Cash, Paper-based and Electronic Payments: A Theoretical Approach*

Francisco Callado†, Jana Hromcová‡ and Natalia Utrero§

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Abstract

In this paper we provide a general equilibrium model that helps explaining payment choice at a retail level: cash, electronic and paper-based instruments. In particular, it provides theoretical foundations to reconcile previous empirical evidence on the matter. Payment structure of a given country can be shaped by the cost of each payment instrument, the degree of technology development and the interest rate. Differences among countries could be based on specific characteristics related to these variables.

Keywords: cash; payments; human capital; cash-in-advance;
JEL classification: E42, E41, O42

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†Universitat de Girona, Departament d'Economia i Empresa

‡Corresponding author: jhromcova@uao.es. Universitat Autònoma de Barcelona, Departament d'Economia Aplicada.

§Universitat de Girona, Departament d'Economia i Empresa

1 Introduction

This paper analyzes the dynamics of the payment instruments choice, in particular the behavior of cash, paper-based and electronic payments. The variables that explain differences between countries are the relative cost of the available payment instruments, degree of technology development and monetary policy.

The way in which payments are made in a given country happens to be a relevant issue at least in two respects. On one side, the cost of a payment system, that could account up to 3% of GDP, Humphrey, Pulley and Vesala (1996 and 2000), is clearly affected by agents' choice since the cost of each payment instrument differs. On the other hand, payment choice could influence the functioning of the financial system and therefore facilitate trade in the real economy. In fact these decisions could have economic consequences since an important part of GDP (around 2/3 in the US) comes from consumer transactions and these are completed with some method of payment, Schreft (2006).

The distribution of payment instruments use is usually different across countries. Some empirical studies have a look at the characteristics of this distribution and also try to analyze and explain the determinants of these differences. Humphrey et. al. (1996) examine the payment systems of 14 developed countries and find that the use of electronic means of payment is clearly increasing in all countries. Besides, they try to explain the possible factors behind the different structures across countries. Among these they find as the most important the degree of payment availability (number of users, terminals, etc) and institutional and cultural differences (income, new instruments, etc.). Figure 3 in Hancock and Humphrey (1998) provides an evidence of how electronic means of payment (credit and debit cards) gain importance with respect to checks and cash. An interesting finding is that the payment choice behavior of consumers in the US is a bit different than that of the rest of developed countries. Although cash use is low, the relevance of electronic payments is clearly lower. Humphrey (2004) deepens the analysis of the US by analyzing the substitution of cash by cards. In Figure 2 in his study we can observe how the relative importance of electronic means of payments (credit and debit cards) increases over time.

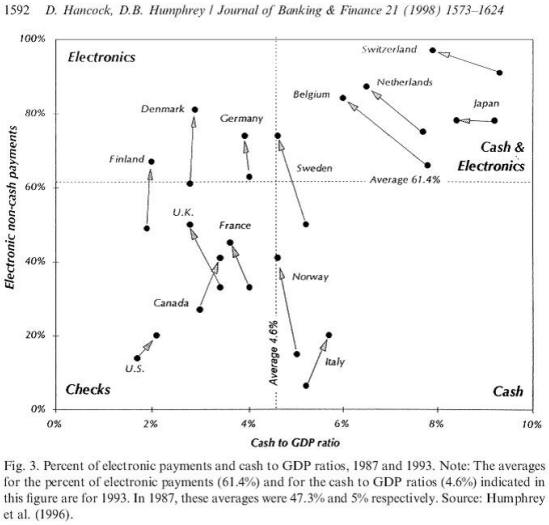


Fig. 3. Percent of electronic payments and cash to GDP ratios, 1987 and 1993. Note: The averages for the percent of electronic payments (61.4%) and for the cash to GDP ratios (4.6%) indicated in this figure are for 1993. In 1987, these averages were 47.3% and 5% respectively. Source: Humphrey et al. (1996).

Hankock and Humphrey (1998), Figure 3

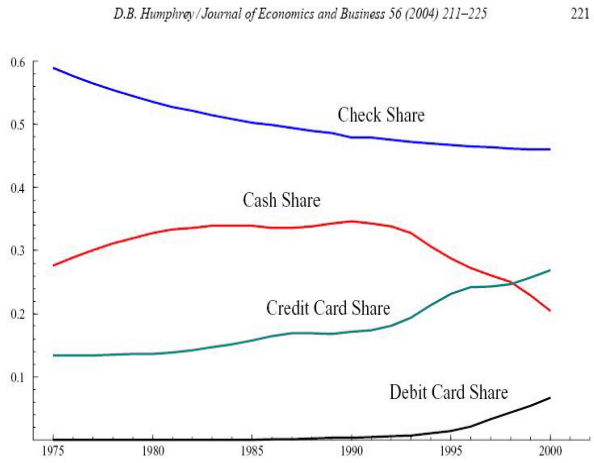


Fig. 2. Shares of cash, consumer checks, credit and debit cards in consumer payments: 1974–2000 (in %).

Humphrey (2004), Figure 2

Humphrey et. al. (2001) include price in trying to explain consumer choices using data from Norway. Although production techniques and consumer needs are very similar across countries, the

usage of payment instruments still differs. They find empirical evidence that technology development and relative prices, together with the relationship between cost and technology, could have a word to say in explaining these differences.

There is a lack of theoretical literature concerning wider choice of payment instruments, see Schreft (2006) for example.[1] There is a need to develop sound theoretical foundations for consumer decisions and use it to analyze their payment choice behavior, Crowe, Schuh and Stavins (2006). Here we propose to fill the gap by focusing on how and why consumers choose which payment instrument they use. We let our consumers use cash, paper-based or electronic payments. A consumer facing the decision on the mean of payment will take into account the relative cost of each instrument. The cost depends on the place and time of the transaction. We adapt for our purpose the model of Hromcová (2008). We consider that a monetary policy and a government action (that favors some of the available means of payments) may alter agents payment instruments choice.

We calibrate the model for yearly US data, 1975-2000. We compare the dynamics of the payment choice in the model and the data. The model can generate very similar behavior to the one found in the data. Collecting data for more countries we will determine the particular parameters responsible for differences in the payment choice dynamics between countries.

The remainder of the paper is organized as follows. The model and its main properties are stated in section 2. In section 3 we describe the balanced growth path behavior. In section 4 we discuss the behavior in the transition. Final conclusions are summarized in section 5.

[1] Many theoretical models which consider two payment instruments where developed. For example, Schreft (1992), Gillman (1993), Aiagari, Braun and Eckstain (1998) show that the choice between cash and a payment via intermediary depends on the mix of the cost of the alternative mean of payment and the monetary policy. Ireland (1994a) points out that the decrease in the use of cash and its substitution by cards is caused by an increase in income. Markose and Loke (2003) show that this substitution is also due to the availability of payment terminals at the point of sale. Ireland (1994b), Marquis and Reffett (1994), English (1999) and Hromcová (2008) relate the choice of payment instruments also to the technological progress.

2 Model

2.1 The Household Problem

We follow closely the specification of the economy in Hromcová (2008). The economy consists of a large number of infinitely lived households. All households have identical preferences, production and trade opportunities.

Households inhabit the following environment: they face continuum of spatially separated markets, which are indexed by $i \in [0, 1]$. All households live in market 0, and the index i indicates the distance from home. In each market i a distinct perishable good is produced and sold in every period. Goods are thus indexed by i, which corresponds to the market of both production and trade. The economy has a representative household with preferences given by the utility function

$$\sum_{t=1}^{\infty} \beta^t \int_0^1 \frac{c_t(i)^{1-\theta} - 1}{1-\theta} di \tag{1}$$

where $c_t(i)$ is defined as the consumption at period t of the good produced in market i and $\theta > 0$ is the inverse of the elasticity of intertemporal substitution. The production and trade is like in Lucas and Stokey (1983). Each household is composed of a worker-shopper pair.

Prior to any trading government fixes the gross nominal interest rate R to be constant in all periods. We will assume that $R \geq 1$. Agents enter the period t with certain amount of monetary balances M_t and the debt B_t, carried over from the previous period, and the human capital stock h_t. A representative worker decides to produce on any of the markets i via the net production function

$$y_t = w h_t l_t \tag{2}$$

where w denotes the marginal product of human capital in goods production, and l_t is the amount of time spent working. Human capital accumulation depends on the time spent studying, on the level of human capital and on the depreciation rate according to

$$h_{t+1} = \phi (1 - l_t) h_t + (1 - \delta) h_t \tag{3}$$

where ϕ is the efficiency of learning parameter and δ is the depreciation rate.

First, the goods market opens and consumption takes place. Worker stays at the market i during the whole period. Shopper visits various markets to acquire consumption goods carrying all the monetary balances of the household.

To acquire consumption goods agents can use government issued money or alternative means of payment. When using non-cash payments, they can choose from paper-based or electronic private securities. All goods purchased with government issued money will be referred to as cash goods. Goods purchased via paper-based payments will be referred to as paper-based credit goods and goods purchased via electronic private securities will be referred to as electronic credit goods. Goods only differ in a way they are acquired.

Nominal monetary balances M_t can be used to buy goods in some of the markets indexed by i. Cash purchases are subject to the liquidity constraint

$$\int_0^1 [1 - \xi_t(i)] c_t(i) di \leq \frac{M_t}{p_t}, \tag{4}$$

where $\xi_t(i) = 0$ if a good is purchased on market i with cash, or $\xi_t(i) = 1$ if a good is purchased on market i via a non-cash payment. The financial intermediary enables paper-based and electronic payments at a cost that is given for each market i and period t.

After the consumption takes place, the monetary holdings of agents are augmented by a lump sum transfer X_t from the government. The amount X_t is endogenously determined in the system according to the given nominal interest rate, so that the money demand is totally satisfied. As the next step the securities market opens. During the securities trading session households choose their currency holdings M_{t+1}. They also purchase (or issue) one-period nominally denominated pure discount bonds paying B_{t+1} units of money at period $t+1$ while they cost $\frac{B_{t+1}}{R}$ units of money at period t. Bonds are in zero net supply. The budget constraint agents are facing can be written

$$\int_0^1 c_t(i)di + \int_0^1 \xi_t(i) \left\{ \ \nu_t(i)\gamma_t^{\varphi}(i) + [1-\nu_t(i)]\,\gamma_t^{e}(i) \ \right\} di + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{Rp_t} \leq wh_t l_t + \frac{M_t}{p_t} + \frac{B_t}{p_t} + \frac{X_t}{p_t}. \tag{5}$$

where $\xi_t(i) = 1$ for a non-cash purchase at market i, $\nu_t(i) = 0$ if the good is purchased on market i via an electronic payment and $\nu_t(i) = 1$ if the good is bought using a paper-based private security. The real cost of the non-cash transaction at time t on market i is $\gamma_t^{\varphi}(i)$ and $\gamma_t^{e}(i)$ for paper-based and electronic payments, respectively.

2.2 Cost of alternative means of payments

We assume that the intermediation cost must be paid by the buyer, as motivated in Ireland (1994b).

To be able to purchase without cash, some resources must be devoted to making the non-cash payment itself available, checking the identity of the buyer and his ability to pay. When the shopper is far away from home (market zero) the communication becomes more difficult, and we assume that the payment to the intermediary increases with i. The process of human capital accumulation gives a potential for the development of new technologies. It also leads to an increase in income per worker and higher consumption. Higher purchase means that more importance will be given to checking the ability of the buyer to pay. The development of new technologies will decrease the processing costs.

The real payment made to the intermediary is characterized by a function that fulfills properties found in some empirical studies, see Hromcová (2008): the intermediation cost is lower in richer countries, the cost of intermediated payment diminishes over time, and the cost elasticity is close to zero (which motivates the proportional intermediation cost). Both paper-based and electronic payments get cheaper as new technology develop.[2]

We specify the intermediation costs for electronic and paper-based payments in the following way

$$\gamma_t^{e}(i) = [\gamma(i) + F]\,\frac{1}{(h_t)^{\alpha_e}} c_t(i), \tag{6}$$

$$\gamma_t^{\varphi}(i) = \gamma(i)\frac{1}{(h_t)^{\alpha_\varphi}} c_t(i). \tag{7}$$

[2] Check processing and connections for electronic transactions get better and cheaper with an increase in existing technologies.

Both paper-based and electronic payments have a time independent cost $\gamma(i)$ that is increasing with a distance from home, similarly as in Gillman (1993), Ireland (1994a) or Hromcová (2003), for example. For the sake of tractability we assign it the functional form used previously in the literature, introduced in Ireland (1994a)

$$\gamma(i) = \frac{i}{1-i}. \tag{8}$$

The fixed cost F is market and time independent and it is attributed to a cost of acquiring and maintaining of a machine that enables electronic transactions. This is a relative cost of paper-based and electronic payments and we interpret it as the government instruments that can motivate the agents to change the relative usage of paper-based and electronic payments.

The time dependent part of the intermediation cost,

$$\frac{1}{(h_t)^{\alpha_j}} c_t^j(i),\ j = \varphi \text{ or } e,$$

embodies properties concerning the proportionality to consumption purchases and the effect of new technologies on the cost. It decreases as the level of knowledge increases and it is proportional to consumption purchases done via intermediary. The function $1/h_t^{\alpha_j}$ reflects the following: as more human capital is accumulated, more knowledge is available, better technologies can be developed and cheaper intermediation services can be offered. The parameter α_j can be interpreted as the degree of knowledge diffusion into the payment system.[3] We assume that $0 < \alpha_j < 1$. For this interval of values, any improvement in the existing technologies makes it more difficult to lower the cost of intermediation services.[4]

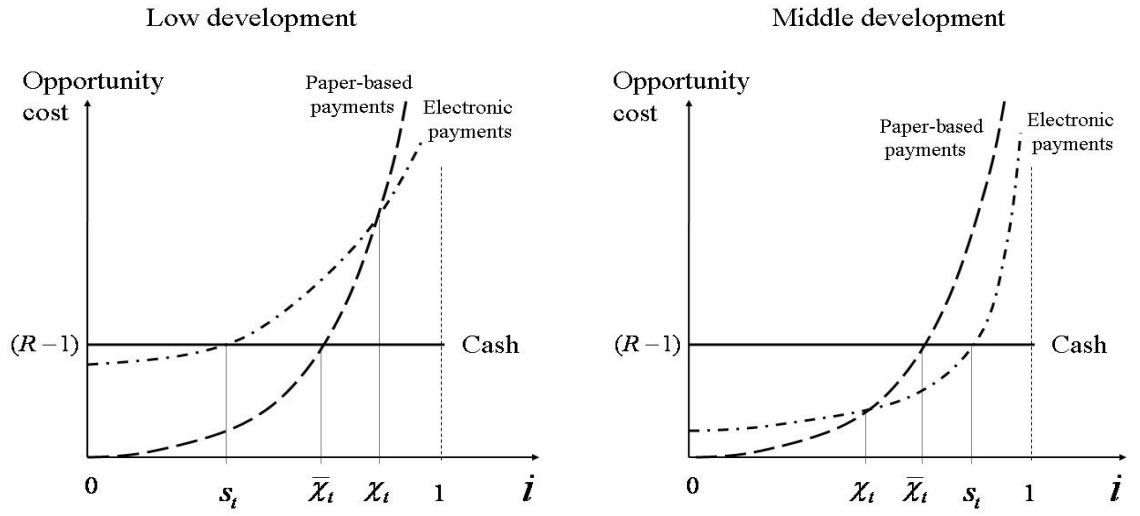
This specification means that for low level of technology the paper-based payments on any market are cheaper than the electronic ones. Because the degree of knowledge diffusion into the electronic payments is higher than the knowledge diffusion into the paper-based transactions, $\alpha_e > \alpha_\varphi$. As the economy grows and new technologies are developed, both payments get cheaper. In the long run the differences between the intermediation cost for employing the electronic and paper-based payments disappear.

The mean of payment will be chosen according to its cost. The opportunity cost of buying with cash is the nominal interest rate $R-1$, which does not vary across markets. The opportunity cost of buying with paper-based or electronic securities is the one corresponding to the intermediation cost $\gamma_t^\varphi(i)$ and $\gamma_t^e(i)$, respectively. We show the behavior of the opportunity cost of using cash, paper-based or electronic payments across markets in Figure 1. For an economy at a low stage of development (low level of human capital) it may happen that no electronic goods will be employed, because they are too expensive. In Figure 1a) we can observe that in all markets with indexes below $\bar{\chi}_t$ paper-based securities have the lowest cost. In all market with indexes above $\bar{\chi}_t$ cash will be employed because the opportunity cost of holding money is lower than the intermediation cost for either non-cash security. For a more developed economy, Figure 1b), paper-based payments are the

[3]For $h_t < 1$ higher α_j increases the intermediation cost for a given level of human capital. We can understand that in our analysis countries capable of performing electronic transactions are developed enough so the initial level of humancapital $h_1 > 1$.

[4]For example a price of a connection to the buyers' bank account (a phone call) is so low that it is really difficult to lower it even more if better technologies are developed, or that cheaper processing of checks is always more difficult to achieve.

cheapest way of acquiring consumption goods in all markets with indexes below χ_t. In all markets with indexes between χ_t and s_t electronic payments have the lowest cost, and in markets with indexes above s_t government issued money are the cheapest.



(a) (b)

Figure 1: Determination of the marginal markets: when $i = \bar{\chi}_t$, agents are indifferent between buying with cash or paper-based securities; when $i = \chi_t$ agents are indifferent between buying via electronic or paper-based transactions; when $i = s_t$ agents are indifferent between buying with cash or via electronic payments. (a) Low development: only cash and paper-based payments are used; (b) Higher development: all three payment instruments are used.

We arbitrarily assume that cash will be used on the marginal markets $\bar{\chi}_t$ and s_t, and electronic payment on the cutoff market χ_t. There will exists a moment in time for which all three marginal markets coincide $s_t = \bar{\chi}_t = \chi_t$. Thus by checking the relationship between s_t and χ_t we can recognize the mentioned two stages of development. One in which no electronic goods will be employed, $s_t < \chi_t$, and the stage of the coexistence of all three types of considered means of payments. The basic characteristic of the second stage is that in the course of time the electronic private securities will drive out of the markets cash and paper-based payments.

Define

$$c_t(i) = \begin{cases} c_t^0(i) & \text{when } \xi_t(i) = 0, \\ c_t^{\varphi}(i) & \text{when } \xi_t(i) = 1 \text{ and } \nu_t(i) = 0, \\ c_t^e(i) & \text{when } \xi_t(i) = 1 \text{ and } \nu_t(i) = 1. \end{cases}$$

The functions $c_t^0(i)$, $c_t^{\varphi}(i)$ and $c_t^e(i)$ characterize the cash, paper-based credit and electronic credit consumption per market i, respectively. Recognizing the two stages of development, we can rewrite the utility function

$$\sum_{t=1}^{\infty} \beta^t \left[\int_0^{\min\{\bar{\chi}_t, \chi_t\}} \frac{c_t^{\varphi}(i)^{1-\theta} - 1}{1-\theta} di + \int_{\min\{\bar{\chi}_t, \chi_t\}}^{\max\{\bar{\chi}_t,\, s_t\}} \frac{c_t^e(i)^{1-\theta} - 1}{1-\theta} di + \int_{\max\{\bar{\chi}_t,\, s_t\}}^{1} \frac{c_t^0(i)^{1-\theta} - 1}{1-\theta} di \right], \qquad (9)$$

the budget constraint

$$\int_0^{\min\{\bar{\chi}_t,\chi_t\}} [c_t^{\varphi}(i)+\gamma_t^{\varphi}(i)]\,di + \int_{\min\{\bar{\chi}_t,\chi_t\}}^{\max\{\bar{\chi}_t,\,s_t\}} [c_t^{e}(i)+\gamma_t^{e}(i)]\,di + \int_{\max\{\bar{\chi}_t,\,s_t\}}^{1} c_t^{0}(i)di + \\ + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{Rp_t} \le wh_tl_t + \frac{M_t}{p_t} + \frac{B_t}{p_t} + \frac{X_t}{p_t} \quad (10)$$

and the cash-in-advance constraint

$$\int_{\max\{\bar{\chi}_t,\,s_t\}}^{1} c_t^{0}(i)di \le \frac{M_t}{p_t}. \quad (11)$$

2.3 Equilibrium

Definition: Given the set of initial conditions h_1, M_1, B_1 and the nominal interest rate R, the equilibrium consists of sequences $\{c_t^0(i),\, c_t^{\varphi}(i),\, c_t^{e}(i),\, l_t,\, h_{t+1},\, M_{t+1},\, B_{t+1},\, \chi_t, \bar{\chi}_t,\, s_t,\, X_t,\, p_t\}_{t=1}^{\infty}$ such that

(a) a representative household is maximizing the discounted utility (9) subject to the budget constraint (10), the cash-in-advance constraint (11) and the condition for accumulation of human capital (3), choosing the sequences $\{c_t^0(i),\, c_t^{\varphi}(i),\, c_t^{e}(i),\, l_t,\, h_{t+1},\, M_{t+1},\, B_{t+1},\, \chi_t, \bar{\chi}_t,\, s_t\}_{t=1}^{\infty}$,

(b) markets for goods, money and bonds clear in every period,

$$\int_0^{\min\{\bar{\chi}_t,\chi_t\}} c_t^{\varphi}(i)di + \int_{\min\{\bar{\chi}_t,\chi_t\}}^{\max\{\bar{\chi}_t,\,s_t\}} c_t^{e}(i)di + \int_{\max\{\bar{\chi}_t,\,s_t\}}^{1} c_t^{0}(i)di + \\ + \int_0^{\min\{\bar{\chi}_t,\chi_t\}} \gamma_t^{\varphi}(i)di + \int_{\min\{\bar{\chi}_t,\chi_t\}}^{\max\{\bar{\chi}_t,\,s_t\}} \gamma_t^{e}(i)di = wh_tl_t, \quad (12)$$

$$M_{t+1} = M_t + X_t, \quad (13)$$

$$B_{t+1} = 0. \quad (14)$$

Let λ_t, η_t, and τ_t be the non-negative Lagrange multipliers associated with the budget constraint (10), the cash-in-advance constraint (11), and the condition for accumulation of human capital (3), respectively. The equations that characterize the equilibrium are the above mentioned market clearing conditions (12), (13), (14) and the first order conditions on all types of consumptions, labor,

capital, nominal balances, nominal bonds and marginal markets indexes, respectively,

$$
\begin{aligned}
c_t^0(i)^{-\theta} &= \lambda_t + \eta_t, && (15)\\
c_t^{\varphi}(i)^{-\theta} &= \lambda_t, && (16)\\
c_t^{e}(i)^{-\theta} &= \lambda_t, && (17)\\
\lambda_t w &= \tau_t \phi, && (18)\\
\tau_t &= \beta\lambda_{t+1} w l_{t+1} + \beta\tau_{t+1}\left\{\phi\left(1-l_{t+1}\right)+(1-\delta)\right\}, && (19)\\
\frac{\lambda_t}{p_t} &= \beta\frac{\lambda_{t+1}+\eta_{t+1}}{p_{t+1}}, && (20)\\
\frac{\lambda_t}{p_t} &= \beta R\frac{\lambda_{t+1}}{p_{t+1}}, && (21)
\end{aligned}
$$

$$
\begin{aligned}
\frac{c_t^0(\bar{\chi}_t)^{1-\theta}-1}{1-\theta} - \frac{c_t^{\varphi}(\bar{\chi}_t)^{1-\theta}-1}{1-\theta} &= -\lambda_t\left[c_t^{\varphi}(\bar{\chi}_t)+\gamma_t(\bar{\chi}_t)\right] + \left(\lambda_t+\eta_t\right)c_t^0(\bar{\chi}_t), && (22)\\
\frac{c_t^e(s_t)^{1-\theta}-1}{1-\theta} - \frac{c_t^0(s_t)^{1-\theta}-1}{1-\theta} &= -(\lambda_t+\eta_t)c_t^0(s_t) + \lambda_t\left[c_t^e(s_t)+\gamma_t^e(s_t)\right] && (23)\\
\frac{c_t^{\varphi}(\chi_t)^{1-\theta}-1}{1-\theta} - \frac{c_t^e(\chi_t)^{1-\theta}-1}{1-\theta} &= -\lambda_t\left[c_t^e(\chi_t)+\gamma_t^e(\chi_t)\right] + \lambda_t\left[c_t^{\varphi}(\chi_t)+\gamma_t^{\varphi}(\chi_t)\right] && (24)
\end{aligned}
$$

Using (15), (20) and (21), we can rewrite the first order condition on cash consumption as

$$c_t^0(i)^{-\theta} = R\lambda_t. \tag{25}$$

From the first order conditions on the marginal markets between paper-based credit goods and cash goods (22), electronic credit goods and cash goods (23) and between paper-based and electronic credit goods (24) we get[5]

$$
\begin{aligned}
\gamma_t^{\varphi}\left[\bar{\chi}(R,h_t)\right] &= \frac{1}{\lambda_t(R)}\left[\frac{c^{\varphi}[\lambda_t(R)]^{1-\theta}-1}{1-\theta} - \frac{c^0[R,\lambda_t(R)]^{1-\theta}-1}{1-\theta}\right] + Rc^0\left[R,\lambda_t(R)\right] - c^{\varphi}\left[\lambda_t(R)\right], && (26)\\
\gamma_t^{e}\left[s(R,h_t)\right] &= \frac{1}{\lambda_t(R)}\left[\frac{c^{e}[\lambda_t(R)]^{1-\theta}-1}{1-\theta} - \frac{c^0[R,\lambda_t(R)]^{1-\theta}-1}{1-\theta}\right] + Rc^0\left[R,\lambda_t(R)\right] - c^{e}\left[\lambda_t(R)\right], && (27)\\
\gamma_t^{e}\left[\chi(R,h_t)\right] &= \gamma_t^{\varphi}\left[\chi(R,h_t)\right]. && (28)
\end{aligned}
$$

[5]The initial level of the Lagrange multiplier on the budget constraint depends on the monetary policy, so we write $\lambda_t = \lambda_t(R)$.

Using the given forms of the cost functions (6) and (7), we get for the respective cutoff markets

$$\bar{\chi}_t = \bar{\chi}(R, h_t) = \frac{\Theta(R)}{\frac{1}{(h_t)^{\alpha_\varphi}} + \Theta(R)}, \tag{29}$$

$$s_t = s(R, h_t) = \frac{\Theta(R) - \frac{1}{(h_t)^{\alpha_e}} F}{\frac{1}{(h_t)^{\alpha_e}} + \Theta(R) - \frac{1}{(h_t)^{\alpha_e}} F}, \tag{30}$$

$$\chi_t = \chi(h_t) = \frac{F}{\frac{(h_t)^{\alpha_e}}{(h_t)^{\alpha_\varphi}} + F - 1} \tag{31}$$

where

$$\Theta(R) = \begin{cases} \ln R & \text{for } \theta = 1, \\ \\ \dfrac{\theta}{1-\theta}\left(1 - \dfrac{1}{R^{\frac{1-\theta}{\theta}}}\right) & \text{for } \theta \neq 1. \end{cases} \tag{32}$$

As can be seen from the goods market equilibrium (12), the current period output is spent between paper-based credit consumption, electronic credit consumption, cash consumption and payments to the intermediary for both types of non-cash purchases. The real monetary balances, which equal the amount of cash consumption purchased in all markets, are

$$m_t = m(R, h_t, \lambda_t) = \begin{cases} \left[1 - \bar{\chi}(R, h_t)\right] c^0\left[R, \lambda_t(R)\right] & \text{for } s_t \leq \chi_t, \text{ or} \\ \\ \left[1 - s(R, h_t)\right] c^0\left[R, \lambda_t(R)\right] & \text{for } s_t > \chi_t \end{cases} \tag{33}$$

where

$$m_t = \frac{M_t}{p_t}. \tag{34}$$

The paper-based credit consumption in all markets is

$$\varphi_t = \varphi(R, h_t, \lambda_t) = \begin{cases} \bar{\chi}(h_t)\ c^\varphi\left[\lambda_t(R)\right] & \text{for } s_t \leq \chi_t, \text{ or} \\ \\ \chi(R, h_t)\ c^\varphi\left[\lambda_t(R)\right] & \text{for } s_t > \chi_t \end{cases} \tag{35}$$

and electronic credit consumption in all markets

$$e_t = e(R, h_t, \lambda_t) = \begin{cases} 0 \text{ for } s_t \leq \chi_t, \text{ or} \\ \\ \left[s(R, h_t) - \chi(R, h_t)\right]\ c^e\left[\lambda_t(R)\right] & \text{for } s_t > \chi_t. \end{cases} \tag{36}$$

The part of output that goes to the intermediary is the sum of payments corresponding to paper-based consumption (solving for the fourth integral in the goods market equilibrium (12)),

$$\Gamma_t^\varphi = \Gamma^\varphi(R, h_t, \lambda_t) = \begin{cases} \left\{-\bar{\chi}(h_t) - \ln\left[1 - \bar{\chi}(h_t)\right]\right\}\ c^\varphi\left[\lambda_t(R)\right] & \text{for } s_t \leq \chi_t, \text{ or} \\ \\ \left\{-\chi(R, h_t) - \ln\left[1 - \chi(R, h_t)\right]\right\}\ c^\varphi\left[\lambda_t(R)\right] & \text{for } s_t > \chi_t. \end{cases} \tag{37}$$

and the one corresponding to electronic credit consumption

$$\Gamma_t^e = \begin{cases} 0 \text{ for } s_t \leq \chi_t, \text{ or} \\ \\ \{[\chi(R,h_t) - s(R,h_t) + \ln(1-\chi_t) - \ln(1-s_t)] + [s_t - \chi_t] F^e\} \frac{1}{(h_t)^{\alpha_e}} c^e [\lambda_t(R)] \text{ for } s_t > \chi_t. \end{cases} \tag{38}$$

The monetary transfer from the government is given by

$$X_t = (\mu_{t+1} - 1) M_t$$

where μ_{t+1} is the gross growth rate of money supply between period t and $t+1$. From (18) and (19) we get the evolution of the marginal utility of consumption

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta(\phi + 1 - \delta). \tag{39}$$

Combining expressions (21), (34) and (39) we can write for the growth rate of money supply

$$\mu_{t+1} = \frac{R}{(\phi + 1 - \delta)} \frac{m_{t+1}}{m_t}. \tag{40}$$

Notice that the constant in (40) is the gross inflation rate

$$\pi_{t+1} = \frac{p_{t+1}}{p_t} = \frac{R}{(\phi + 1 - \delta)}. \tag{41}$$

3 Balanced Growth Path

To perform our analysis we will assume that human capital grows at a positive rate, i.e.

$$\frac{h_{t+1}}{h_t} > 1.$$

To characterize the properties of the economy when it reaches the balanced growth path we look at the behavior of variables as time goes to infinity. From (29)-(31) we see that when the human capital accumulates at a positive rate and $h_t \to \infty$ the cutoff index between the electronic and cash goods approaches unity, $s(R, h_t) \to 1$, and the one between paper-based payments and cash goods also $\bar{\chi}(R, h_t) \to 1$, while the cutoff index between the paper-based and electronic payments is approaching zero, $\chi(h_t) \to 0$. That means that the electronic payments are used in more and more markets and the cash and paper-based private securities are less and less employed.

Let us look at the long run growth rates of all types of consumption and payments to the intermediary. When taking the limits of the growth rates of these variables as time goes to infinity we get the following: cash consumption and the payment to the intermediary for electronic transactions in the long run grow at the rate

$$\lim_{t\to\infty}\left(\frac{m_{t+1}}{m_t}\right) = \lim_{t\to\infty}\left(\frac{\Gamma^e_{t+1}}{\Gamma^e_t}\right) = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{1}{\theta}}\left(\frac{h_t}{h_{t+1}}\right)^{\alpha_e}, \tag{42}$$

paper-based credit consumption

$$\lim_{t\to\infty}\left(\frac{\varphi_{t+1}}{\varphi_t}\right) = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{1}{\theta}}\left(\frac{h_t}{h_{t+1}}\right)^{\alpha_e-\alpha_\varphi},$$

the intermediation cost for paper-based payments in the long run does not grow

$$\lim_{t\to\infty}\left(\frac{\Gamma^\varphi_{t+1}}{\Gamma^\varphi_t}\right) = 1 \tag{43}$$

and the electronic credit consumption grows at the rate

$$\lim_{t\to\infty}\left(\frac{e_{t+1}}{e_t}\right) = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{1}{\theta}}.$$

Goods market equilibrium

$$wl_t = \frac{m_t}{h_t} + \frac{\varphi_t}{h_t} + \frac{e_t}{h_t} + \frac{\Gamma^\varphi_t}{h_t} + \frac{\Gamma^e_t}{h_t} \tag{44}$$

implies that credit consumption in the long run must grow like human capital. That means that

$$\frac{h_{t+1}}{h_t} = \left(\frac{\lambda_t}{\lambda_{t+1}}\right)^{\frac{1}{\theta}}. \tag{45}$$

Plugging (45) into (42) and using (39), (3) and (40) we can summarize the results in the following proposition.

Proposition 1 *On the balanced growth path cash consumption and the payment for electronic transactions grow at the rate*

$$\left[\beta\left(\phi+1-\delta\right)\right]^{\frac{1-a_e}{\theta}},$$

paper-credit consumption grows at

$$\left[\beta\left(\phi+1-\delta\right)\right]^{\frac{1-a_e+a_\varphi}{\theta}}$$

electronic-credit consumption and human capital grow at the rate

$$\left[\beta\left(\phi+1-\delta\right)\right]^{\frac{1}{\theta}},$$

labor is given by

$$\frac{\left(\phi+1-\delta\right)-\left[\beta\left(\phi+1-\delta\right)\right]^{\frac{1}{\theta}}}{\phi}$$

and the growth rate of money supply is

$$R\beta^{\frac{1-a_e}{\theta}}\left(\phi+1-\delta\right)^{\frac{1-a_e-\theta}{\theta}}.$$

Proof. The labor on the balanced growth path is obtained by equating the condition for the human capital accumulation (3) and the long run growth rate of capital. ■

4 Transitional Dynamics

4.1 Numerical Example

In order to characterize the dynamics we rewrite the equilibrium equations. We define

$$\hat{m}_t = m_t\lambda_t^{\frac{1}{\theta}},\ \hat{\varphi}_t=\varphi_t\lambda_t^{\frac{1}{\theta}},\ \hat{e}_t=e_t\lambda_t^{\frac{1}{\theta}} \tag{46}$$

$$\hat{\Gamma}_t^{\varphi} = \Gamma_t^{\varphi}\lambda_t^{\frac{1}{\theta}},\ \hat{\Gamma}_t^{e}=\Gamma_t^{e}\lambda_t^{\frac{1}{\theta}}. \tag{47}$$

Define

$$\hat{G}\left(R,h_t\right)=\hat{m}_t+\hat{\varphi}_t+\hat{e}_t+\hat{\Gamma}_t^{\varphi}+\hat{\Gamma}_t^{e}. \tag{48}$$

Then the goods market equilibrium (12) can be rewritten as

$$h_t=\hat{G}\left(R,h_t\right)\frac{1}{wl_t\lambda_t^{\frac{1}{\theta}}}. \tag{49}$$

Time spent working l_t can be written as a function of the growth rate of human capital using the equation (3). Using (48), (49), (3) and (39) we can characterize the entire equilibrium by a second order difference equation in human capital

$$\frac{h_{t+1}}{h_t}=\frac{\hat{G}\left(R,h_{t+1}\right)}{\hat{G}\left(R,h_t\right)}\left(\frac{\phi-\frac{h_{t+1}}{h_t}+1-\delta}{\phi-\frac{h_{t+2}}{h_{t+1}}+1-\delta}\right)\left[\beta\left(\phi+1-\delta\right)\right]^{\frac{1}{\theta}}. \tag{50}$$

4.1.1 Numerical Technique

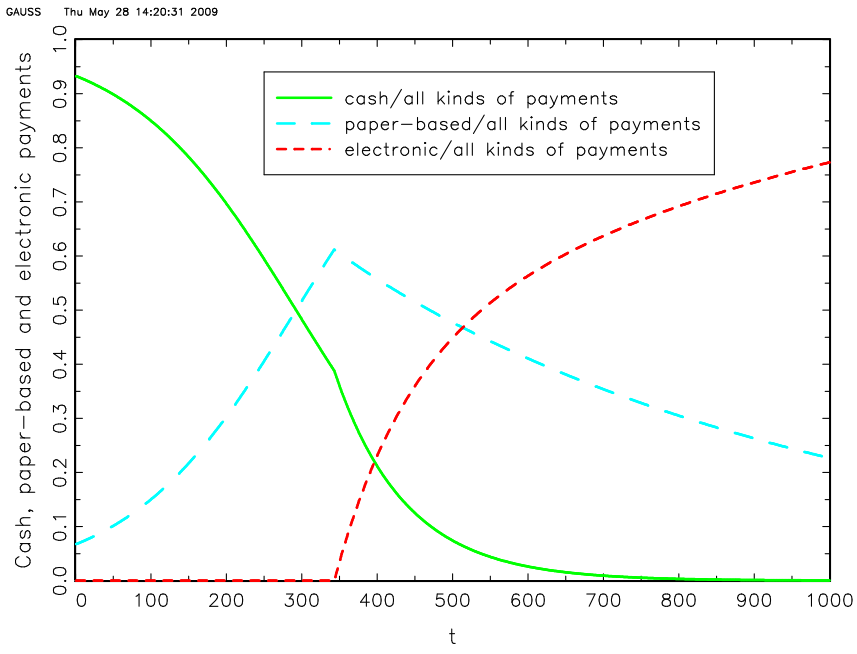
In order to illustrate the behavior of the economy in the transition in more detail, we will use a numerical example. Because we know the long run characteristics, we will use backward induction to solve the difference equation in human capital (50). We assume that for some high enough level of human capital, say $h_{T+1} \approx 10^{13}$, the economy is on the balanced growth path, $\frac{h_{T+2}}{h_{T+1}} = \lim_{t\to\infty}\left(\frac{h_{t+1}}{h_t}\right) = \beta\left(\phi+1-\delta\right)^{\frac{1}{\theta}}$. Then we apply the Newton Raphson method to find h_t knowing h_{t+1} and h_{t+2} for all t. In this way we obtain a numerical policy function $h_{t+1} = H(h_t, R)$. Given the initial level of human capital and the policy function, the behavior of all other variables in the economy can be calculated from the equations (29)-(38) and (46)-(47).

4.1.2 Calibration

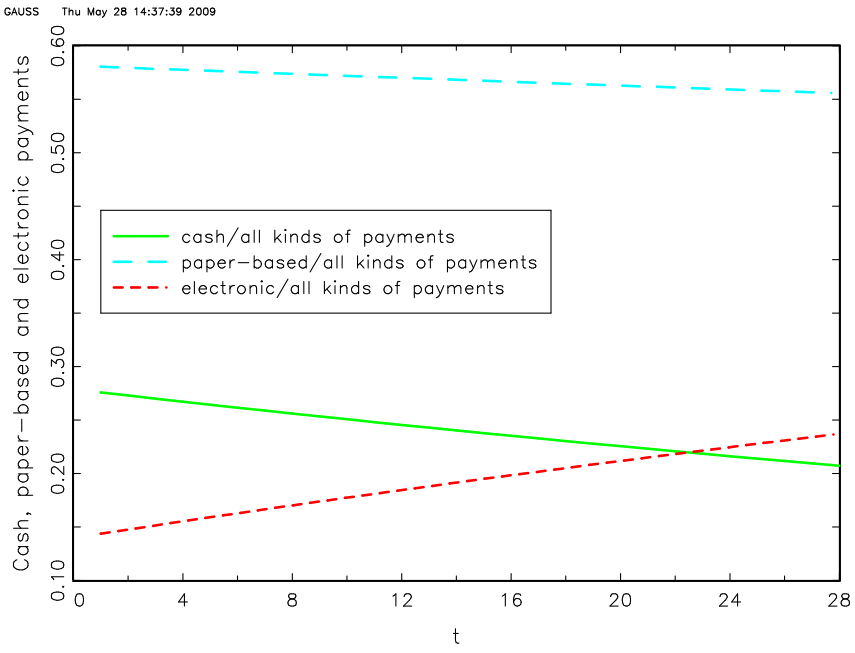
We calibrate the model to the yearly US data, period 1975-2000. We will consider that one period in the model corresponds to one year. We set the long run growth rate of the economy to $g^* = 1.015$, the growth rate of output per worker found in the data. The baseline monetary policy is set to $R = 1.07$, as implied by the average value of 1 year treasury bill rate. The depreciation of human capital will be set to $\delta = 0.025$ as in Gillman, Kejak, Valentinyi (1999). The discount factor in the utility function $\beta = 0.995$. Marginal product of capital in goods production $w = 1$. Other parameters are set to match as closely as possible the average gross inflation rate, $\pi^{\text{data}} = 1.046$ and the average gross growth rate of money supply in the data, $\mu^{\text{data}} = 1.055$. We set the efficiency of learning to $\phi = 0.05$, which implies that on the balanced growth path 20% of time is devoted to working and the rest to studying. The degree of knowledge diffusion into the paper-based and electronic payment is set to $\alpha_\varphi = 0.6$ and $\alpha_e = 0.7$. These values as set in accordance with the growth rates of checks and cash found in the data. The relative cost between electronic and paper-based payments is set to $F = 1$. It leads to the initial ratio of paper-based to electronic payments as found in the data. The inverse of the intertemporal elasticity of substitution is endogenous, $\theta = 1.3$. The mentioned parametrization implies $\pi^{\text{model}} = 1.044$ and $\mu^{\text{model}} = 1.049$.

4.2 The dynamics of the payment instrument choice

We simulate the behavior of the economy using the solution of the above mentioned second order difference equation. In Figure 2 we plot the relative usage of each instrument in the course of time. Figure 2a) shows the relative choice of payment instruments over a very long period of time. The initial value of human capital in that case is $h_0 = 1$. We begin at a very low stage of development during which only two means of payments coexist, cash and paper-based ones. With the development of new technologies paper-based payments become relatively cheaper and the fraction of markets where this payment method is chosen increases, meanwhile the fraction of markets where cash is employed decreases.



(a)



(b)

Figure 2: Evolution of the fraction of cash, paper-based and electronic payments as a percentage of all kinds of transactions for $\alpha_\varphi = 0.6$, $\alpha_e = 0.7$ and $F = 1$. (a) Behavior of the economy over 1000 years, $h_0 = 1$; (b) Behavior of the model economy which corresponds to the US, 1975-2000, $h_0 = 275$.

Electronic payments emerge at a certain level of technology (in this particular case displayed in Figure 2, $h_{343} = 172$). Since that point agents can choose from three payment instruments. As the cost of electronic payments becomes cheaper and cheaper with the development of new technologies, paper-based instruments and government money are driven away from the economy. However, the process of transformation of the payment system is rather slow. In Figure 2b), which is a detail picked from Figure 2a), we can find similar behavior as observed in the US in the period 1975-2000 (Figure 2 in Humphrey (2004), depicted in the introduction). In the electronic payments age, the fraction of markets where cash and paper-based payments are employed is shrinking, but it does not mean that non-electronic payments are not growing any more. For $\alpha_e < 1$ and $\alpha_e - \alpha_\varphi < 1$, cash and paper-based credit consumptions both grow at positive rates, as well as the intermediation payments for the non-cash transactions, see Figure 3. We also plot the evolution of human capital. We can see that its growth rate does not vary much over time. Figure 3 helps us to see that in the long run the growth rate of electronic credit consumption converges to the one of human capital, and the growth rate of cash consumption converges to the growth rate of intermediation payments for electronic transactions.[6]

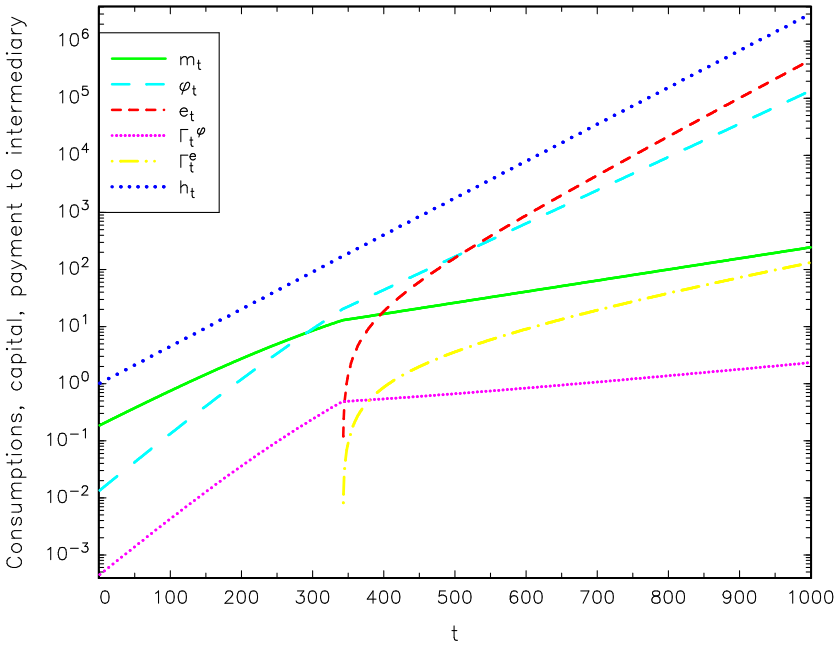


Figure 3: Evolution of the cash consumption, paper-based credit consumption, electronic credit consumption, payments to intermediary for both non-cash transactions and human capital for $\alpha_\varphi = 0.6$, $\alpha_e = 0.7$ and $F = 1$.

In Figure 4 we plot the relationship between the ratio of cash/all instruments and electronic/non-cash instruments. The arrows show the transformation that the payment system undergoes in 25 years period of time. We compare the results delivered by the model with the US data.

[6] This is a result found analyticaly for the balanced growth path.

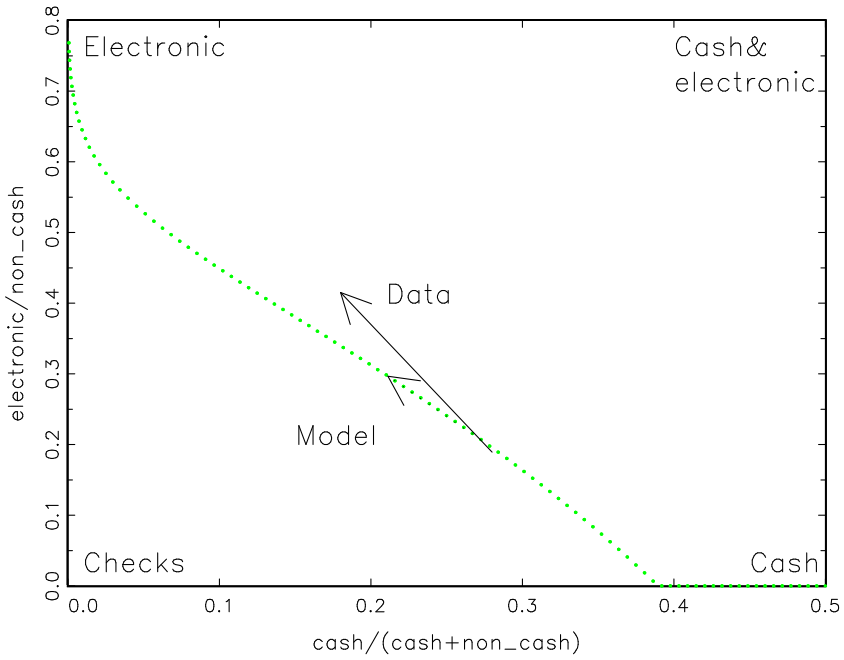


Figure 4: Comparison of the transformation of the payment system in the model and the US data, baseline parameters.

4.2.1 Effects of monetary policy

We can illustrate the effect of a variation of the nominal interest rate, R, in Figure 1. Imagine that the opportunity cost of holding money increases. The 'cash line' in Figure 1 moves up. Agents will want to economize on their money holding, substituting away from money. In the low stage of development the fraction of markets where paper-based payment instruments are used increases and where cash is employed decreases, i.e. $\frac{d\bar{\chi}_t}{dR} > 0$. On the higher level of development paper-based payments will be used on the same fraction of markets as under low nominal interest rate, because the cutoff market χ_t is not affected directly by the monetary policy, see also the equation (31). This means that the shift away from cash will go towards electronic credit consumption. The behavior of the economy under different monetary policies is illustrated in Figure 5. In Figure 5b) we show the dynamics of the relative usage of each payment instrument for two different monetary policies described above.

The emergence of electronic payments is accompanied by an increase in the growth rate of the economy. As more resources are channeled (unproductively) to the financial intermediary, agents want to compensate for the loss by working more (and studying less). The change in the working time is relatively higher that the change in the accumulation of human capital, and it implies a jump in the growth rate of the economy, see Figure 5c).

For a given level of technology, higher nominal interest rate implies less cash transactions. The arrows in Figure 5a) all begin at the same level of human capital. The higher is the opportunity cost of holding money, the higher is the initial grade of substituting away from cash. Nevertheless, the further transformation (over the next 25 years) is slowing down with inflation (and higher interest rate). It is a kind of convergence behavior. The higher is the initial distance of a country from its

balanced growth path (the balanced growth path transformation of the payment system), the more rapid is the convergence towards it.

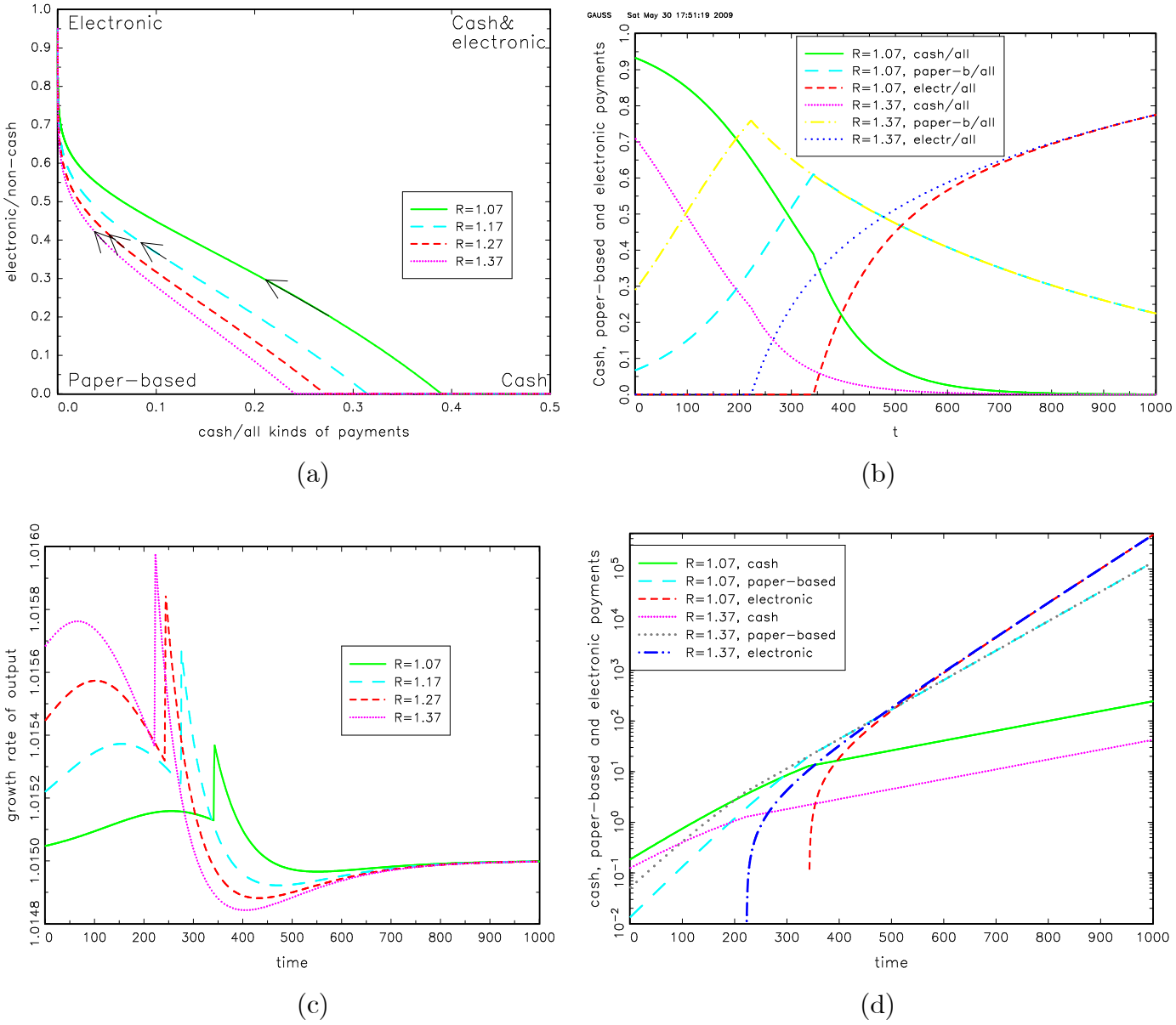


Figure 5: Behavior of the model under different monetary policies for baseline parameters; $R = 1.07$, 1.17, 1.27 and 1.37 correspond to the inflation rate of 4.4%, 14%, 24% and 34% respectively; (a) Substitution of cash by paper-based and electronic payments; (b) Evolution of the fraction of cash, paper-based and electronic payments as a percentage of all kinds of transactions; (c) Evolution of the growth rate of the economy; (d) Evolution of cash, paper-based and electronic payments.

In Figure 5d) we can see that the long run level of cash consumption is affected by the opportunity cost of holding money, as implied also by the equation (33) and (25). The initial dependence of paper-based and electronic credit consumption on R tends to disappear in the long run, as the fraction of

markets where electronic transactions are employed increases to 1, $\lim_{t\to\infty}\chi_t = 1$, and the fraction of markets where paper-based transactions are employed decreases to 0, $\lim_{t\to\infty} s_t = 0$ for all R.

4.2.2 Effects of government incentives to a usage of electronic means of payments

The effect of a decrease in the relative cost for electronic and paper-based payments, F, can be again deduced from Figure 1. Moving the 'electronic payments line' downward we can see that $\frac{ds_t}{dR} > 0$ and $\frac{d\bar{\chi}_t}{dR} < 0$. Electronic payments become relatively cheaper and it will be profitable for agents to choose this instrument at lower level of development. The substitution of cash and paper-based transactions by electronic ones will begin sooner. The behavior of the economy for different levels of fixed cost is illustrated in Figure 6. We can see that the initial position of a country and the speed of transformation can be affected in an important way by the relative cost of electronic and paper-based instruments, see the arrows in Figure 6a).[7] If the government wanted to alter the agents payment instrument choice, it could subsidize the fixed cost for electronic transactions. Similarly to the previous case, the emergence of the electronic transactions results in an increase of the growth rate of the economy. The long run level of cash transactions will not be affected. Since the emergence of electronic instruments, the level of paper-based transactions is lower due to lower relative cost. This difference persists in the long run. The long run level of electronic transactions converges for different fixed costs, as all initial differences disappear, $\lim_{t\to\infty}\chi_t = 1$ and $\lim_{t\to\infty} s_t = 0$ for all F.

5 Conclusions

This paper tries to improve previous literature in several ways. First it helps in covering the need for theoretical works by presenting a general equilibrium model on payment choice at a retail level. Second, it also provides theoretical foundations to reconcile previous empirical evidence on payment choice among cash, electronic and paper-based instruments. Payment structure of a given country can be shaped by the interest rate, the degree of technology development and the cost of each payment instrument. And third, the model is able to explain why countries with similar payment alternatives and consumer needs for payment instruments end up with very different usage of these mechanisms.

[7] Again, all arrows begin at the same level of human capital and show the changes in the payment choice decisions over 25 years period.

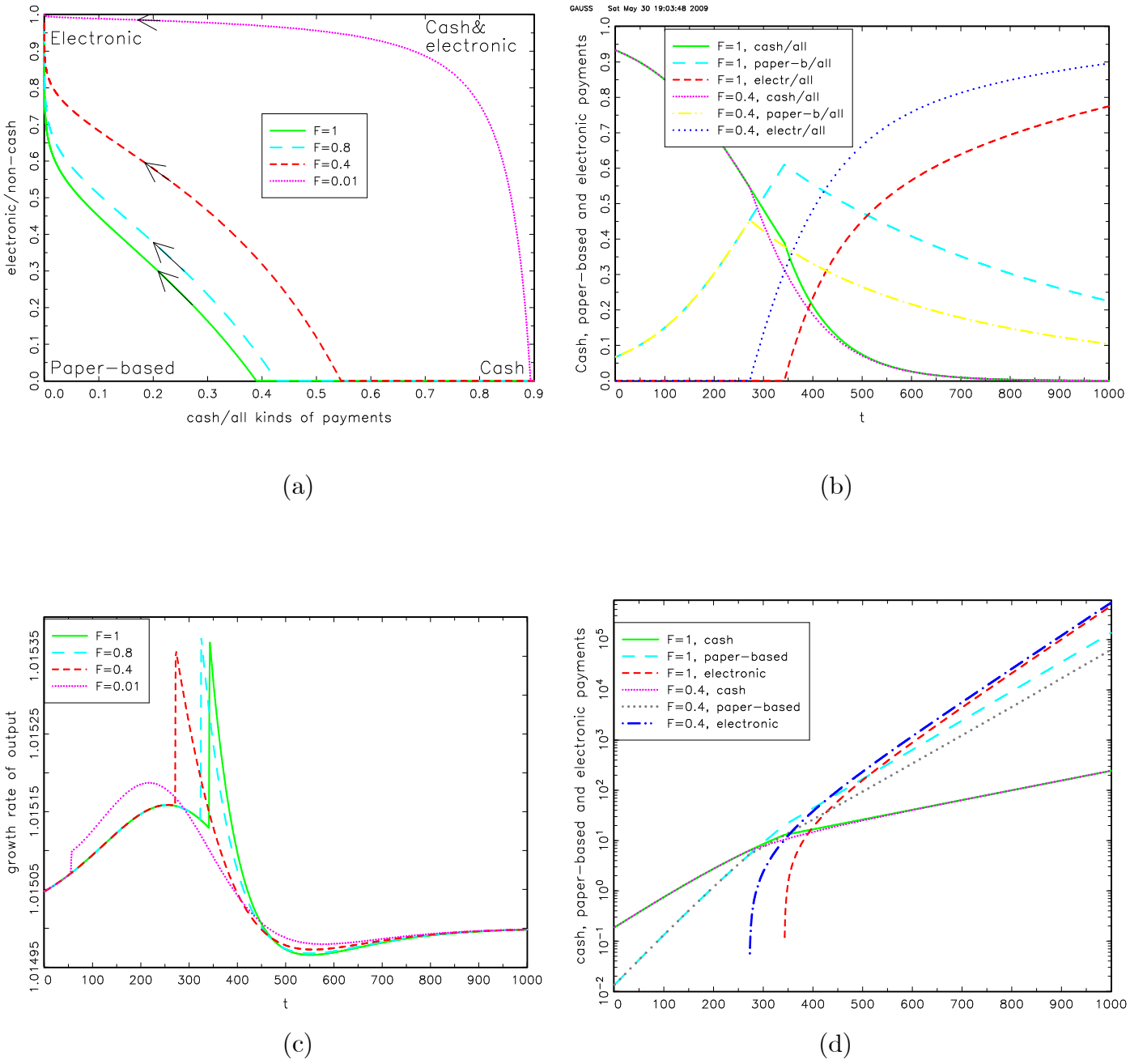


Figure 6: Behavior of the model under different fixed costs in the intermediation function for electronic payments, $F = 1,\ 0.8,\ 0.4$ and 0.01, for baseline parameters; (a) Substitution of cash by paper-based and electronic payments; (b) Evolution of the fraction of cash, paper-based and electronic payments as a percentage of all kinds of transactions; (c) Evolution of the growth rate of the economy; (d) Evolution of cash, paper-based and electronic payments.

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