005): "An Anatomy of International Trade: Evidence from French Firms," mimeo, University of Minnesota.

Falvey, R. E., and H. Kierzkowski (1987): "Product Quality, Intra-industry Trade and (Im)perfect Competition," in H. Kierzkowski (ed.) *Protection and Competition in International Trade*, Basil Blackwell, Oxford.

Fieler, A. C. (2008): "Quality Differentiation in International Trade: Evidence from the Demand and Supply Sides," mimeo, NYU.

Flam, H. and E. Helpman (1987): "Vertical Product Differentiation and North–South Trade," *American Economic Review*, 77, 810–822.

Grossman, G.M., and E. Helpman (1991): *Innovation and growth in the global economy*, MIT Press, Cambridge, Mass.

Hallak, J.C. (2006): "Product Quality and the Direction of Trade," *Journal of International Economics*, 68, 238-265.

Hummels, D., and P. Klenow (2005): "The Variety and Quality of a Nation's Exports," *American Economic Review*, 95, 704–723.

Johnson, R.C. (2007): "Trade and Prices with Heterogeneous Firms," mimeo, UC-Berkeley.

Kehoe, T. J., and K. J. Ruhl (2002), "How Important is the New Goods Margin in International Trade?" mimeo, University of Minnesota and Federal Reserve Bank of Minneapolis.

Khandelwal, A. (2009): "The Long and Short (of) Quality Ladders," *Review of Economic Studies*, forthcoming.

Kugler, M. and E. Verhoogen (2008): "The Quality-Complementarity Hypothesis: Theory and New Evidence from Colombia," mimeo, Columbia University.

Melitz, M. (2003): "The Impact of Trade on Intra Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695-1726.

Murphy, K. and A. Shleifer (1997): "Quality and Trade," *Journal of Development Economics*, 53, 1–15.

Schott, P. (2004): "Across-Product versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics*, 119, 647–678.

Stokey, N. (1991): "The Volume and Composition of Trade between Rich and Poor Countries," *Review of Economic Studies*, 58, 63–80.

Tybout, J. (2003): "Plant and Firm Level Evidence on the 'New' Trade Theories," in *Handbook of International Trade*, ed. by E.K. Choi and J. Harrigan. Basil Blackwell, Oxford.

Verhoogen, E. A. (2008): "Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector," *Quarterly Journal of Economics*, 123, 489-530.

APPENDIX

Firms' Profit Maximization First Order Conditions

Each firm maximizes profits $\pi_{ki}(j)=x_{ki}(j)[q_{ki}(j)P(j)-c_{ki}(q,j)]$ with respect to $x_{ki}(j)$ and $q_{ki}(j)$, taking as given industry inverse-demand function (5) and the other firms' output and quality choices. For firm $k_i(j)$: $\frac{\partial P(j)}{\partial x_{ki}(j)} = -\frac{P(j)q_{ki}}{\sum_{g=H,F}\sum_h q_{hg}(j)x_{hg}(j)}$ and $\frac{\partial P(j)}{\partial q_{ki}(j)} = -\frac{P(j)x_{ki}}{\sum_{g=H,F}\sum_h q_{hg}(j)x_{hg}(j)}$.

Hence profit maximization yields the following FOC:

$$\text{(A.1)}\quad \frac{\partial \pi_{ki}(j)}{\partial x_{ki}(j)} = q_{ki}(j)P(j) + x_{ki}(j)q_{ki}(j)\frac{\partial P(j)}{\partial x_{ki}(j)} - c_{ki}(q,j) = 0 = P(j) - s_{ki}(j)P(j) - \frac{c_{ki}(q,j)}{q_{ki}(j)};$$

$$\text{(A.2)}\quad \frac{\partial \pi_{ki}(j)}{\partial q_{ki}(j)} = P(j) + q_{ki}(j)\frac{\partial P(j)}{\partial q_{ki}(j)} - \frac{\partial c_{ki}(q,j)}{\partial q_{ki}(j)} = 0 = P(j) - s_{ki}(j)P(j) - \frac{\partial c_{ki}(q,j)}{\partial q_{ki}(j)}.$$

Then, (A.1) and (A.2) yield (7) and (8) in the main text.

Proof of Proposition 2.1

First, let us characterize the functions $\psi_{Fj}(P(j))$ and $\psi_{Hj}(P(j)Ta(j)/w)$. Note that (10) implies that the number of active firms $\bar{k}_i(j)$ is increasing in $P(j)$. Moreover, taking into account expression (7) we have that ψ_{Fj} is continuous in $P(j)$ and satisfies:

(*i*) if $P(j) \le e/z_0 = e$, then $\psi_{Fj}(P(j)) = 0$;

(*ii*) if $P(j) > e$, then $\psi_{Fj}(P(j)) > 0$ and strictly increasing;

(*iii*) since $\lim_{P(j)\to\infty} \psi_{Fj}(P(j)) > 1$, there exists $\bar{P}$, $\bar{P} > e$, such that $\psi_{Fj}(\bar{P}) = 1$.

Note that $\bar{P}$ does not depend on any parameter specific to industry j due to the normalization of foreign industry parameters. Similarly, it is easy to check that ψ_{Hj} is also a continuous and increasing function of the ratio $P(j)Ta(j)/w$. Moreover,

$$\psi_{Hj}(P(j)Ta(j)/w) = \begin{cases} 0 & \text{if and only if} \quad P(j)Ta(j)/w \le e; \\ 1 & \text{if and only if} \quad P(j)Ta(j)/w = \bar{P}. \end{cases}$$

Now, the following claim completes de proof of Proposition 2.1 by showing how the equilibrium price P_j^* is determined as a function of any possible value of the ratio $Ta(j)/w$.

Claim 2.1.A: *For any T, w, and a(j), there is an industry equilibrium price* P_j^*. *Moreover,* P_j^* *is a continuous and decreasing function of the ratio Ta(j)/w, satisfying:*

$$\begin{cases} P_j^* = \overline{P} & \text{if } Ta(j)/w \in (0\,, e/\overline{P}], \quad \text{impying } \psi_{Hj} = 0\,, \psi_{Fj} = 1; \\ \overline{P} > P_j^* > e & \text{if } Ta(j)/w \in (e/\overline{P}\,, \overline{P}/e), \quad \text{impying } \psi_{Hj} > 0\,, \psi_{Fj} > 0; \\ P_j^* = \overline{P}w/Ta(j) \le e & \text{if } Ta(j)/w \in [\overline{P}/e\,, \infty); \quad \text{impying } \psi_{Hj} = 1, \psi_{Fj} = 0. \end{cases}$$

Moreover, if $Ta(j)/w \in (e/\overline{P}\,, \overline{P}/e)$ *then* $0 > \frac{\Delta P_j^*}{P_j^*} / \frac{\Delta(Ta(j)/w)}{Ta(j)/w} > -1$.

The Claim is proven as follows:

(*i*) For $0 < Ta(j)/w \le e/\overline{P}$ we must have $P_j^* = \overline{P}$ since for any ratio $Ta(j)/w$ in that interval, $P(j) < \overline{P}$ would imply $\psi_{Fj}<1$ and $\psi_{Hj}=0$ (and $P(j) > \overline{P}$ implies $\psi_{Fj}>1$). Hence $\psi_{Hj}=0$ and $\psi_{Fj}=1$.

(*ii*) Now consider $Ta(j)/w \ge \overline{P}/e$. We then have $P_j^* = \overline{P}w/[Ta(j)] \le e$, since: (1) for a price lower than this expression we would have $\psi_{Hj}<1$ and $\psi_{Fj}=0$; (2) for a price higher than this expression we would have $\psi_{Hj}>1$. Hence $\psi_{Hj}=1$ and $\psi_{Fj}=0$. Note that at the initial point of the interval defining this case, $Ta(j)/w = \overline{P}/e$, this result implies $P_j^* = e$.

(*iii*) Consider now the intermediate values $Ta(j)/w \in (e/\overline{P}, \overline{P}/e)$. Starting from $Ta(j)/w = e/\overline{P}$ and $P_j^* = \overline{P}$, any increase in $Ta(j)/w$ raises home-country market share above 0. This must be compensated by a reduction in P_j^* so that (10) can be satisfied by means of a reduction in the foreign market share. Moreover, the relative increase $\Delta(Ta(j)/w)/(Ta(j)/w)$ must be larger (in absolute terms) than the relative reduction in the equilibrium price $\Delta P_j^*/P_j^*$ (otherwise the home country share would be either unaffected or reduced while the foreign share is reduced). This will be the case until ψ_{Fj} turns out equal to 0 as $Ta(j)/w$ reaches $e/\overline{P}$. □

Proof of Proposition 2.2

Consider equation (11) in the text:

$$\int_0^1 l_i(j)dj = \int_0^1 \sum_{k=0}^{\infty} e \frac{x_{ki}(j)}{[T_i a_i(j) z_k]^{1-\sigma}} dj = \frac{1}{w_i} \int_0^1 \sum_{k=0}^{\infty} P_j^* q_{ki}(j) x_{ki}(j) e \frac{w_i}{P_j^* T_i a_i(j) z_k} dj$$

$$= \frac{1}{w_i} Y_W \int_0^1 \sum_{k=0}^{\infty} s_{ki}(j)[1 - s_{ki}(j)] dj = 1; \quad i = H, F.$$

Recall that market shares $s_{ki}(j)$ are functions of the ratio T/w and prices P_j^*; and that, in turn, prices are also functions of the ratio T/w. Hence we define: $\Psi_i(T/w) \equiv \int_0^1 \sum_{k=0}^{\infty} s_{ki}(j)[1 - s_{ki}(j)] dj$. Dividing the last two terms above for $i=F$ by the same expression for $i=H$ yields:

(A.3) $$w\frac{\Psi_F(T/w)}{\Psi_H(T/w)}=1.$$

This condition can be used to substitute for one of the labor-market equilibrium conditions (12). Wages satisfying (A.3) guarantee that country shares in production match relative labor supplies, whereas (12) for either H or F can be used to insure that world income Y_W adjusts to the scale that is consistent with the absolute size of world labor supply.

Foreign market shares $s_{kF}(j)$ are increasing in P_j^* and therefore decreasing in T/w. To the contrary, domestic shares are increasing in T/w (since they positively depend on this ratio and since the elasticity of P_j^* with respect to T/w is lower than 1; see the last statement in Claim 2.1.A in the proof of Proposition 2.1). Then, note that $a(1)\equiv a_1=\min(a(j))$ and $a(0)\equiv a_0=\max(a(j))$. Hence from Claim 2.1.A (in the proof of Proposition 2.1) we have that $T/w=e/(\bar{P}a_0)$ implies $\psi_{Hj}=0$ all j; and $T/w=\bar{P}/(a_1e)$ implies $\psi_{Fj}=0$ all j.

Now, assume the distribution of efficiencies is such that no firm has more than half the market share in any industry (e.g., $s_{0i}(j)<0.5$, all j; this assumption is not necessary but greatly simplifies the proof). It is then easy to check that:

$$\Psi_H(T/w)\equiv\int_0^1\sum_{k=0}^{\infty}s_{kH}(j)\left[1-s_{kH}(j)\right]dj:\begin{cases}T/w=e/(\bar{P}a_0) & \Rightarrow\Psi_H(T/w)=0;\\ e/(\bar{P}a_0)<T/w\le\bar{P}/(a_1e) & \Rightarrow\Psi_H(T/w)>0\ \text{and increasing}\end{cases}$$

$$\Psi_F(T/w)\equiv\int_0^1\sum_{k=0}^{\infty}s_{kF}(j)\left[1-s_{kF}(j)\right]dj:\begin{cases}e/(\bar{P}a_0)\le T/w<\bar{P}/(a_1e) & \Rightarrow\Psi_F(T/w)>0\ \text{and decreasing}\\ T/w=\bar{P}/(a_1e) & \Rightarrow\Psi_F(T/w)=0.\end{cases}$$

Note that, as integrals of the continuous functions s_{ij} in T/w, $\Psi_H(T/w)$ and $\Psi_F(T/w)$ are also continuous in T/w. Define the function $\Phi(T/w):(e/(\bar{P}a_0),\bar{P}/(a_1e)]\rightarrow[0,\infty)$ as $\Phi(T/w)\equiv\Psi_F(T/w)/\Psi_H(T/w)$. This function is characterized as follows:

$$\begin{cases}\lim_{T/w\rightarrow e/(\bar{P}a_0)}\Phi(T/w)=\infty\\ \Phi(T/w)\ \text{is strictly decreasing in}\ T/w & \text{if}\ e/(\bar{P}a_0)<T/w<\bar{P}/(a_1e);\\ \Phi(T/w)=0 & \text{if}\ T/w=\bar{P}/(a_1e)\end{cases}$$

Now, for any given $T\in(0,\infty)$, the product $w\cdot\Phi(T/w)$ can be seen as a function of w in the interval $[a_1e/\bar{P}T,\bar{P}a_0T/e)$, which is again continuous in w and satisfies:

$$w\cdot\Phi(T/w)\quad\begin{cases}\lim_{w\rightarrow\bar{P}a_0T/e}w\cdot\Phi(T/w)=\infty.\\ \text{strictly increasing in}\ w & \text{if}\ \bar{P}a_0T/e>w>a_1e/\bar{P}T;\\ =0 & \text{if}\ w=a_1e/\bar{P}T\end{cases}$$

Therefore for any T>0 there exists w^*, $a_1 e / \bar{P}T < w^* < \bar{P} a_0 T / e$, satisfying the general equilibrium condition $w^* \cdot \Phi(T/w^*)=1$. Moreover, since $\Phi(T/w)$ is decreasing in T/w, $w^*=w^*(T)$ is increasing in T. □

Proof of Proposition 3.1

To prove this proposition, first consider the following intermediate result:

Claim 3.1.A *(differentiability of country market shares and prices)***:** *With a continuous number of firms, $\psi_{ij}(P(j)T_i a_i(j)/w_i)$ is differentiable. Furthermore,* $\frac{\partial \Psi_H(T/w)}{\partial (T/w)} > 0$ *and* $\frac{\partial \Psi_F(T/w)}{\partial (T/w)} < 0$.

To prove this Claim, define $m \equiv P(j)T_i a_i(j)/w_i$ and consider firm and country market shares as functions of m. Note from expression (7) in the main text that each firm's market share $s_{ki}(j)$ is continuous and differentiable at all points m>0, except at the point where the firm switches from being inactive to being active. At this point, the firm's market share is continuous but not differentiable:

$$\lim_{m_h \to m^-} \frac{\partial s_{\bar{k}i}(j)}{\partial m} = 0 \neq \lim_{m_h \to m^+} \frac{\partial s_{\bar{k}i}(j)}{\partial m} = \frac{1}{m}.$$

However, the country's market share ψ_{ij} is differentiable at all points:

$$\lim_{m_h \to m^-} \frac{\partial}{\partial m} \psi_{ij}(m_h) = \lim_{m_h \to m^-} \frac{\partial}{\partial m} \int_0^{\bar{k}_{ij}} \left(1 - \frac{e}{m_h g(k)} \right) dk$$
$$= \lim_{m_h \to m^-} \left[\int_0^{\bar{k}_{ij}} \frac{\partial}{\partial m} \left(1 - \frac{e}{m g(k)} \right) dk + \left(1 - \frac{e}{m g(\bar{k}_i(j))} \right) \frac{\partial \bar{k}_i(j)}{\partial m} \right] = \int_0^{\bar{k}_{ij}} \frac{e}{m^2 g(k)} dk = \lim_{m_h \to m^+} \frac{\partial}{\partial m} \psi_{ij}(m_h).$$

Therefore, $\psi_{ij}(m)$ is differentiable with respect to m, for all m>0. Moreover, $\partial \psi_{ij}(m)/\partial m>0$.

Now, assuming to simplify the proof that the distribution of efficiencies is such that no firm controls half or more of the market in any industry (i.e., $s_{0i}(j)<0.5$, all j), we have:

$$\frac{\partial \Psi_F(T/w)}{\partial (T/w)} = \int_0^1 \left(\frac{d \sum_{k=0}^{\infty} s_{kF}(j)[1 - s_{kF}(j)]}{d(T/w)} \right) dj = \int_0^1 \left(\frac{\partial \sum_{k=0}^{\infty} s_{kF}(j)[1 - s_{kF}(j)]}{\partial P_j^*} \frac{dP_j^*}{d(T/w)} \right) dj < 0.$$

$$\frac{\partial \Psi_H(T/w)}{\partial (T/w)} = \int_0^1 \left(\frac{d \sum_{k=0}^{\infty} s_{kH}(j)[1 - s_{kH}(j)]}{d(T/w)} \right) dj = \int_0^1 \left(\frac{\partial \sum_{k=0}^{\infty} s_{kH}(j)[1 - s_{kH}(j)]}{\partial (P_j^* T/w)} P_j^* \left[1 + \frac{dP_j^*}{d(T/w)} \frac{T/w}{P_j^*} \right] \right) dj > 0.$$

Where the last inequality takes into account that $0 \geq \frac{\Delta P_j^*}{P_j^*} / \frac{\Delta(T/w)}{T/w} \geq -1$ for all *j*, with strict inequalities for an open interval of industries (see Claim 2.1.A). This completes the proof of Claim 3.1.A.

Now, to check (*i*) in Proposition 3.1 use Claim 3.1.A to differentiate expression (A.3) with respect to *T*, which yields:

$$\text{(A.4)} \quad 0 < \frac{dw^*}{dT}\frac{T}{w^*} = \frac{\varepsilon_\Phi}{\varepsilon_\Phi - 1} < 1; \text{ where } \varepsilon_\Phi \equiv \frac{\partial\left[\Psi_F(T/w)/\Psi_H(T/w)\right]}{\partial(T/w)}\frac{T/w}{\Psi_F(T/w)/\Psi_H(T/w)} < 0.$$

To check (*ii*), note first that both countries have strictly positive market shares if $Ta(j)/w \in (e/\bar{P}, \bar{P}/e)$ (see Claim 2.1.A). Differentiating $P_j^*(Ta(j)/w)$ with respect to *T* and using Claim 2.1.A and (A.4) yields:

$$\text{(A.5)} \quad -1 < \frac{dP_j^*}{dT}\frac{T}{P_j^*} = \frac{\partial P_j^*}{\partial(T/w)}\frac{T/w}{P_j^*}\left[1 - \frac{dw}{dT}\frac{T}{w}\right] < 0 \text{ for } j \text{ such that } a(j) \in (\frac{ew}{\bar{P}T}, \frac{\bar{P}w}{eT}).$$

Then, for industries where the home country is not a producer or it is the unique producer we have:

$$\begin{cases} \dfrac{dP_j^*}{dT}\dfrac{T}{P_j^*} = 0 & \text{for } j \text{ such that } \quad a(j) \in (0, \dfrac{ew}{\bar{P}T}]; \\ \dfrac{dw}{w} - \dfrac{dP_j^*}{P_j^*} = \dfrac{dT}{T} & \text{for } j \text{ such that } \quad a(j) \in [\dfrac{\bar{P}w}{eT}, \infty). \end{cases}$$

Finally, to check (*iii*) consider country *i*'s income:

$$Y_i = \int_0^1 \left[\int_0^{\bar{k}} P(j) q_{ki}(j) x_{ki}(j) dk\right] dj = \Upsilon_W \int_0^1 \psi_{ij} dj.$$

The elasticity of relative income with respect to home aggregate efficiency is given by:

$$\frac{dY_H}{dT}\frac{T}{Y_H} - \frac{dY_F}{dT}\frac{T}{Y_F} = \int_0^1 \left[\frac{d\psi_{Hj}}{dT}\frac{T}{\psi_{Hj}}\right] dj - \int_0^1 \left[\frac{d\psi_{Fj}}{dT}\frac{T}{\psi_{Fj}}\right] dj > 0.$$

The sign follows from the arguments above in this Proposition implying $d\psi_{Hj}/dT \geq 0$ and $d\psi_{Fj}/dT \leq 0$; with strict inequalities for industries where both countries have positive production. Thus, country *i*'s share in world income υ_i is increasing in its aggregate efficiency. □

TABLE 1: US Import Unit Value and Exporter Revealed Comparative Advantage

Results from running regressions pooling data from all the 72 commodities at the 5-digit level in industry 84 of the SITC (articles of apparel and clothing accessories)

	Dependent variable is log unit value			
	(1)	(2)	(3)	(4)
Log *PCGDP*	0.482***	0.492***	0.489***	0.480***
	(0.009)	(0.011)	(0.011)	(0.011)
Log *RCA*	0.050***	0.052***		
	(0.004)	(0.004)		
Log RCA^{UE}			0.065***	
			(0.004)	
Log *QRCA*				0.024***
				(0.004)
Controls for exporter size and distance	No	Yes	Yes	Yes
Number of commodities	72	72	72	72
Total number of observations	7679	7679	7608	7670
Adj. R^2	0.623	0.625	0.629	0.618

Results of regressing unit values of US imports on exporter PPP per capita income (*PCGDP*) and three different measures of exporter revealed comparative advantage (see equation (12). The three measures are standard revealed comparative advantage (*RCA*), revealed comparative advantage US-excluded (RCA^{UE}), and quantity revealed comparative advantage (*QRCA*)). Each column corresponds to a different specification. Standard errors shown in parenthesis are robust to heterokedasticity. As controls for exporter size and distance, I use exporter PPP GDP, and the average distance between the US and the exporter (see equation (12)). Estimations pool data from 2005 and 2006. All specifications include fixed time effects as well as 72 commodity fixed effects.

TABLE 2: Import Unit Value and Exporter Revealed Comparative Advantage

Results from running independent regressions for each commodity at the 5-digit level in industry 84 of the SITC (articles of apparel and clothing accessories). Four different specifications.

Measure of Revealed Comparative Advantage	Additional controls	% of Positive Coefficients out of the 72 commodities			% of Negative Coefficients out of the 72 commodities		Adj. R^2
		P-value *Any*	P-value *<10%*	P-value *<5%*	P-value *<10%*	P-value *<5%*	
Log *RCA*	No	83.3	48.6	41.7	1.4	1.4	29.6
Log *RCA*	Yes	81.9	48.6	43.1	2.8	1.4	32.3
Log RCA^{UE}	Yes	90.3	59.7	52.8	1.4	1.4	33.0
Log *QRCA*	Yes	75.0	30.6	26.4	5.6	1.4	31.0

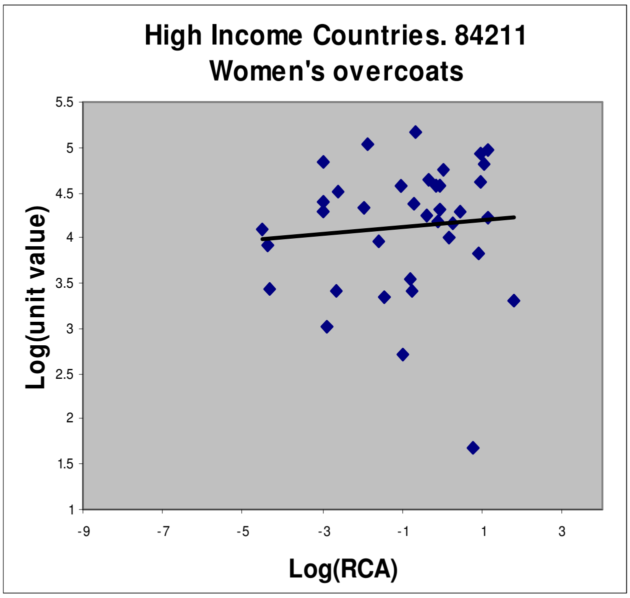
Number of Commodities: 72

Average number of observations per commodity: 105

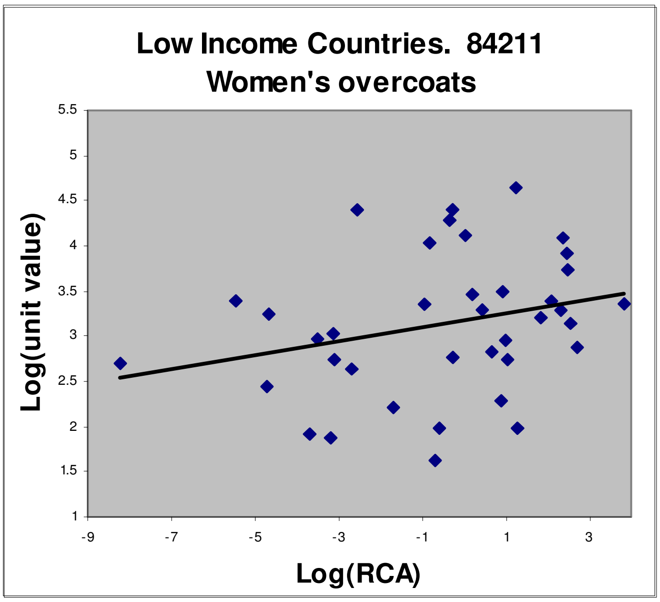
Results of regressing unit values of US imports on exporter PPP per capita income (*PCGDP*) and three different measures of exporter revealed comparative advantage (see equation (12). The three measures are standard revealed comparative advantage (*RCA*), revealed comparative advantage US-excluded (RCA^{UE}), and quantity revealed comparative advantage (*QRCA*)). Each of the four rows in the table shows the results corresponding to a different specification. In rows 2-4, the additional controls are average distance between the exporter and the US, and exporter PPP GDP. The same four equations were independently estimated for each of the 72 commodities. The results in each row show the percentage of the 72 regressions (one for each commodity) that yielded a positive estimated coefficient for the measure of revealed comparative advantage, as well as the percentage of the 72 commodities that yielded a statistically significant coefficient with either sign. All coefficients on exporter *PCGDP* were positive and significant at the 1-percent level in the regressions of all the commodities except two (i.e., in 97.2% of all the regressions). All variables are in logs. Estimations pool data from 2005 and 2006 and include fixed time effects.

Figure 1. Revealed Comparative Advantage and Unit Values for several 5-digit commodities of the SITC-rev3

(a)

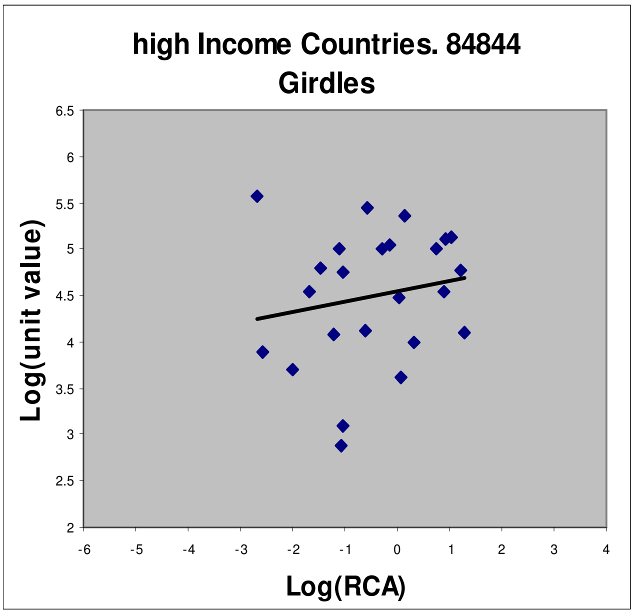


(b)

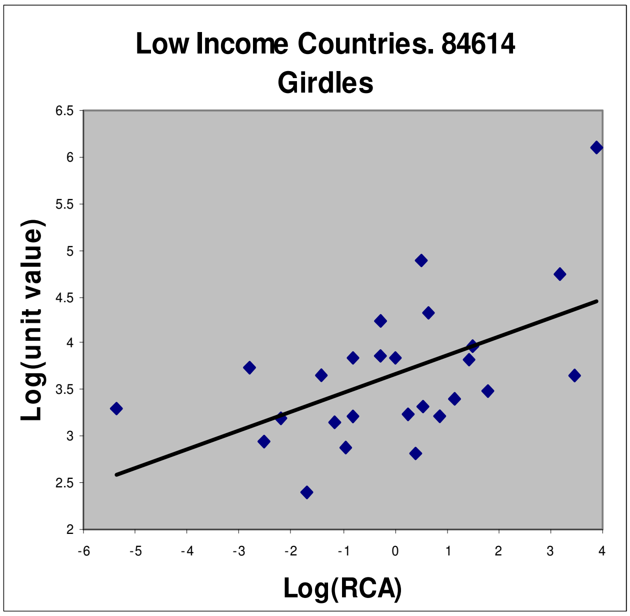


S3-84211: Women's or girls' overcoats, raincoats, car coats, capes, cloaks and similar articles

(c)

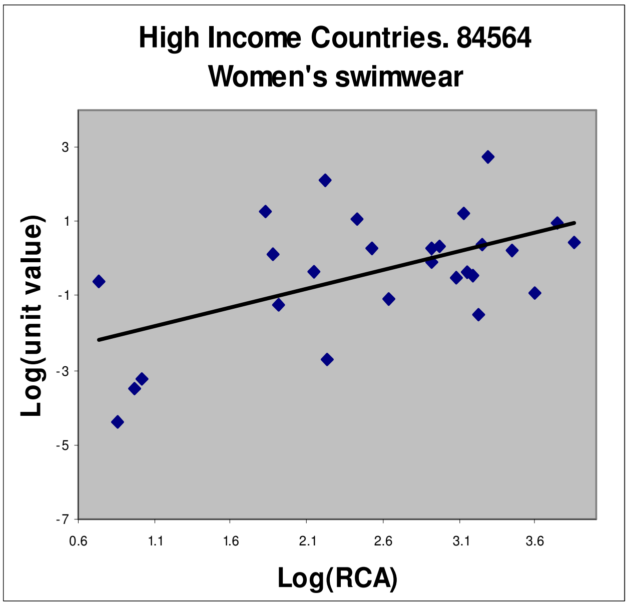


(d)

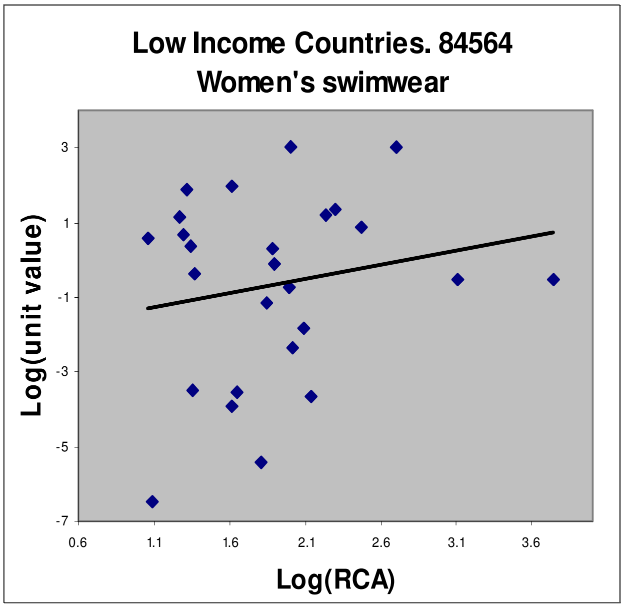


S3-84552: Girdles, corsets, braces, suspenders, garters and similar articles

(e)

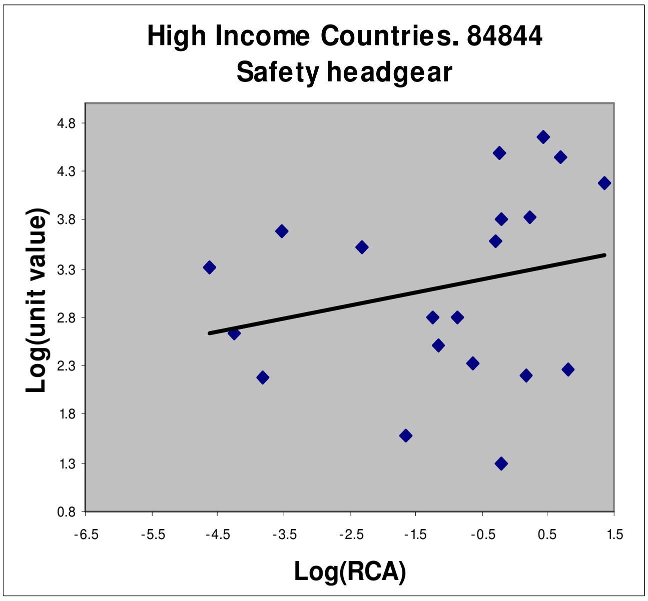


(f)

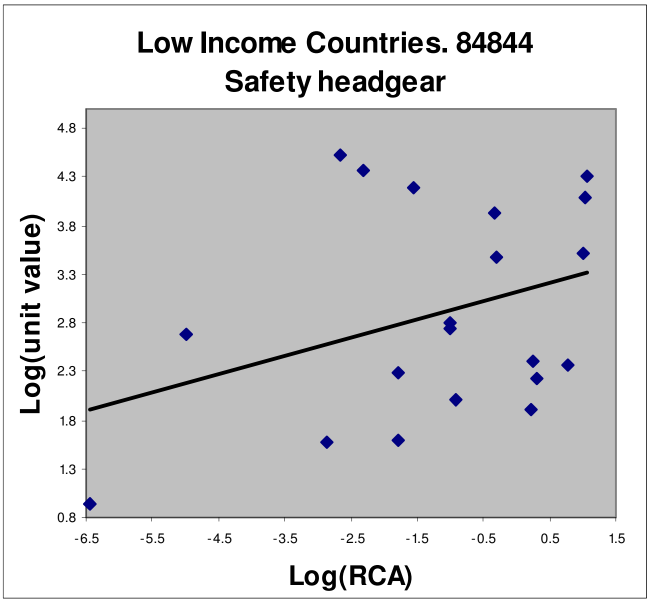


S3-84564: Swimwear, women's or girls', knitted or crocheted

(g)



(h)



S3-84844: Safety headgear, whether or not lined or trimmed

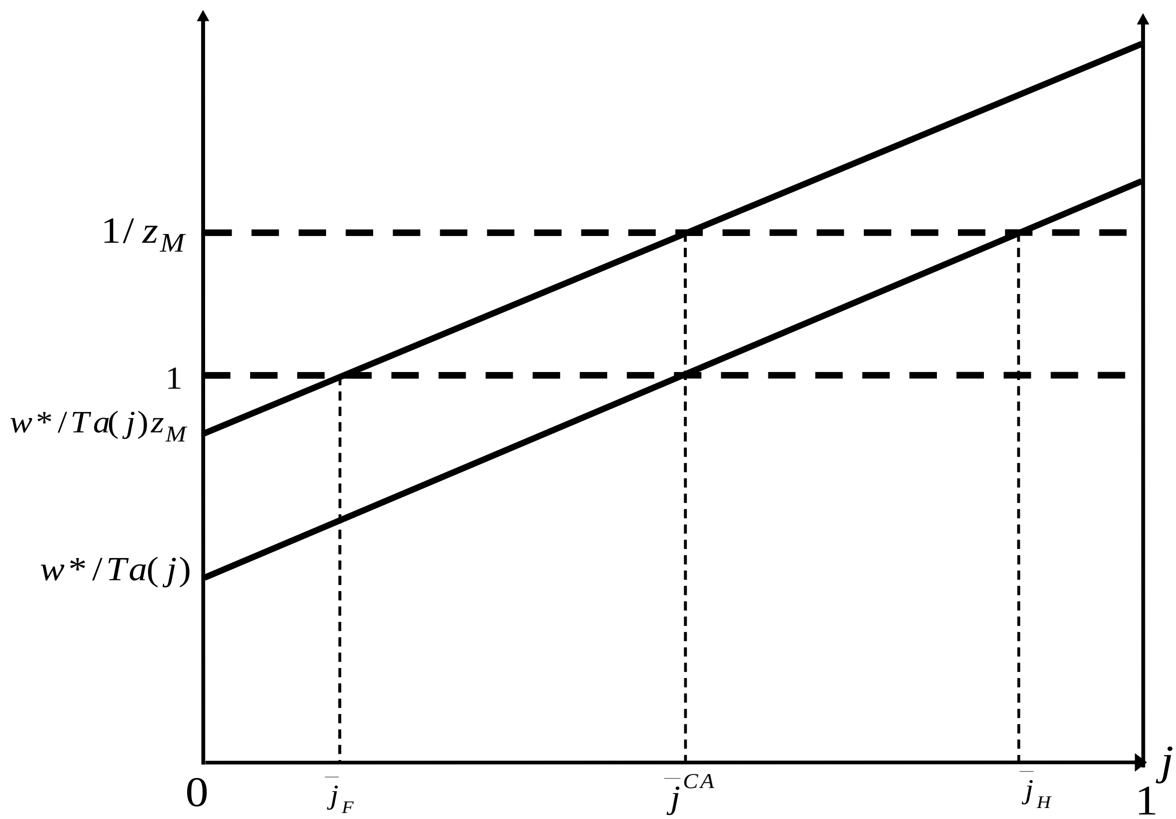


Figure 3: Specialization across goods

For each industry j, w^*/Ta^j is the unit cost of producing one unit of quality by the home-country's most efficient firm in that industry (see expression (9) and recall that the most efficient firm in each industry has $z_0 = 1$). $w^*/Ta^j z(M)$ is the unit cost of the home-country's marginal firm (firm M) in that industry. Thus, solid lines represent unit costs of the home country's most efficient firms and the marginal firms across industries. In turn, dashed lines represent the unit costs of the most efficient firms and the marginal firms across industries in the foreign country (due to normalization, the foreign-country most efficient firm in each industry has always unit cost equal to 1; and that the foreign-country marginal firm in each industry has unit cost $1/z(M)$). For industries such that $j \geq \bar{j}_H$ the home country's output is zero. For industries such that $j \leq \bar{j}_F$ the foreign country's output is zero. The home country has a comparative advantage in industries such that $j \geq \bar{j}^{CA}$.

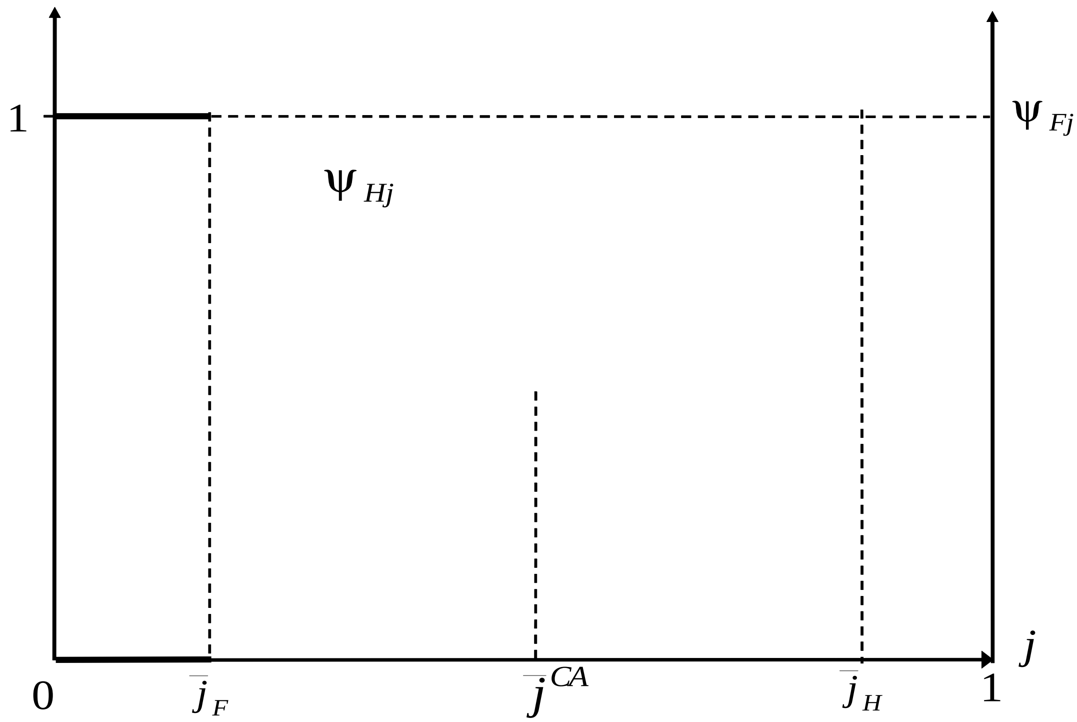
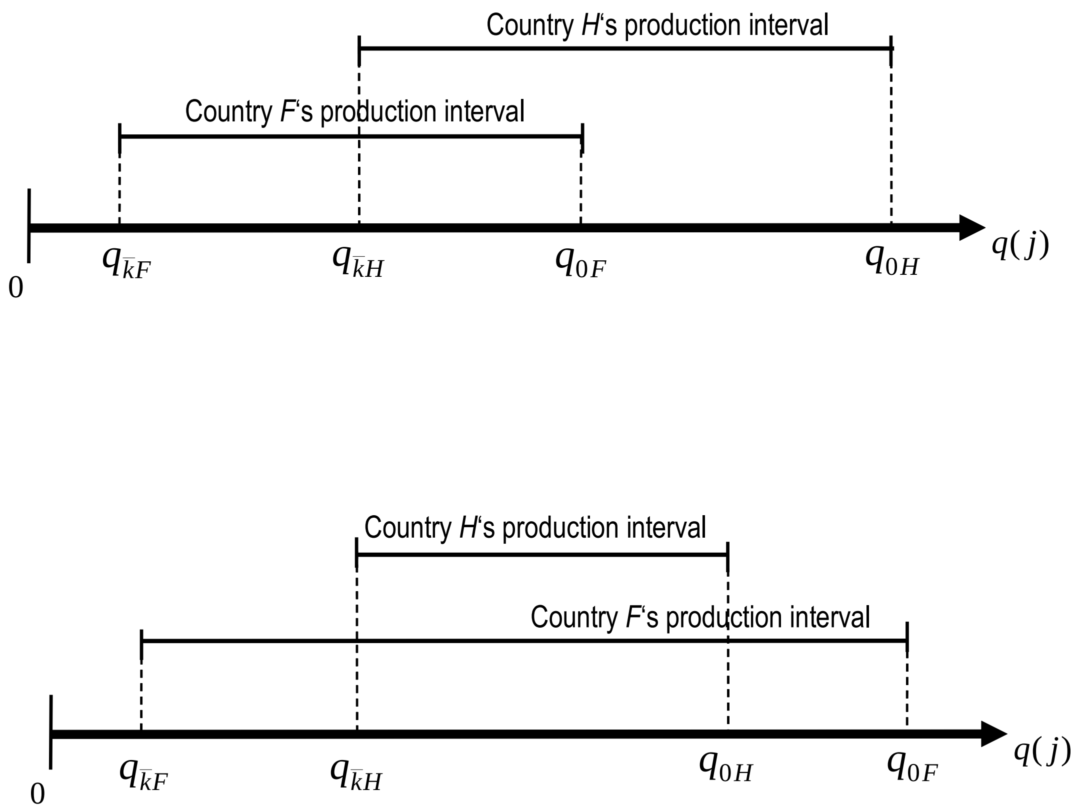


Figure 4: Country market shares across industries

For industries such that $j \geq \overline{j}_H$ the home country's market share is zero. For industries such that $j \leq \overline{j}_F$ the foreign country's market share is zero.



Figures 5a and 5b: Interval of qualities produced by each country in a given industry

Figures 5a shows the case of an industry in which the richer country (H) has an absolute advantage. Figures 5b show the case in which the poorer country (F) has an absolute advantage.

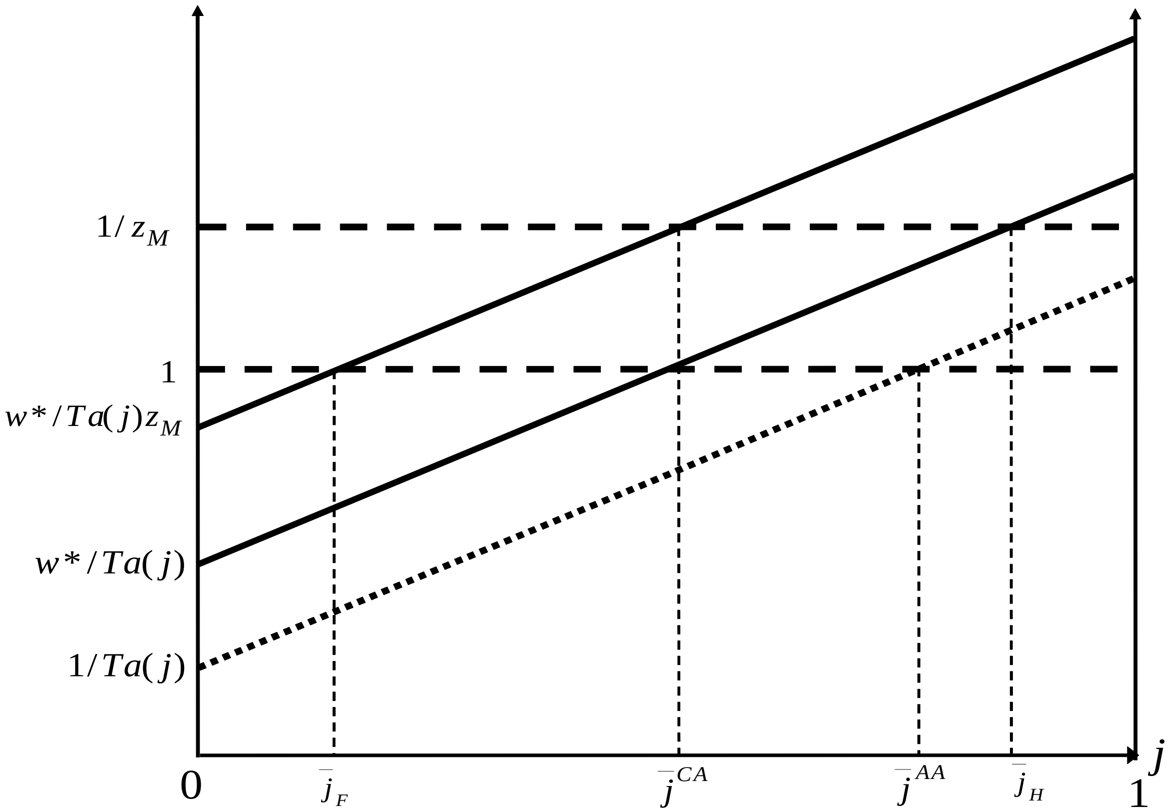
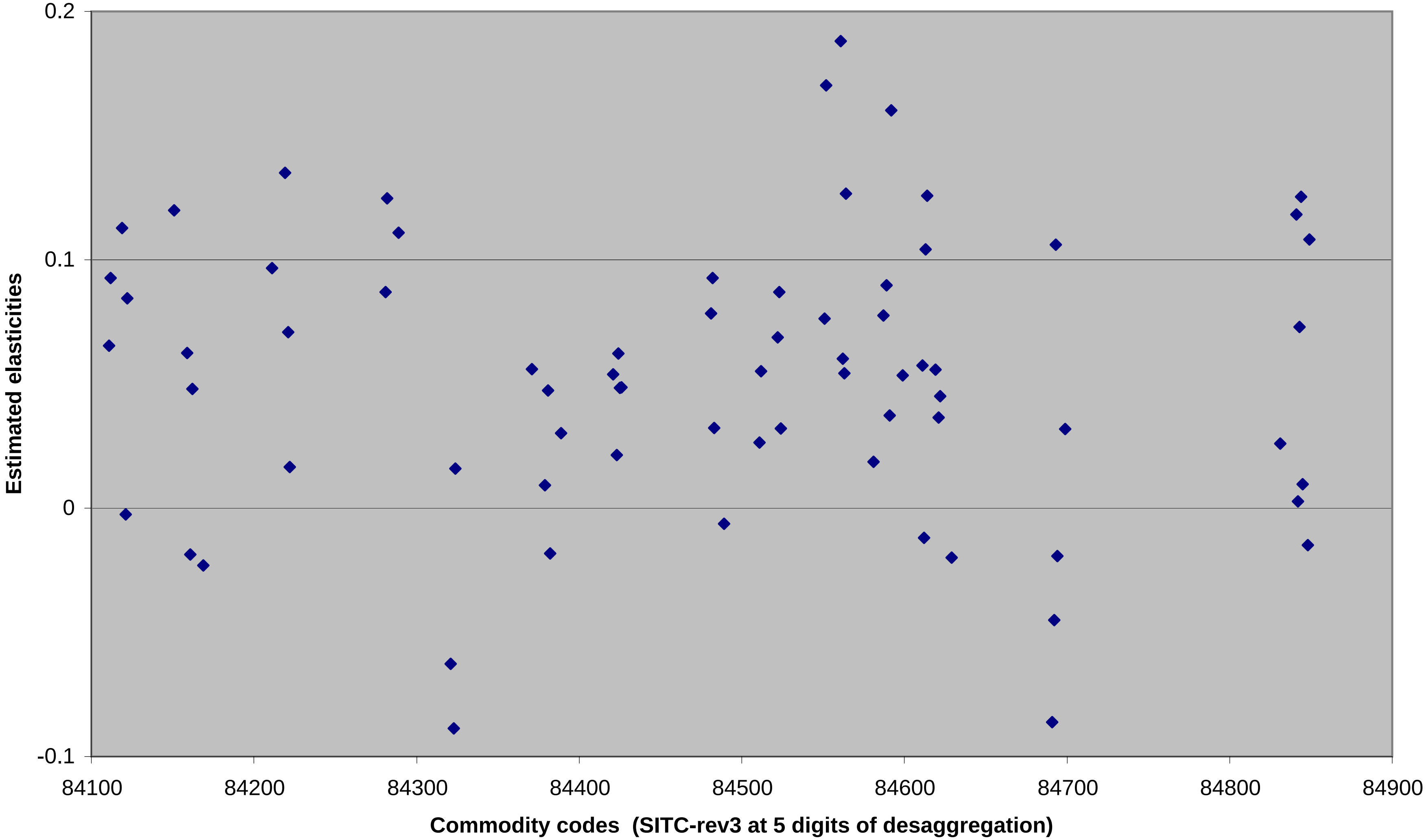


Figure 6: Specialization across goods and quality specialization within goods.

This figure adds the inverse of country H's efficiency across industries $1/Ta^j$ (the dotted line) to the lines in Figure 3. Country H is assumed to have a higher wage level than country F (i.e., $w^* > 1$) For industries between $\bar{j}_F$ and $\bar{j}_H$, both countries have positive production. To the left (respectively, right) of $\bar{j}^{AA}$, country H (resp. country F) has an absolute advantage and produces the highest qualities. To the left (resp. right) of $\bar{j}^{CA}$, it has a cost advantage and produces the widest set of qualities. To the right of $\bar{j}_F$, country F has a positive output and produces the lowest qualities because it has the lowest wage (in spite of also producing the highest qualities to the right of $\bar{j}^{AA}$).

Figure 7a: The Effect of Exporter Revealed Comparative Advantage on Import Unit Values by the US

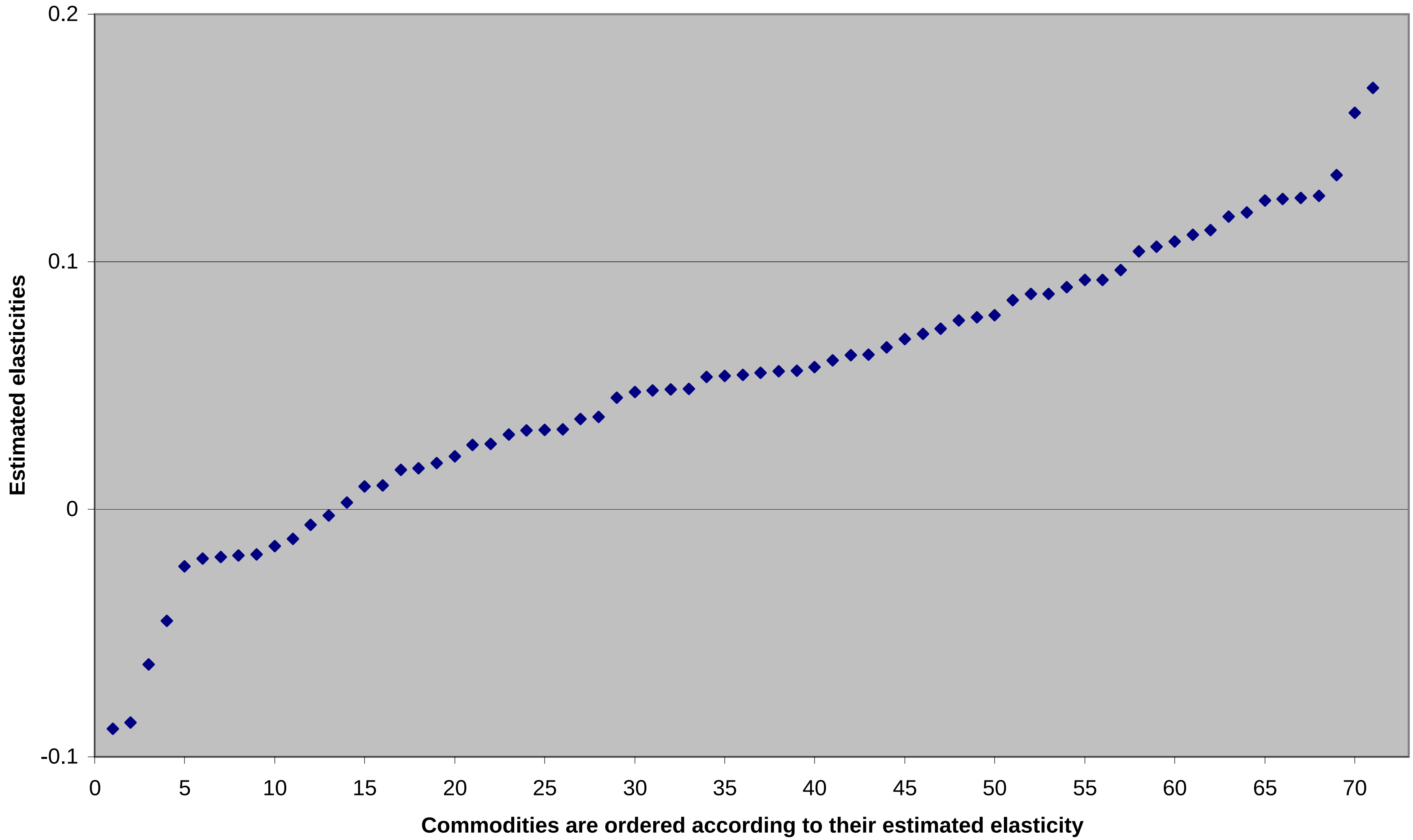
Estimated elasticites for 72 commodities



Note: The estimating equations also include (log of exporter) per capita GDP, Distance to the US, and GDP.

Figure 7b: The Effect of Exporter Revealed Comparative Advantage on Import Unit Values by the US

Estimated elasticites for 72 commodities



Note: The estimating equations also include (log of exporter) per capita GDP, Distance to the US, and GDP.