Time to Consume, Quality, and Growth

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Abstract

Consumption requires time, while higher-quality goods provide more utility per unit of time. Thus, since time is limited, higher income is increasingly spent upgrading consumption quality instead of raising quantities. After analyzing consumer quantity/quality choices, the paper investigates growth implications. As a country develops, quality growth becomes increasingly important as a component of GDP growth. Technical progress is quality-biased. Lower income inequality and luxury consumption taxes raise the scale of output while reducing average quality. This is positive for growth at early stages of development (but may be negative at later stages). Results are broadly consistent with evidence on GDP growth composition, trade patterns of vertical specialization, and the impact of inequality on growth.

Keywords: Time Allocation, Product Quality, Growth, Inequality, Progressive Consumption Taxes. **JEL Classification:** O11, O15, D11.

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1 Introduction

For most consumption goods, production and expenditure can be seen as involving two dimensions: quantity and quality. For example, car companies may increase their output by increasing the number of cars they produce or by raising their quality, consumers may increase their expenditure in restaurants by dining out more often or by going to better restaurants, etc. Most macroeconomic models do not distinguish between these two dimensions. However, the distinction is increasingly important as suggested by the evidence presented below. This paper provides a new simple microeconomic analysis of consumer quantity/quality choices and then explores some important macroeconomic implications for growth. In this second respect, the paper considers four issues: the share of quality growth as component of GDP growth; the quality bias of technical progress; the potential of progressive consumption taxes as a growth policy; and the effect of inequality on growth through the quantity/quality channel. These issues are analyzed within a simple, tractable model.

Although measuring quality is admittedly a difficult endeavor, the importance of quality growth as a component of GDP growth is becoming apparent. Bils and Klenow (2001) estimate that annual quality growth averaged 3.7 percent for consumer durable goods in the US over 1980-1996 (about 60 percent of this growth would be wrongly accounted for as inflation by the Bureau of Labor Statistics, BLS). This growth would be even larger for a more recent period. Bils (2005) and (2009) estimates an annual quality growth for durables of about 5 percent since 1988. Additional calculations suggest that quality growth could account for a large portion of GDP growth. Moulton and Moses (1997) use data for 1995 and estimate that BLS methods may account for as much as one percent quality growth that year. To this one percent we must add unmeasured quality growth. The Boskin Commission Report (1996) argued that unmeasured quality change was the most important source of CPI upwards bias in the US and would be responsible for an approximate effect of 0.6 percent per year.[1] However, backwards extrapolation of recent estimations of CPI bias appears to be unrealistic (Gordon 2005), so that the importance of quality growth as a component of

[1] The literature on the upward bias in the measurement of CPI inflation due to unmeasured quality growth is extensive. For example, Gordon (2006) estimates that, even after the thorough methodological revisions recently put in place by the BLS, there is still an upward bias in the measurement of CPI annual inflation of at least one percentage point, which is due to a large extent to unmeasured quality upgrading. See Bils and Klenow (2001) and Bils (2005) for further references.

GDP growth would have increased over time.

Recent international trade literature provides evidence of the increasing importance of distinguishing the quantity and quality dimensions. In fact, this distinction has become essential in describing current patterns and consequences of international trade. On the demand side, trade is characterized by richer countries importing relatively more from countries that produce higher-quality goods (Hallak 2006). Along the same line, country pairs with similar income distributions tend to show similar quality distributions of imports (Choi, Hummels and Xiang 2009). This is consistent with the positive relationship between income and the consumption of higher quality. On the supply side, horizontal country specialization across goods is losing importance relative to vertical (quality) specialization within goods. That is, both low- and high-wage countries are increasingly exporting the same kinds of goods though of different qualities. Moreover, richer countries tend to export higher quality within each good (Schott 2004; Hummels and Klenow 2005).[2] In other words, richer countries have a comparative advantage in producing higher quality.

The starting point of this paper is the analysis for consumer quantity/quality choices, which can be outlined as follows. Consumption requires time. Individuals need time to listen to a concert, travel for pleasure, or play with a video game. Since time is limited, this reduces the possibility of increasing utility by increasing the quantity of consumption (i.e., by increasing the number of units of the goods being consumed). In turn, higher-quality goods provide higher utility per unit of time allocated to consumption though at a higher monetary cost. The consequence of this is that higher income is decreasingly spent augmenting the quantity of consumption and increasingly spent upgrading its quality.[3,4] Most

[2] Furthermore, Khandelwal (2009) shows that the ability of the US to maintain employment and output in face of competition from low-wage countries is increasing in the importance of the quality dimension in each industry.

[3] In other words, obtaining utility through consumption has both a time and a monetary cost. Higher-quality goods provide utility at higher monetary cost but cheaper time cost. Thus, richer individuals consume higher quality.

[4] Some durable goods such as washing machines and vacuum cleaners save time for individuals. However, these goods are better seen as capital goods used for household production instead of as consumption goods (e.g., people enjoy the cleanliness produced by a vacuum but do not tend to enjoy spending time vacuuming the house). See for example the theoretical and quantitative analysis in Greenwood, Seshadri and Yorukoglu (2005). The concept of household production was introduced by Becker (1965). The allocation of time to different types of consumption, leisure, and home production have been investigated by Mark Aguair and

models considering the quality dimension of goods typically assume a positive relationship between income and the demand for quality. However, this positive relationship is taken as a fact without exploring the underlying reasons. This paper shows that the positive relationship between income and the demand for quality can be explained as a consequence of the time constraint and the complementarity between consumption and time.[5]

Then, the paper explores the growth implications of this analysis of consumer quantity/quality decisions. The first natural implication is that, as a country develops, the quality growth component of GDP growth becomes increasingly larger relative to the quantity growth component. This is consistent with the estimates of the great importance of quality growth in recent times cited above and with the unlikeliness that those estimates could be extrapolated backwards.

Endogenous growth is introduced in the model by assuming a two-dimensional learning-by-doing process. Note that most learning processes tend to have a double dimension: individuals and firms simultaneously learn to perform tasks *faster* and *with more precision and quality*. However, the relative importance of dynamic productivity gains in each of these two dimensions may vary across time. In the model, both labor efficiency and learning have two parameters, each linked to one of the two dimensions of production (quantity and quality). In this context, the model predicts that technical progress is increasingly biased in favor of reducing the relative cost of producing higher quality. This prediction is consistent with the cited international trade evidence showing that richer countries have a comparative advantage in producing higher-quality goods. It is also shown that the model can be calibrated to produce reasonable long run paths of GDP growth. Nonetheless, the simple endogenous growth model built in this paper seems to overestimate the quality component of GDP growth. Thus, an important task for future research is to build and calibrate more flexible models that can do a better job at matching the quantity and quality components

Erik Hurst in a series of papers (see, for example, Aguair and Hurst 2007 and 2008).

[5] The complementarity between consumption and time was pointed out long time ago by Becker (1965). However, the literature has paid little attention to the implications of this complementarity. Recent important exceptions are Goolsbee and Klenow (2006), and Hall and Jones (2007). The underlying key argument in this second paper is closer to the one in the present paper: since individuals marginal utility of per-period consumption falls quickly, as individuals get richer they spend an increasing share of income on extending life expectancy. Still, the connection between the time constraint and the demand for higher-quality goods, which is the starting point in this paper, seems to have been completely neglected in the literature.

of GDP growth, as more thorough estimates of these components become available.

The model has sharp and intuitively appealing implications for policy and for the growth impact of income inequality. The model predicts that the quantity/quality composition of output affects the intensity and quality bias of technical progress and, therefore, affects growth. The quantity/quality composition of output is determined by consumer choices which in turn can be influenced by progressive consumption taxes. As a result, progressive consumption taxes can be used as a growth policy. It is then shown that growth can be enhanced at early stages of development (when efficiency and production are small) by shifting the quantity/quality composition of GDP in favor of a larger scale. The reason is that learning in both the quantity and the quality dimensions of efficiency go to zero as the scale of production goes to zero. Charging higher taxes on higher-quality goods helps increase production scale and heightens growth at early stages. Thus, the model provides an argument for taxing luxury goods in developing countries. However, the positive impact of progressive consumption taxes can change to negative at later stages of development. The reason is that at later stages, output has already reached a large scale whereas high efficiency at producing high quality becomes more valuable as consumers get richer.

Finally, inequality also affects growth through a quality channel. The mechanism is similar to the one for progressive consumption taxes. Since preferences are non-homothetic with respect to quantity/quality pairs, income distribution affects the composition of output. In turn, the composition of output affects the intensity and quality bias of technical progress. The model predicts that higher inequality hinders growth at early stages of development. The reason is that higher inequality raises average quality of GDP but reduces the volume of output. And, as already noted, reaching a large scale of output is more effective to foster technical progress at early stages of development, than producing high quality. Nonetheless, again, the sign of the inequality-growth relationship can change at more advanced stages of development as quality-biased technical progress becomes more valuable.

The actual impact of inequality on growth is still a debated empirical issue. However, the predictions here are consistent with most of the recent empirical evidence, as discussed in more detail in the corresponding section below. The mechanism analyzed in this paper is complementary to other mechanisms previously analyzed in the literature. Hence, from the empirical point of view, we are interested in checking the relationship between inequality and growth after controlling for those other mechanisms. The analysis in Barro (2000) and (2008) serves this purpose. After controlling for the variables involved in most of the mechanisms previously suggested in the literature (e.g., schooling, investment ratio, rule of law), Barro still finds a significant non-linear impact of inequality and growth. Specifically, he finds a negative impact of inequality on growth among the group of less developed countries, which vanishes or even turns into a positive impact among the group of richer countries. This coincides with the predictions in this paper. Also, there is some scattered historical evidence supporting a favorable contribution of lower income inequality to early industrialization, although its quantitative importance is unclear.[6]

The paper is organized as follows. The next section reviews some additional related literature on growth. Section 3 lays down the partial equilibrium analysis of consumer quantity/quality decisions. Section 4 embeds this analysis into an exogenous technical-change growth model. It explores the quantity/quality composition of GDP growth along different stages of development. Technical change is endogenized in Section 5 assuming a two-dimensional learning-by-doing process in a model where technical progress can be quality biased. Section 6 explores the potential role of progressive consumption taxes as a growth policy. Section 7 introduces agent heterogeneity to study how income inequality affects growth through the output composition mechanism. Section 8 summarizes and concludes.

[6]More unequal economies would spend a larger share of income in artisan production of luxury goods consumed by the elite, which has a limited scope for productivity gains. In contrast, more equal distributions of income would be more favorable to the development of industrial mass production (of lower-quality goods). For example, the United States developed a larger spectrum of mass-production industries than England (where production was often oriented towards higher quality) in the first half of the nineteenth century. This seemingly was the consequence of a large demand by a wide number of middle class farmers and was at the root of its economic superiority in the twentieth century (Rosenberg 1972). The data by Milanovic, Lindert and Williamson (2008) are also consistent with the first industrial countries having relatively low *inequality extraction ratios* at the time of industrialization (the inequality extraction ratio is a measure of actual inequality with respect to the maximum feasible inequality given the economy's resources). The model in this paper captures in a simple way the positive relationship at early stages of development between more equality, mass production, and fast increase of productivity; as opposed to high income inequality, a large share of output oriented towards small scale (artisan) production of high quality goods for the elite, and slow increase of productivity.

2 Related Growth Literature

This paper can be framed within the related growth literature as follows. Economies expand their per capita consumption as they get richer by consuming more units of each good (the intensive margin), a wider set of goods (the extensive margin), and higher average quality (the quality margin). Most of the growth literature focuses on the intensive margin. Grossman and Helpman (1991) and Greenwood and Uysal (2005), among others, have analyzed the extensive margin from different perspectives. This paper focuses on the quality margin in conjunction with the intensive margin.

There is some important work related to the quality margin. Grossman and Helpman (1991) analyze quality-improving innovations across the existing set of consumption goods. However, different qualities are assumed to be perfect substitutes. Hence all individuals consume the cheapest quality-adjusted good and there is no room for a relationship between income and the demand for quality. The closest growth model with quality differentiated consumption goods is Stokey (1988). In Stokey (1988), as in this paper, consumption expands in both the quantity and the quality dimensions, and a positive relationship between income and the quality of consumption is obtained. Nonetheless, both the formulation and the applications here are different. A primary difference is that preferences in Stokey (1988) do not have enough structure to generate a specific pattern of the quantity/quality composition of growth. That is, output quality is increasing over time, but the importance of quality growth as a component of GDP growth could be decreasing. The time-to-consume constraint considered here introduces such a structure. There is also an emphasis in this paper on building a very tractable model that is easily amenable to quantitative analysis.

Demand non-homotheticities create a link between income distribution and the composition of output, which in turn may affect technical progress and growth. There is a string of literature exploring these links to which the analysis in Section 7 of this paper is also related. Murphy, Shleifer and Vishny (1989) is a static model where lower inequality may help overcome the indivisibilities involved in modern industrial technologies. Zweimüller (2000), Matsuyama (2002), and Foellmi and Zweimüller (2006) are closer to this paper. They build dynamic models where income distribution affects efficiency and technical progress. However, their setting is very different from the one here. All these papers assume hierarchical preferences across a growing set of goods. Each individual consumes one or zero units of each good, and richer individuals consume all the goods that are consumed by poorer individuals, plus some additional goods. Hence these papers focus on the extensive margin of consumption. The model in this paper is instead set in a simpler quantity/quality space of preferences and a two-dimensional technology space. These differences bring about a distinctive mechanism and specific results about the interaction between inequality and growth.[7]

3 Time to Consume and the Demand of Quality

This section analyzes consumer quantity/quality choices assuming that consumption requires time. It develops the basic argument in a simple form that can be useful for growth analysis in the remaining sections. In order to clarify the assumptions, it is convenient to first consider an economy where there is only a single quality variety of the good.

3.1 One Good and a Single Quality

There is a single good which is produced in a single quality variety. Utility depends on the number of units x being consumed and the time allocated to their consumption. Allocating more time to consuming a given unit of the good increases the utility it provides. It is assumed that the individual allocates consumption time evenly across all the units being consumed. The amount of time per unit of consumption is denoted by ω.[8] Total time allocated to consumption is assumed to be exogenous and normalized to be 1.[9] Consumer

[7] Two other related papers are the following. Mani (2001) investigates how the interaction between inequality and output composition affects human capital accumulation. Zweimüller and Brunner (2005) consider an economy with a quality-differentiated good and a non-differentiated good in order to analyze how income distribution affects the intensity of innovative activities in the quality-differentiated good. Depending on the equilibrium regime, the three classes of consumers (poor, middle class, and rich) may or may not consume the same quality of the differentiated good; and lower inequality may have a positive effect of innovation. See Foellmi and Zweimüller (2006) for further references on related literature.

[8] This assumption could be derived from a framework where the consumer chooses how much time to allocate to the consumption of each unit of the good, and where the marginal utility of the time allocated to each unit is decreasing.

[9] The complementarity between consumption and time has important implications for the allocation of time between work and consumption (and leisure). However, the main points in this paper can be made without considering individuals' labor supply decisions. Hence, I simplify on this. See Alcalá (2009) for an analysis of labor supply with consumption and time complementarity as well as quality differentiation.

utility is given by the following C^2 function:

$$W = W(x, \omega); \tag{1}$$

$$\lim_{x \to 0, \omega \to \infty} W = 0, \quad W_x \geq 0, \quad W_\omega \geq 0;$$

where subscripts on W indicate partial derivatives. Clearly, $\lim_{x \to 0, \omega \to \infty} W = 0$ above is just a normalization. Consumer maximizes (1) subject to income and time-to-consume constraints:

$$x \leq y, \tag{2a}$$

$$1 = x \cdot \omega, \tag{2b}$$

$$x \geq 0, \quad \omega \geq 0; \tag{2c}$$

where the price of the good is normalized to be 1 and y denotes income.

The analysis will require some additional assumptions. As in Hall and Jones (2007), it is convenient to set them in terms of the elasticities of utility, which in this model are $\eta_x(x, \omega) \equiv \frac{x}{W} W_x$ and $\eta_\omega(x, \omega) \equiv \frac{\omega}{W} W_\omega$. I will consider the following

Assumption 1 $\lim_{x \to 0, \omega \to \infty} \eta_x(x, \omega) > 0$ *and* $\lim_{x \to 0, \omega \to \infty} \eta_\omega(x, \omega) = 0$. *Also,* $\frac{\partial \eta_x(x,\omega)}{\partial x} \leq 0$, $\frac{\partial \eta_\omega(x,\omega)}{\partial \omega} \leq 0$, $\frac{\partial \eta_\omega(x,\omega)}{\partial x} \geq 0$, *and* $\frac{\partial \eta_x(x,\omega)}{\partial \omega} \geq 0$, *with at least one strict inequality.*

The assumption $\lim_{x \to 0, \omega \to \infty} \eta_\omega(x, \omega) = 0$ implies that increasing per-unit time-to-consume is useless when the individual has nothing to consume. In turn, $\frac{\partial \eta_\omega(x,\omega)}{\partial x} \geq 0$ and $\frac{\partial \eta_x(x,\omega)}{\partial \omega} \geq 0$ convey the hypothesis of complementarity between time and consumption. Note that the time constraint implies $\frac{d\omega}{dx} = -1/x^2$. Taking this into account, we have $\frac{dW}{dx} \frac{x}{W} \equiv \eta_W(x) = \eta_x(x, \omega) - \eta_\omega(x, \omega)$. Hence, Assumption 1 implies $\lim_{x \to 0, \omega \to \infty} \eta_W(x) > 0$ and $\frac{d\eta_W(x)}{dx} < 0$, which will be useful below.[10]

If consumption requires time, it also seems reasonable to assume that there is a limit to the number of goods that individuals can enjoy per unit of time.[11] If time and consumption are perfect complements (for example, watching movies), individuals may not have the time

[10]Note that sufficient conditions for $\frac{\partial \eta_x(x,\omega)}{\partial x} \leq 0$ and $\frac{\partial \eta_\omega(x,\omega)}{\partial \omega} \leq 0$ are $\rho_x \equiv -x W_{xx}/W_x \geq 1$ and $\rho_\omega \equiv -\omega W_{\omega\omega}/W_\omega \geq 1$, respectively.

[11]It may be argued that consumers can enjoy different goods at the same time. For example, one can simultaneously enjoy wearing nice shoes, listening to music, driving a comfortable car, etc. However, at every point in time, the existing number of different types of goods is finite. One can then make the argument taking as the unit of consumption the whole finite bundle of goods that can be consumed simultaneously.

to consume more units even if they would like to do so should they have more time. Or, more generally, if time and consumption are only imperfect complements, total utility may decrease if a limited amount of time is allocated to the consumption of too many units of the good.[12] In other words, for a quantity of consumption x sufficiently large, the time allocated to consuming each unit would be so small that the impact on utility of further increasing the quantity of consumption (and therefore, further reducing the time to consume each unit) would be zero or negative. Any of the following assumptions will be sufficient for the results that follow.

Assumption 2a $\lim_{x\to\infty,\omega\to 0}\eta_x(x,\omega)\leq 0$ *and* $\lim_{x\to\infty,\omega\to 0}\eta_\omega(x,\omega)>0$.

Assumption 2b $\lim_{x\to\infty,\omega\to 0}\eta_x(x,\omega)<\infty$ *and* $\lim_{x\to\infty,\omega\to 0}\eta_\omega(x,\omega)=\infty$.

The key implication of any of the two assumptions is that $\lim_{x\to\infty,\omega\to 0}\left[\eta_x(x,\omega)-\eta_\omega(x,\omega)\right]<0$. Hence, since $\lim_{x\to 0,\omega\to\infty}\left[\eta_x(x,\omega)-\eta_\omega(x,\omega)\right]>0$, by continuity, there exists $\overline{x}$, $0<\overline{x}<\infty$, such that $\eta_x(\overline{x},\frac{1}{\overline{x}})-\eta_\omega(\overline{x},\frac{1}{\overline{x}})=0$. Then, note that $\frac{dW(\overline{x},\frac{1}{\overline{x}})}{dx}=0$. Therefore, if Assumptions 2a or 2b hold, utility maximization implies $\lim_{y\to\infty}x^*(y)=\overline{x}$. Thus, as a result of the complementarity between consumption and time, individuals may seem satiated (in the sense that they stop consuming more units even if their income constraint is not binding) even if preferences do not display any satiation point; i.e., even if $W_x(x,\omega)>0$, for any $x>0$ and $\omega>0$.

Recall that the arguments in this subsection are made for an economy where individuals do not have the possibility of increasing the quality of the goods they consume. In the next subsection, it is shown that consumers' response to the time-to-consume constraint is to spend an increasing share of their income upgrading the quality of consumption instead of increasing its quantity.

3.2 A Continuum of Qualities

Let us now introduce quality. There is a single good which can be consumed along a continuum of quality varieties $q\in[0,\infty)$. Utility depends on the number of units x being

[12] For example, individuals may not feel satiated by the number of books they read or the cities they visit. However, they may prefer taking more time to read one book and visit one city rather than rushing to read more books and visit more cities.

consumed, their quality q, and per-unit of consumption time ω. Higher-quality varieties provide more utility than lower-quality varieties when allocating the same amount of time to their consumption. I simplify by considering a separable utility function of the form:

$$U(x,\omega,q) = W(x,\omega)\cdot V(q); \tag{3}$$
$$V(0)=0, \quad V_q>0, \quad V_{qq}\le 0;$$

where $W(x,\omega)$ is the same function as in the previous subsection. As before, the time allocated to consumption is taken as exogenous and normalized to be 1. Individuals maximize (3) subject to

$$x\cdot p(q)\le y, \tag{4a}$$
$$x\cdot\omega = 1; \tag{4b}$$
$$x\ge 0, \quad \omega\ge 0; \tag{4c}$$

where $p(q)$ is price as a function of quality. Since marginal utility of quality is positive, the budget constraint is always satisfied with equality. Utility maximization yields the following first order condition:

$$\eta_x(x,\omega)-\eta_\omega(x,\omega)=\frac{\eta_V(q)}{\eta_p(q)}; \tag{5}$$

where $\eta_V(q)\equiv\frac{q}{U}U_q=\frac{q}{V}V_q$ and $\eta_p(q)\equiv\frac{\partial p}{\partial q}\frac{q}{p}$.

Note that quality has no natural units of measurement. It is only a preference order over the different varieties of the same good, such that if two different varieties have quality indexes q' and q'', then $q'>q''$ means that one unit of variety q' is always preferred to one unit of q''. Thus, quality indexes may be rescaled using an increasing transformation to obtain a simple pattern of prices as a function of qualities. This facilitates the exposition. Specifically, by an appropriate relabelling of qualities and without loss of generality, we can assume the following pattern of prices:[13]

$$p(q)=e^{q/\gamma}; \quad \gamma>0. \tag{6}$$

[13]To see that this, consider the original set of quality varieties to be $\{Q\}=[0,\infty)$. Quality varieties are assumed to be indexed (labelled) according to an increasing preference order such that if $q'>q''$, then one additional unit of variety q' is always preferred to one additional unit of q''. Let $P(Q)$ be the price of quality Q. $P(Q):[0,\infty)\to[1,\infty)$ is assumed to be continuous, increasing, differentiable, and satisfying $P(0)=1$. Relabel varieties as $q\in[0,\infty)$ using the one-to-one increasing mapping $\phi(Q)=q$ defined as $\phi(Q)\equiv\gamma\ln P(Q)$. Hence, $Q=P^{-1}(e^{q/\gamma})$. Therefore, the price of each relabelled quality is given by $p(q)\equiv P(P^{-1}(e^{q/\gamma}))=e^{q/\gamma}$. It may be noted that the parameter γ does not play any role in this section,

The function $V(q)$ has to be (re-)defined taking into account this relabelling of qualities. Using (6), expression (5) becomes:

$$\eta_x(x,\omega) - \eta_\omega(x,\omega) = \gamma \frac{V_q(q)}{V(q)}. \tag{7}$$

Substituting with (4b) into (7) implicitly defines a consumer-optimal mapping between x and q, $\psi(x) = q$. This mapping is a continuous function $\psi(x) : (0, \overline{x}) \to (0, \infty)$, if Assumption 2a or 2b holds or $\psi(x) : (0,\infty) \to (0,\infty)$ otherwise. Differentiating (7) with respect to x and q (after using (4b) to substitute for ω), and using Assumption 1 yields:

$$\begin{aligned} \frac{dq}{dx} &\equiv \psi' \\ &= \left[-\frac{\partial \eta_x(x,\omega)}{\partial x} + \frac{\partial \eta_x(x,\omega)}{\partial \omega}\frac{1}{x^2} - \frac{\partial \eta_\omega(x,\omega)}{\partial \omega}\frac{1}{x^2} + \frac{\partial \eta_\omega(x,\omega)}{\partial x} \right] \frac{1}{\gamma} \frac{V(q)}{V_q(q)^2/V(q) - V_{qq}(q)} > 0. \end{aligned} \tag{8}$$

The mapping $q = \psi(x)$ is depicted in Figure 1. In turn, given the individual's income $y > 0$, the frontier of consumer's attainable pairs (x, q) is obtained by substituting with (6) into the budget constraint (4a) with equality. This yields $q = \ln(y/x)$. The intersection of $q = \psi(x)$ and $q = \ln(y/x)$ determines the consumer's optimal pair (x^*, q^*) (see Figure 1). Clearly, for any $y > 0$ the two schedules cross in the positive quadrant. Moreover, higher y implies higher x^* and q^*. Hence, we have the following:

Proposition 1 *Let Assumption* 1 *hold. As income rises, the quality of consumption increases.*

As argued in the Introduction, quality growth seems to be an increasingly important component of GDP growth. The result in Proposition 1 is not enough to explain this. The following proposition conveys the stronger message that as income rises, the share of any income rise that is spent upgrading the quality of consumption is increasingly larger (i.e., $x\frac{\partial p}{\partial q}\frac{dq}{dy}$ is increasingly larger, whereas the share spent augmenting consumption quantity, $p(q)\frac{dx}{dy}$, is increasingly smaller; $x\frac{\partial p}{\partial q}\frac{dq}{dy} + p(q)\frac{dx}{dy} = 1$). Recall that Assumption 2a or 2b imply that there is an upper bound $\overline{x}$ to the optimal quantity of consumption. Hence any of these assumptions implies that, eventually, income increases are entirely spent upgrading the quality of consumption. See Figure 1 where Assumption 2a or 2b implies that $\psi(x)$ has an asymptote $\lim_{x \to \overline{x}} \psi(x) = \infty$.

so that we could just take $\gamma = 1$. However, it will play an important role in the endogenous growth model below so that it seems convenient to introduce it from the very beginning.

Proposition 2 *Let Assumption* 1 *and Assumption* 2a *or* 2b *hold. For any* s, $0 < s < 1$, *there is an income level* $y(s)$ *sufficiently large such that for* $y > y(s)$, *a share larger than* s *of any income rise is spent upgrading the quality of consumption.*

Proof. See Appendix. ■

4 The Quantity/Quality Composition of Growth

The partial equilibrium analysis of consumer decisions in the previous section has a straightforward implication for the dynamics of the quantity/quality composition of growth: quality growth is an increasingly important component of GDP growth. This section develops this implication and also sets the framework for the following sections.

4.1 Technology

There is a single representative agent and a single good that can be produced along a continuum of qualities $q \in [0, \infty)$. Labor is the only factor of production and there are constant returns to scale producing any quality. Producing higher quality requires more labor per unit of output. Output at time t, x_t, when producing quality q_t is given by

$$x_t = \frac{A_t L}{F(q_t)}.$$

where L is the labor input, A_t is a general efficiency parameter that evolves over time, and $F(q)$ is a continuous, differentiable, strictly increasing function function $F(q) : [0, \infty) \to [1, \infty)$ that satisfies $F(0) = 1$. Labor supply is assumed to be constant and is normalized $L = 1$. By an appropriate relabelling of qualities and without loss of generality, we can assume $F(q) = e^{q/\gamma}$. Hence, the production function becomes:[14]

$$x_t = \frac{A_t}{e^{q_t/\gamma}}. \tag{9}$$

Denote the wage at t by w_t. Assuming perfectly competitive markets, prices are

$$p_t(q) = \frac{e^{q/\gamma}}{A_t} w_t. \tag{10}$$

[14] To see this, consider the original set of quality varieties to be $\{Q\} = [0, \infty)$. Relabel quality varieties as $q \in [0, \infty)$ using the one-to-one increasing mapping $\varphi(Q) = q$ defined as $\varphi(Q) \equiv \gamma \ln F(Q_t)$. Hence, $Q = F^{-1}(e^{q/\gamma})$. Therefore, substituting in the production function yields $x_t = A_t L_t / F(Q_t) = A_t L_t / e^{q_t/\gamma}$. Flam and Helpman (1987) may have been the first paper using the production function in expression (9).

In this section, technical progress is exogenously given by the growth of A_t, which is assumed to be positive: $g(A_t) > 0$ for every t.

4.2 The composition of GDP growth

Consider the same consumer problem (3)-(4) as in the previous section, where prices are now given by (10). As in the previous section, from utility maximization we can obtain a consumer-optimal continuous, positively valued, and increasing mapping $q_t = \psi(x_t)$. Given A_t, expression (9) determines the production-possibility frontier in the quantity/quality space, whereas $q_t = \psi(x_t)$ determines consumers' choices among (x_t, q_t) pairs. See Figure 2, where $q_t = \psi(x_t)$ is drawn with an asymptote at $\overline{x}$ as implied by Assumption 2a or 2b. The intersection between (9) and $q_t = \psi(x_t)$ determines the instantaneous equilibrium pair (x_t, q_t) at time t. Clearly, for any $A_t > 0$ there is a unique pair $(x_t, q_t) > (0, 0)$ solving this system. Moreover, as A_t grows, x_t and q_t will increase. Eventually, there is no growth in x_t.

Denote GDP at time t at current prices by y_t:

$$y_t \equiv x_t \, p_t(q_t).$$

Therefore, GDP growth at constant prices for each quality variety $p_t(q)$ is given by $g(y_t) = g(x_t) + \frac{\partial p_t(q)/\partial q}{p_t(q)} \dot{q}_t$. Using $(\partial p_t/\partial q)/p_t = 1/\gamma$ from (10), yields:

$$g(y_t) = g(x_t) + \frac{q_t}{\gamma} g(q_t). \tag{11}$$

The shares of quantity and quality growth are, respectively, $g(x_t)/g(y_t)$ and $\frac{q_t}{\gamma} g(q_t)/g(y_t)$. Since $\lim_{A_t \to \infty} y_t = \infty$ and since Proposition 2 implies $\lim_{y_t \to \infty} \frac{(dx/x)}{(dy/y)} = 0$, under assumptions 1 and 2a or 2b we have $\lim_{A_t \to \infty} \frac{(dx/x)}{(dy/y)} = 0$. Therefore, $\lim_{A_t \to \infty} g(x_t)/g(y_t) = 0$. Hence, using continuity, we have the following:

Proposition 3 *Let Assumption* 1 *and Assumption* 2a *or* 2b *hold. For any* s, $0 < s < 1$, *there is a level* $A(s)$ *of development in terms of technical efficiency such that for* $A > A(s)$, *the share of quality growth in total GDP growth is larger than* s *(i.e.,* $\frac{q_t}{\gamma} g(q_t) > s \cdot g(y_t)$*).*

This result is consistent with the apparent increasing importance of quality growth as a component of GDP growth that was suggested in the Introduction.

5 Endogenous Growth with Quality-Biased Technical Progress

As noted in the Introduction, recent international trade literature shows that richer countries tend to have a comparative advantage in producing higher quality goods. Since richer countries have more advanced technologies, a potential explanation for this fact is that technical progress tends to be quality biased. That is, as a country develops, efficiency producing the higher qualities increases faster than efficiency producing the lower qualities. This section builds a model where technical progress shows this bias.

The model introduces endogenous growth in the simple form of learning by doing. Repetition of tasks improves skills and knowledge that in turn help both the speed and quality of work (i.e., experience helps produce goods faster and better). Hence technical progress comes along two dimensions: (i) general efficiency in producing any quality variety (which is captured in the production function by the variable A_t); and (ii) relative efficiency in producing higher qualities (which is captured in the production function by the variable γ_t). It is assumed that, for any given quantity of output, the higher is the quality being produced, the more quality-biased is the resulting learning.[15]

5.1 The Model

Technology Consider the same production function as before except that now both technological parameters A_t and γ_t are subject to progress due to learning-by-doing. Labor supply is constant and normalized to be 1. Thus, output at time t when producing quality q_t is given by:

$$x_t = \frac{A_t}{e^{q_t/\gamma_t}}; \tag{12}$$

As already noted, A_t is a general efficiency parameter, whereas γ_t governs the relative efficiency in producing higher-quality goods. Technical progress in general efficiency A_t evolves over time as a result of a standard learning by doing process, as in Krugman (1987) and

[15] Learning by doing may be considered as an initial simple formulation for endogenous technical progress. Still, a model with intentional R+D would likely bring about similar qualitative results on the increasing quality bias of technical progress. The reason is that incentives for quality-biased R+D (as opposed to quality neutral R+D) are likely to increase as consumers spend a larger share of any income rise on upgrading the quality of consumption.

Lucas (1988):

$$\dot{A}_t = \theta_1 x_t - \delta_1 A_t; \quad \theta_1 > \delta_1 > 0. \tag{13}$$

In turn, γ_t is linked to both the quantity and quality being produced. For example, accumulated experience in sewing shirts in a clothing factory may bring about not only a higher number of sewed shirts per unit of labor but also improvements in skills and techniques for carrying out more accurate seams. Or, in package delivery, accumulated experience may help develop techniques to reduce misplacements (which is a quality characteristic), besides increasing the number of deliveries per worker. In general, repeating a task helps improve not only the speed at which the task is performed but also the quality of the result. Moreover, the higher the output quality being targeted in a production process, the more likely it is that learning will be quality-biased. Still, as the number of units being produced tends to zero, learning would also tend to zero even if the quality being produced is very high. These qualitative circumstances are embedded in a simple way in the following law of motion for γ_t:

$$\dot{\gamma}_t = \theta_2 q_t x_t - \delta_2 \gamma_t; \quad \theta_2 > \delta_2 \geq 0. \tag{14}$$

Obsolescence parameters δ_1 and δ_2 may be justified in terms of a succession of finitely lived representative agents whose skills have to be replaced.[16] At any rate, results are obtained assuming that δ_1 and δ_2 are small. In fact, the same central results can be obtained with $\delta_1 = \delta_2 = 0$. However, $\delta_1 > 0$ brings about the existence of a steady state with constant rates of growth, which seems interesting to consider.

Utility I now simplify by considering a particular case of the utility function in previous sections that delivers an explicit solution. The representative agent maximizes the following instantaneous utility function subject to the same constraints (4a)-(4b)-(4c) as before:

$$U_t = \frac{\omega_t}{e^{\sigma/x_t}} q_t. \tag{15}$$

As in the previous section, the production function (12) implies that perfect-competition prices are given by $p_t(q) = w_t e^{q/\gamma_t}/A_t$. This, together with the first order conditions of utility maximization, yields the following optimal relationship between quantity and quality of consumption:

$$\frac{q_t}{\gamma_t} = \frac{x_t}{\sigma - x_t}. \tag{16}$$

[16]Knowledge, experience, and skills are embodied in individuals that are subject to a life cycle.

5.2 Equilibrium and Growth

Instantaneous Equilibrium Define $z_t \equiv q_t/\gamma_t$. This change in variable will allow solving first a one-state variable (A_t) system with two control variables (x_t and z_t), instead of dealing with a two-state variable system with two control variables (A_t and γ_t; and x_t and q_t, respectively). Expressions (12) and (16) are now rewritten as:

$$x_t = \frac{A_t}{e^{z_t}}; \tag{17}$$

$$z_t = \frac{x_t}{\sigma - x_t}. \tag{18}$$

Given A_t, expression (17) sets the feasible (x_t, z_t) pairs at time t, whereas (18) determines consumer choice among those pairs (see Figure 3). Note that $x/(\sigma - x)$ is increasing in $x > 0$ with $\lim_{x \to 0} \frac{x}{\sigma - x} = 0$, whereas $\ln(A_t/x)$ is decreasing in x. Clearly, for any $A_t > 0$ there is a unique pair $(x_t, z_t) > (0, 0)$ solving (17)-(18). Both x_t and z_t are continuous and strictly increasing in A_t. Then, given γ_t, $q_t = z_t\gamma_t$ determines output quality q_t.

GDP Growth GDP at time t is $y_t \equiv x_t p_t(q_t)$. GDP growth at time t at constant prices $p_t(q)$ is given by $g(y_t) = g(x_t) + \frac{\partial p_t(q)/\partial q}{p_t(q)}\dot{q}_t$. Since $(\partial p_t/\partial q)/p_t = 1/\gamma_t$ and using also (12), we have

$$g(y_t) = g(x_t) + \frac{q_t}{\gamma_t} g(q_t) = g(A_t) + \frac{q_t}{\gamma_t} g(\gamma_t). \tag{19}$$

This expression provides two approaches to GDP growth. From the point of view of the composition of output, GDP grows in both the quantity and the quality dimensions. From the point of view of the source, GDP grows due to general as well as quality-biased technical progress. The importance for GDP growth of consumption quality upgrading $g(q_t)$ and quality-biased efficiency improvements $g(\gamma_t)$ depends on the elasticity of prices with respect to quality $(\partial p_t/\partial q)q_t/p_t$, $(\partial p_t/\partial q)q_t/p_t = q_t/\gamma_t \equiv z_t$. Clearly, there is no reason to expect $g(x_t)$ to be equal to $g(A_t)$ or $g(q_t)$ to be equal to $g(\gamma_t)$ (in fact, this will only be the case in the steady state). The behavior of GDP growth from the point of view of the composition of output is the same as in the model in Section 4. Hence the remainder of this section focuses on the analysis of GDP growth from the point of view of technical progress characteristics.

From (17), (13), and (14) we have:[17]

$$g(A_t) = \frac{\theta_1}{e^{z_t}} - \delta_1. \tag{20}$$

[17] Note from (21) that, given the parameters of the economy and initial technological conditions, non-negative growth rates of γ_t (and of GDP) require δ_2 to be sufficiently low.

$$g(\gamma_t) = \theta_2 z_t x_t - \delta_2. \quad (21)$$

Steady State This economy has a unique steady state with strictly positive growth. In this steady state, the volume of output is constant whereas quality grows at a constant positive rate. The following proposition characterizes this using asterisks to denote steady state values.

Proposition 4 *For δ_2 sufficiently small, the model has a unique steady state with constant values $A^* > 0$, $x^* > 0$, $z^* > 0$, $g_q^* = g_\gamma^* > 0$, and $g_y^* = z^* g_\gamma^*$. Moreover, g_y^* is increasing in θ_1 and θ_2, and decreasing in δ_1 and δ_2.*

Proof. Substituting with $g_A^* = 0$ in (20) yields $z^* \equiv (q/\gamma)^* = \ln(\theta_1/\delta_1) > 0$. With this, expressions (17) and (18) yield $x^* = \sigma \frac{\ln(\theta_1/\delta_1)}{1+\ln(\theta_1/\delta_1)}$ and $A^* = x^* \frac{\theta_1}{\delta_1}$. Then, $g(z_t) = g(q_t) - g(\gamma_t)$ implies $g_q^* = g_\gamma^*$, and (21) yields $g_q^* = g_\gamma^* = \theta_2 z^* x^* - \delta_2$ (which is positive for δ_2 sufficiently small). Finally, (19) yields $g_y^* = z^* g_\gamma^*$. ∎

In Figure 3, the $z = \ln(A_t/x)$ schedule shifts over time towards the North-East until it crosses the $z = x/(\sigma - x)$ schedule at point $z^* = \ln(\theta_1/\delta_1)$, which corresponds to the steady state.

Transitional Dynamics The system (17)-(18)-(20) can be solved independently from the rest of equations. Denote the initial condition by A_0. For $A_0 < A^*$, the schedule (17) at $t = 0$ crosses (18) to the South-West of (x^*, z^*) in Figure 3. It is easy to see that the economy converges monotonically to its steady state.[18]

Now, using (19), we can characterize GDP growth from the point of view of the quality bias of technical progress at different stages of development.

Proposition 5 *Technical progress is mostly quality-neutral at early stages of development (which are characterized by low levels of general efficiency A), but becomes increasingly quality-biased over time.*

Proof. See Appendix ∎

[18] Given $0 < A_0 < A^*$, (17)-(18) imply $0 < x_0 < x^* < \sigma$, $0 < z_0 < z^* = \ln(\theta_1/\delta_1)$. Then, $g(x_t) = g(A_t)\frac{\sigma - x_t}{2\sigma}$ (which can be obtained using (20), (17), and (18)) implies $g(A_t) > 0$ and $g(x_t) > 0$. Then, $g(z_t) = g(x_t)\,[1 + z_t]$ implies $g(z_t) > 0$. Therefore the system converges monotonically to (A^*, x^*, z^*).

This changing nature of technical progress occurs in parallel to the shift of demand towards higher-quality goods. As noted, this prediction is consistent with the comparative advantage shown by richer countries in producing higher-quality goods (Schott 2004).

5.3 A Numerical Illustration

In spite of its simplicity, this framework may deliver reasonable long run paths for per capita GDP growth. This section provides a numerical exercise.[19] The Neoclassical model predicts decreasing growth rates, whereas standard endogenous growth models usually predict constant rates. However, the secular pattern of growth in advanced economies since the industrial revolution has been one of increasing growth rates that seem to have stabilized somewhat below $2-$percent in the last half century. For example, estimates of per worker output growth in England at the beginning of the ninetieth century go from $0.35-$percent to $1.3-$percent (see Feinstein 1981, and Crafts and Harley 1992). On the other hand, US per capita growth in the recent decades has averaged about $1.7-$percent (though, as discussed in the Introduction, this number may underestimate actual growth due to unmeasured quality growth). In this subsection the model is discretized and calibrated so that it roughly matches the long run per capita growth path of the most advanced economies in the last two centuries along those numbers. The calibration generates an initial per capita GDP growth of $0.72-$percent (which is roughly consistent with the mean of the cited estimates for England at the beginning of the XIXth century) and yields a steady state growth of $1.8-$percent (which is consistent with the overall observed trend in the last decades).

Parameter values are set as follows. Since skills and knowledge are embodied in individuals, annual depreciation rates $\delta_1 = \delta_1 = 0.025$ seem reasonable. Furthermore, I set $\sigma = 3$, $\theta_1 = 0.054$, $\theta_2 = 0.048$. Finally, the initial value A_0 is chosen such that $x_0 = 1$. Figure 4 draws the annual rates of growth of GDP for 300 years. As noted, calibrated GDP growth starts at an annual rate of $0.72-$percent and then shows a rapid acceleration, doubling after 120 years and surpassing $1.68-$percent after 150 years.

GDP growth is also divided in Figure 4 into the quantity growth component, $g(x_t)$, and the quality growth component, $z_t g(x_t)$. Consistent with Proposition 4, quantitative growth

[19] Notwithstanding, the stripped-down simplicity of the technical progress functions assumed in the model prevents obtaining quantitatively plausible long run paths for the division of growth between quantity and quality growth.

is the main source of growth at initial stages. Then, it is gradually substituted by quality growth. Although an empirical assessment of the secular paths of these two components of growth does not yet exist, their paths in Figure 4 seems implausible (the calibration seems to overrate quality growth in late periods). Future work may find it necessary to introduce more flexible functional forms for technical progress in order to generate more reasonable secular paths of these two components, as estimates of these paths become available. Introducing population growth will also enhance the importance of quantity growth.[20]

6 Growth Policy: Progressive Consumption Taxes

Progressive consumption taxation (i.e., taxing higher-quality or *luxury* goods at higher rates) has been discussed from different perspectives such as the impact on aggregate savings and the distribution of wealth.[21] This section discusses its possible role as a growth policy through its influence on the composition of demand. Note that different quantity/quality compositions of demand and output give rise to different technical progress in terms of intensity and quality bias. Moreover, the *value* of quality-biased technical progress changes along different stages of development: as individuals become richer and consume higher quality, quality-biased technical progress becomes more valuable for GDP growth. In this context, distortionary consumption taxes (or subsidies) can be used to influence the quantity/quality composition of demand in order to enhance growth by appropriately directing technical progress.

Influencing Demand Composition At any point in time, the government can influence the instantaneous equilibrium pair (x_t, z_t) by using a non-linear tax/subsidy scheme. Consider the following tax/subsidy scheme parameterized by $\tau > 0$: $p_{\tau,t}(q) = [p_t(q)]^{\tau}$; where $p_{\tau,t}(q)$ is the after-tax/subsidy price of quality q at time t. Clearly, $\tau > 1$ (respectively, $\tau < 1$) involves a tax/subsidy scheme relatively unfavorable (respectively, favorable) to higher-quality goods. Consumers are assumed to finance these subsidies or receive the yields of this tax scheme in a lump sum. The production function in (17) together with perfect competition yields $p_{\tau,t}(q) = \eta \left[w_t e^{q_t/\gamma_t}/A_t\right]^{\tau}$. Plugging in after-tax prices into the income

[20]Note that population growth would be directly added to quantity GDP growth and would likely reduce the quality bias of technical progress.

[21]See for example Frank (2005) and references therein.

constraint and maximizing utility yields a new version of expression (18):

$$z_t \tau = \frac{x_t}{\sigma - x_t}. \tag{22}$$

This expression together with (17) determines the new instantaneous equilibrium. Now, taking derivatives in (22) yields:

$$\frac{dz_t}{d\tau} = \frac{x_t [\sigma - x_t]}{\sigma [\partial x_t / \partial z_t] - \tau [\sigma - x_t]^2} < 0; \tag{23}$$

where from (17) we have $\partial x_t / \partial z_t = -x_t$. Therefore, as expected, tax/subsidy schemes that tax higher-quality goods at higher rates (or subsidize them at lower rates) reduce the quality of consumption and increase quantity. This can be used to affect technical progress and growth.

Enhancing Growth At early development stages, the small scale of output may be the most important impediment for faster technical progress. Therefore, reducing the demand for quality and raising the scale of output may enhance GDP growth. However, promoting quality-biased technical progress may be optimal at later stages of development when consumers are richer and efficiency in producing higher quality becomes more valuable. This is in fact the case in this model. To see this, consider the impact of a tax/subsidy scheme τ on GDP growth. From (19), we have:

$$\frac{dg(y_t)}{d\tau} = \left[\frac{dg(A_t)}{dz_t} + z_t \frac{dg(\gamma_t)}{dz_t} + g(\gamma_t) \right] \frac{dz_t}{d\tau}. \tag{24}$$

Changing τ affects GDP growth through three components: the growth of A_t, the growth of γ_t, and the GDP-growth impact of γ_t growth which is given by z_t. As shown in (23), higher τ reduces the quality of output and increases its scale. This is always positive for $g(A_t)$ but may have an uncertain effect on $g(\gamma_t)$, and has a negative effect on z_t. The relative importance of these three effects changes over time. Early stages of development are characterized by a low general efficiency A_t, resulting in an output consumption x_t close to zero and minimal quality. Increasing the scale of output at these stages brings about a sizable increase in general efficiency A growth, whereas the alternative of increasing quality would not bring about much quality-biased technical progress because this also requires a large scale of output, which is hurt by increasing quality (see (20) and (21)). Moreover, quality-biased technical progress is of little value because individuals are consuming very low quality at these stages (hence z_t is low in (19)). Hence, a tax/subsidy scheme that shifts GDP towards larger output has a positive net effect on GDP growth at early development stages. However, the positive effect of a progressive tax scheme on $g(A_t)$ becomes weaker at later stages, as A_t increases. Meanwhile, the negative effect on $g(\gamma_t)$ becomes larger, while the growth value of being more efficient in producing higher quality (which is given by z_t) also increases. Therefore, for some parameter values, a tax/subsidy scheme that shifts consumption towards higher quality and lower quantity may have a positive effect on GDP growth at more advanced stages of development.

Proposition 6 *At early stages of development, progressive consumption taxes enhance GDP growth. This may not be the case at later stages.*

Proof. Consider the initial stages of development, which are characterized by A_t being close to 0. As already noted, from (17)-(18) we have $\lim_{A_t \to 0} x_t = \lim_{A_t \to 0} z_t = 0$. Now consider the three components in (24). From (20) we have $\lim_{A_t \to 0} dg(A_t)/dz_t = -\theta_1/e^{z_t} < 0$; whereas from (21) we have $\lim_{A_t \to 0} dg(\gamma_t)/dz_t = 0$ and $\lim_{A_t \to 0} g(\gamma_t) \leq 0$. Since $dz_t/d\tau < 0$ (see (23)), this yields $\lim_{A_t \to 0} dg(y_t)/d\tau > 0$. Now, continuity of these derivatives implies that at least for an initial interval of time such that A_t is still sufficiently low, GDP growth is increasing in the tax scheme τ. ■

Note that since the arguments in the proof do not use the assumptions on preferences, this proposition holds for any utility function as long as $dz_t/d\tau < 0$. As already noted, once output has reached a sufficiently large scale, growth could be more effectively enhanced by favoring the quality dimension of consumption. That is, for some parameters of the model, a subsidy to the production of higher quality goods, $\tau < 1$, may enhance growth. This would be the case, for example, for δ_1 and δ_2 close to zero and $\ln(\theta_1/\delta_1) < 2$.[22] This, of course, requires θ_2 being strictly positive. In the case $\theta_2 = 0$, which corresponds to a standard learning-by-doing model where technical progress is not quality biased, progressive taxation always enhances growth. The reason is that, in that case, only the quantity of output has a positive effect on technical progress.

[22] In such a case, we have $z^* < 2$. The argument is completed using (24) to get $dg(y_t)/d\tau = \left[-\frac{\theta_1}{e^{z_t}} + \theta_2 \sigma \frac{z_t^2}{1+z_t}(2 - z_t) - \delta_2\right] \frac{dz_t}{d\tau}$, substituting with $\theta_1/e^{z^*} = \delta_1$, and applying continuity around the steady state.

7 Inequality and Growth: The Quality Channel

This section analyzes the impact of income inequality on growth channeled through the quantity/quality composition of output. In this model, the distribution of income affects the quantity/quality output composition, which in turn influences the intensity and quality bias of technical progress. This in turn affects growth. Given the technology at a point in time, higher inequality involves higher average quality of output but smaller scale. Since at early stages of development a larger scale tends to be more beneficial to growth than higher average quality, higher inequality hinders growth during these stages. Nonetheless, the sign of this effect can change at later stages: when the scale of production is already large and when consumers prefer upgrading the quality of consumption rather than augmenting its quantity, increasing the quality bias of technical progress may help GDP growth. As a result, higher inequality, which raises the quality bias of technical progress, may have a positive impact on growth.

7.1 The Model with Income-Heterogeneous Agents

Income inequality can be introduced in a simple way by assuming that different individuals supply different *efficiency units of labor*. Let ℓ_i be individual i's inelastic supply of efficiency units of labor; $i = 1, ..., I$. Efficiency units of labor are normalized so that total supply is equal to one: $\sum_i \ell_i \equiv L = 1$. All individuals have the same utility function (15) assumed in the previous two sections. I adopt the strong simplification that every period each individual's consumption is equal to her income. Technology is also the same as in previous sections, except that now labor in the production function is measured in efficiency units.

Each efficiency unit of labor earns wage w_t at time t. GDP is denoted by Y_t. Since $L = 1$, we have $w_t = Y_t$. Different individuals may now consume different qualities since their incomes may be different. Denoting by x_{qt} the production of quality q at time t, GDP is now given by:

$$Y_t \equiv \sum_q x_{qt} p_t(q). \tag{25}$$

Denote individual i's income by y_{it} and her share in aggregate income by β_i. In principle, we have $y_{it} \equiv \ell_i w_t = \beta_i Y_t$. But we may also consider the existence of a zero-sum redistributive scheme of taxes and transfers across indididuals. In such a case, β_i is interpreted as agent i's share in aggregate income after income redistributions, and y_{it} as disposable income.

Individual i's quantity of consumption at time t is x_{it}, q_{it} is its quality, and $z_{it} \equiv q_{it}/\gamma_t$. Given A_t, individual i's budget constraint at time t is $\beta_i Y_t = x_{it} p_t(q_{it}) = x_{it} \frac{e^{z_{it}}}{A_t} w_t$. Thus, since $w_t = Y_t$, individual i's attainable pairs satisfy:[23]

$$x_{it} = \frac{A_t}{e^{z_{it}}} \beta_i. \tag{26}$$

Intersection of (26) with expression (18) from utility maximization determines individual i's equilibrium at time t (which, in turn, given γ_t, determines her quality choice q_{it}). See Figure 5 where (26) is drawn for two individuals m and n such that $\beta_m > \beta_n$. Clearly, $\beta_m > \beta_n$ implies $(x_{mt}, z_{mt}) > (x_{nt}, z_{nt})$.

Using (26) to substitute in (25) yields $Y_t = \sum_i \frac{A_t}{e^{q_{it}/\gamma_t}} \beta_i p_t(q_i)$. Hence, GDP growth at constant prices is given by:

$$g(Y_t) = g(A_t) + g(\gamma_t) \textstyle\sum_i \beta_i z_{it}. \tag{27}$$

Finally, expressions (20) and (21) become:

$$g_{At} = \frac{\theta_1}{A_t} \textstyle\sum_i x_{it} - \delta_1; \tag{28}$$

$$g_{\gamma t} = \frac{\theta_2}{\gamma_t} \textstyle\sum_i x_{it} q_{it} - \delta_2. \tag{29}$$

7.2 Inequality and Growth

The growth impact of inequality is analyzed by considering the instantaneous growth effect of a change in the distribution of the β_is (with $\sum_i \beta_i = 1$), with given technology and aggregate labor supply.[24] Specifically, I consider the impact of higher inequality defined as a mean-preserving spread on the distribution of the β_is; i.e., a redistribution from individuals with lower β_i to individuals with higher β_i. This involves analyzing the sign of $\frac{dg(Y_t)}{d\beta_m} - \frac{dg(Y_t)}{d\beta_n}$, for $\beta_m > \beta_n$. In turn, this accounts to analyzing the sign of $\frac{d^2 g(Y_t)}{d\beta_i^2}$, where a negative value would imply a negative impact of inequality on growth.

[23] Note that β_i may be seen as the share of the aggregate labor supply (in efficiency units) that individual i is able to buy in order to produce her consumption bundle.

[24] The analysis may be seen as comparing growth for two different economies with the same technology and aggregate labor endowments but different distribution of efficiency units of labor, or as considering the growth effect of an income redistribution through a government lump-sum tax/transfer scheme.

From (27) the derivative of GDP growth with respect to β_i is given by

$$\frac{dg(Y_t)}{d\beta_i} = \frac{dg(A_t)}{d\beta_i} + \frac{dg(\gamma_t)}{d\beta_i} \sum_i \beta_i z_{it} + g(\gamma_t) \frac{d \sum_i \beta_i z_{it}}{d\beta_i}. \tag{30}$$

In this expression, $g(A_t)$ is increasing and concave in β_i. The reason is that $g(A_t)$ only depends on the sum of quantities being produced $\sum_i x_{it}$ and consumption quantities are concave in income. Thus, increasing inequality is negative for $g(A_t)$. In turn, $g(\gamma_t)$ (which depends on $\sum_i q_{it} x_{it}$) is increasing in β_i and initially convex, but can change to concave before the steady state is reached. Similarly, $\sum_i \beta_i z_{it}$ is also convex in β_i for low z_{it}. Hence the sign of the relationship between inequality and growth through the quantity/quality mechanism may change over time as a country develops.

However, inequality has a definite negative effect at early stages of development. The reason is that, at these stages, the *growth value* of $g(\gamma_t)$ (which is given by $\sum_i \beta_i z_{it}$; see the second right-hand term of expression (30)) tends to be null because the preference for quality at low income levels is very low (note that both x_{it} and z_{it} are close to zero when A_t is close to zero, even for the richer individuals). In other words, being able to produce higher quality at low cost has little value at early stages of development. Moreover, $g(\gamma_t)$ in the last term of (30) is also close to zero since the $x_{it} z_{it}$ terms are close to zero. Therefore, only the negative effect of inequality on $g(A_t)$ is quantitatively relevant at early stages. Conversely, at more advanced stages of development, the effect on $g(A_t)$ becomes small whereas the value of raising quality efficiency γ_t increases. Therefore, the sign of the inequality/growth relationship may change to positive depending on the specific value of the parameters.[25]

Proposition 7 *At early stages of development, inequality has a negative impact on growth through the quantity/quality mechanism. The sign of this impact may change at later stages.*

Proof. See Appendix. ■

Another intuitive way to explain this proposition is as follows. More unequal poor economies spend a larger share of GDP in the production of luxury goods consumed by the elite. This limits the possibilities of generating aggregate productivity gains for two potential reasons. First, achieving significant technical progress though learning-by-doing

[25] Clearly, if $\theta_2 = \delta_2 = 0$, the quality being produced has no impact on technical progress. Hence, growth is always maximized by maximizing quantity at every point in time, which in turn implies that inequality is always negative for growth.

requires a large scale of production. However, producing for the elite in a poor country implies that the scale of production is small. And second, technical progress will tend to be strongly quality biased. However, technical progress in producing higher-quality goods may be of little value for producing the varieties consumed by the majority of the population (who, even in poor unequal societies, may absorb a large share of GDP).

As pointed out in the Introduction, the prediction in this proposition is consistent with most empirical evidence on the relationship between inequality and growth. Work on the relationship between inequality and growth tends to be impaired by reverse causality problems and the lack of reliable data. To reduce these problems, Easterly (2007) instruments for inequality using some physical characteristics of a countries' land. He finds an overall negative impact on growth. Still, there is evidence suggesting that this impact may be non-linear and may even turn positive for the group of richer countries. In fact, Deininger and Squire (1998) found that inequality reduces income growth in poor countries but not in rich ones. In turn, Forbes (2000) found a positive relationship between inequality and growth using a sample that included mostly developed countries. This non-linear effect between inequality and growth may be explained by other mechanisms different from the quality/quantity mechanism explored here. For example, it may be explained by the negative consequences of credit constraints on human capital (Galor and Moav 2004), as long as human capital is the relatively more scarce production factor in poor countries (as a result, for example, of international capital mobility) and credit constraints are loosened as countries become richer. However, Barro (2000) and (2008) still finds a non-linear effect of inequality on growth after controlling for schooling as well as other possible mechanisms suggested by the literature (such as the investment ratio and rule of law). Specifically, he finds a negative impact of inequality on growth among the group of less developed countries, which vanishes or even turns into a positive impact among the group of richer countries, as predicted by the model in the paper.

8 Concluding Comments

Recent empirical work in several areas such as real GDP growth measurement and international trade has uncovered the increasing importance of the quality dimension of output. This paper explores the consumer time-constraint foundations of the demand for quality and some of its macroeconomic implications. The starting point is that consumption requires time and that higher-quality goods provide higher utility per unit of time though at a higher monetary cost. It is then shown that additions to income are increasingly spent upgrading the quality of consumption. The paper provides a very tractable model where macroeconomic issues related to the quality dimension of goods can be analyzed. Four topics receive a first-bite examination: the share of quality growth as a component of GDP growth, the quality bias of technical progress, the role of progressive consumption taxes as a growth policy, and the impact of inequality on growth through the quantity/quality mechanism. In spite of its simple structure, the growth model in this paper can deliver reasonable quantitative paths for the secular growth of GDP in advanced economies. Notwithstanding, the calibration put forward seems to largely overestimate the quality component of GDP growth. Building and calibrating models that can match the quantity and quality components of long run GDP growth –as more thorough estimates of these components become available– and that can be linked to recent patterns of international trade and technical progress seems an interesting area for future research.

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9 Appendix: Proofs of Propositions

9.1 Proof of Proposition 2

Using the income constraint, note that the share of an income rise that is spent increasing the quantity of consumption is $p(q)\frac{dx}{dy} = \frac{dx}{dy}\frac{y}{x}$. Denote the optimal quantity of consumption as a function of y by $x^*(y)$. Since Assumption 2a or 2b imply $\lim_{y\to\infty} x^*(y) < \infty$, we have $\lim_{y\to\infty}\left(\frac{dx^*(y)}{dy}\frac{y}{x^*(y)}\right) = 0$. Therefore, the share of an income rise that is spent upgrading the quality of consumption $x\frac{\partial p}{\partial q}\frac{dq}{dy}$ tends to 1. Then, by continuity of the maximizer, for any s, $0 < s < 1$, there is an income level $y(s)$ sufficiently large such that $y > y(s)$ implies $x\frac{\partial p}{\partial q}\frac{dq}{dy} > s$.

9.2 Proof of Proposition 5

First consider the instantaneous equilibrium at initial stages of development (i.e., for $A(t)$ close to zero). Note that $\lim_{A_t\to 0} g(A_t) = \theta_1 - \delta_1 > 0$. In turn, $\lim_{A_t\to 0} z_t = 0$ and $\lim_{A_t\to 0} x_t = 0$; and therefore, $\lim_{A_t\to 0} g(\gamma_t) = \lim_{A_t\to 0} \theta_2 z_t x_t - \delta_2 \leq 0$. Also note that growth rates are continuous in time. Hence, for $A(t)$ sufficiently close to zero we have $g(A_t) > g(\gamma_t)$. Therefore, at early stages of development, quality-neutral technical progress g_A is the main source of growth.

Now consider later stages of development. Recall that z_t increases monotonically to its steady state. Hence expression (20) implies that $g(A_t)$ decreases monotonically from $g_A(0) > 0$ to $g_A^* = 0$. On the other hand, since z_t and x_t increase monotonically towards their steady state values, (21) implies that $z_t g(\gamma_t)$ increases monotonically towards $z_t^* g_\gamma^*$ which is strictly positive.

9.3 Proof of Proposition 7

Early stages of development are characterized by low levels of general efficiency A. Furthermore, increases in inequality are defined as income redistributions from individuals with lower β_i to individuals with higher β_i. Thus, to prove Proposition 7 I will show that for A sufficiently small, $d^2g(Y_t)/d\beta_i^2$ in expression (27) is negative.

Before turning to that derivative, first consider the derivatives of x_{it}, z_{it}, and $x_{it}z_{it}$ with respect to β_i. The time subscript does not play any role in the following and therefore it is suppressed. Given A, x_i and z_i can be solved from the system (18)-(26). Using this system (and taking into account that in equilibrium $x_i = \sigma\frac{z_i}{1+z_i}$), the derivatives of x_i and z_i with respect to β_i yield

$$\frac{dx_i}{d\beta_i} = \frac{A}{e^{z_i}}\frac{(\sigma - x_i)^2}{(\sigma - x_i)^2 + \sigma x_i} > 0, \quad \frac{d^2x_i}{d\beta_i^2} = -\frac{A}{e^{z_i}}\frac{(z_i + z_i^2 + 1)(z_i + 2)(z_i + 1)\sigma^3}{(\sigma + z_i\sigma + z_i^2)^3}\frac{dz_i}{d\beta_i} < 0;$$

$$\frac{dz_i}{d\beta_i} = \frac{A}{e^{z_i}}\frac{(z_i + 1)^2}{(z_i + z_i^2 + 1)\sigma} > 0, \quad \frac{d^2z_i}{d\beta_i^2} = -\frac{A}{e^{z_i}}\frac{(2z_i + z_i^2 + 3)(z_i + 1)z_i}{(z_i + z_i^2 + 1)^2\sigma}\frac{dz_i}{d\beta_i} > 0.$$

$$\frac{d(x_iz_i)}{d\beta_i} = \sigma\frac{(z_i + 2)z_i}{(z + 1)^2}\frac{dz_i}{d\beta_i} > 0, \quad \frac{d^2(x_iz_i)}{d\beta_i^2} = -\frac{A}{e^{z_i}}\frac{(z_i^3 + 2(z_i^2 + z_i - 1))(z_i + 1)}{(z_i + z_i^2 + 1)^2}\frac{dz_i}{d\beta_i}.$$

The sign of the last expression depends on the size of z_i. Now, consider the second derivative of expression (27) with respect to β_i:

$$\frac{d^2g(Y_t)}{d\beta_i^2} = \frac{d^2g(A)}{d\beta_i^2} + \frac{d^2g(\gamma)}{d\beta_i^2}\sum\nolimits_i\beta_iz_i + 2\frac{dg(\gamma)}{d\beta_i}\left[\beta_i\frac{dz_i}{d\beta_i} + z_i\right] + g(\gamma)\left[\beta_i\frac{d^2z_i}{d\beta_i^2} + 2\frac{dz_i}{d\beta_i}\right] = H\frac{dz_i}{d\beta_i};$$

where

$$\begin{aligned} H \equiv & -\theta_1\frac{1}{e^{z_i}}\frac{(z_i + z_i^2 + 1)(z_i + 2)(z_i + 1)\sigma^3}{(\sigma + z_i\sigma + z_i^2)^3} + \theta_2\frac{A}{e^{z_i}}\frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2}(1 + z_i)\sum\nolimits_i\beta_iz_i \\ & + 2\theta_2\sigma z_i\frac{2 + z_i}{(1 + z_i)^2}\left[\beta_i\frac{dz_i}{d\beta_i} + z_i\right] + g(\gamma)\left[\beta_i\frac{A}{e^{z_i}}\frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2}(1 + z_i) + 2\right] \end{aligned}$$

Taking into account that $\lim_{A\to 0} x_i = \lim_{A\to 0} z_i = 0$, we have

$$\lim_{A\to 0} H = -2(\theta_1 + \delta_2) < 0.$$

Note that x_i, z_i and all the derivatives above are continuous in A. Therefore, for A sufficiently close to 0 we have $d^2g(Y_t)/d\beta_i^2 < 0$.

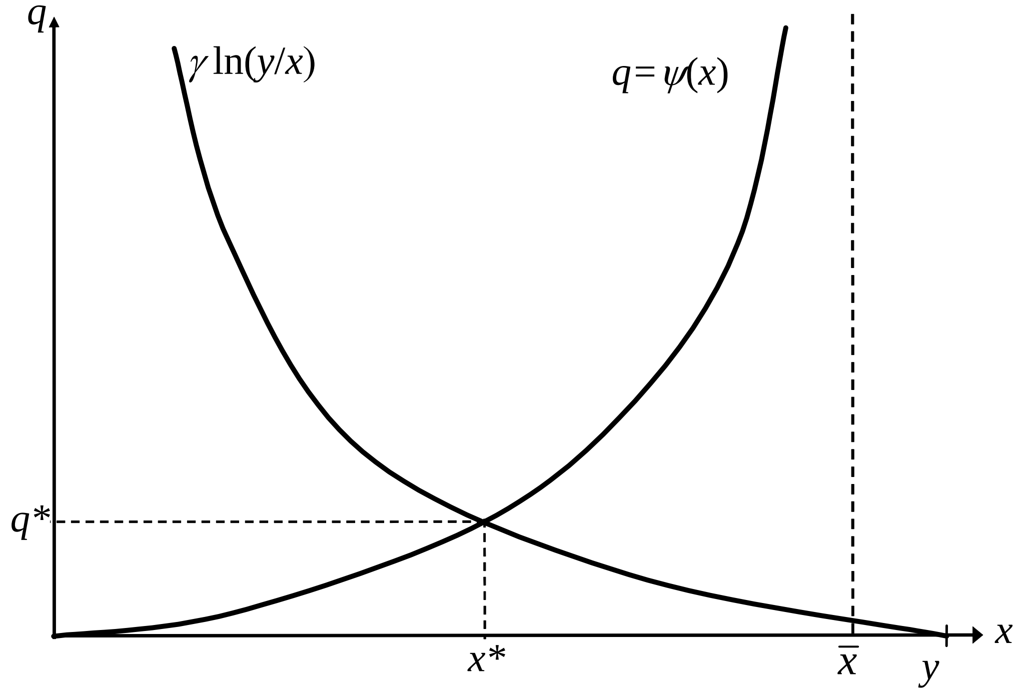


Figure 1:

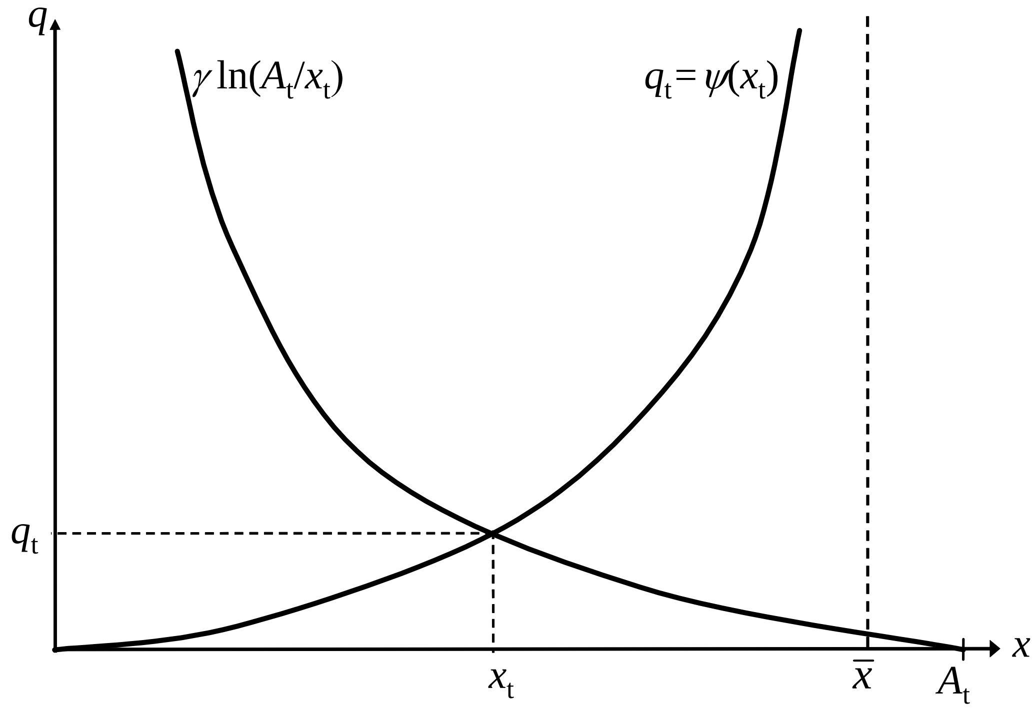


Figure 2

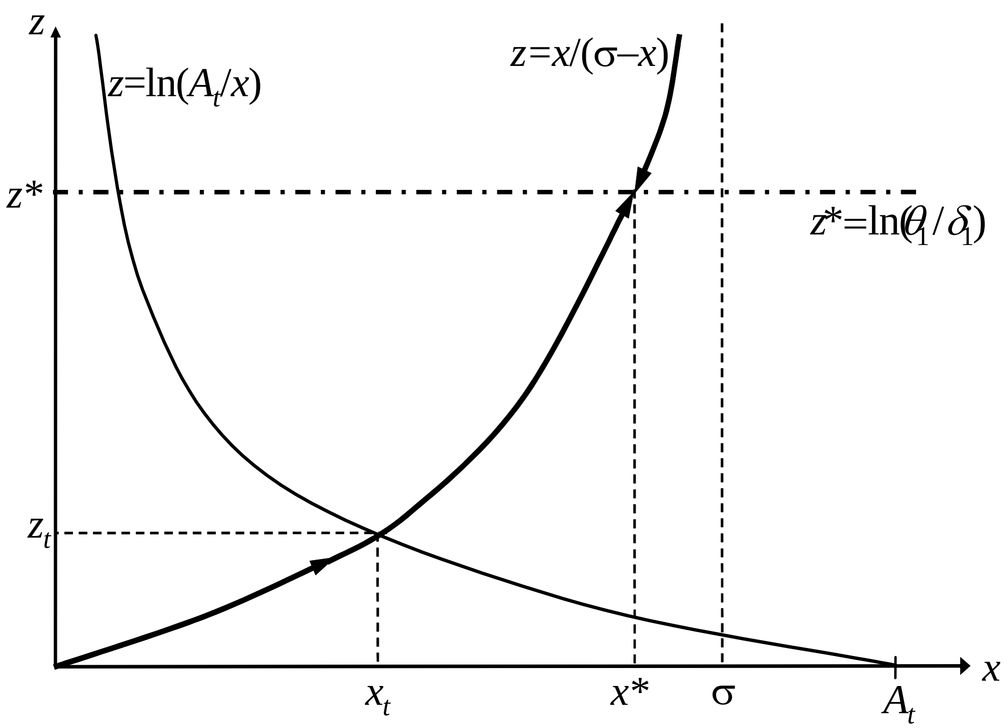
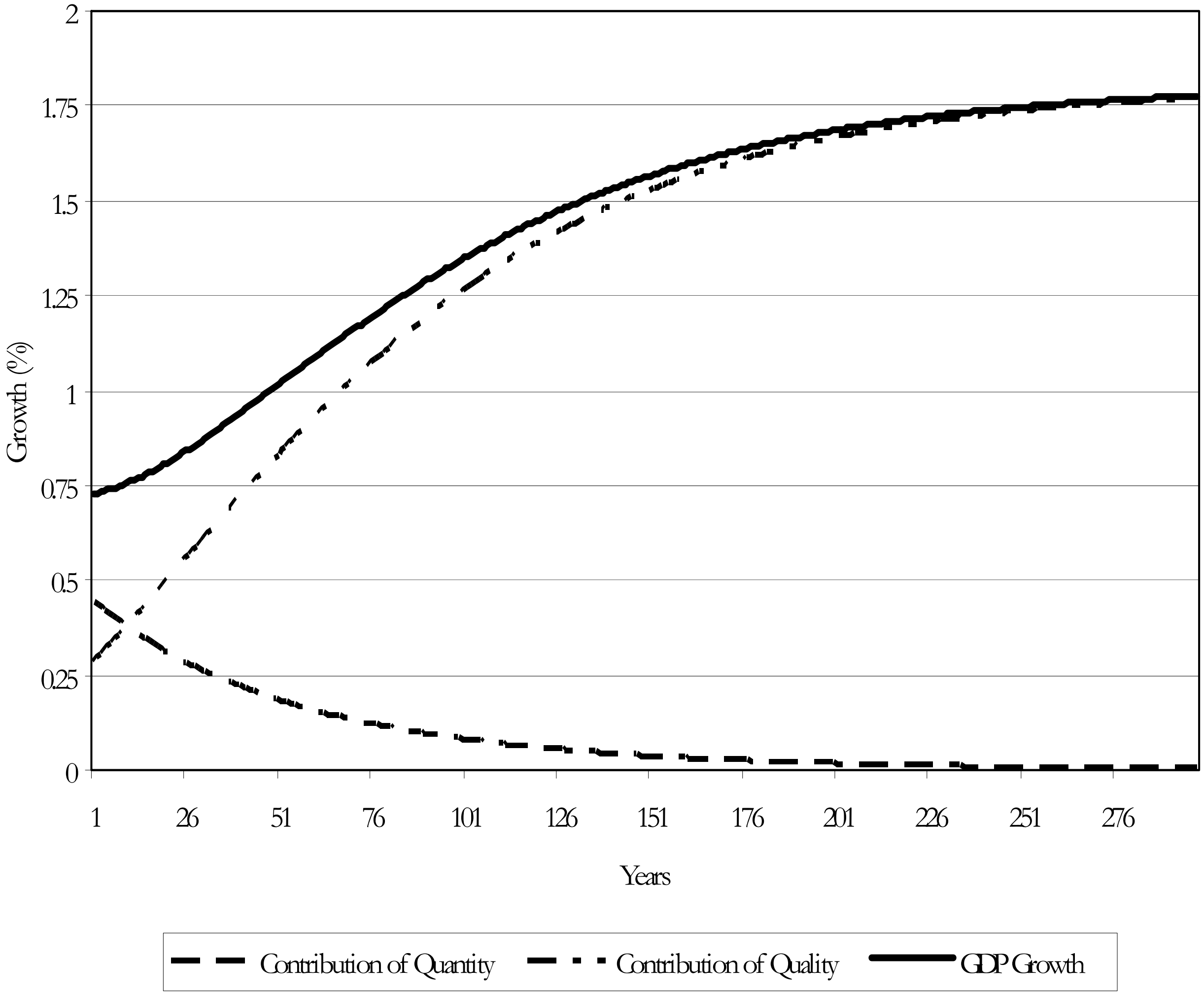
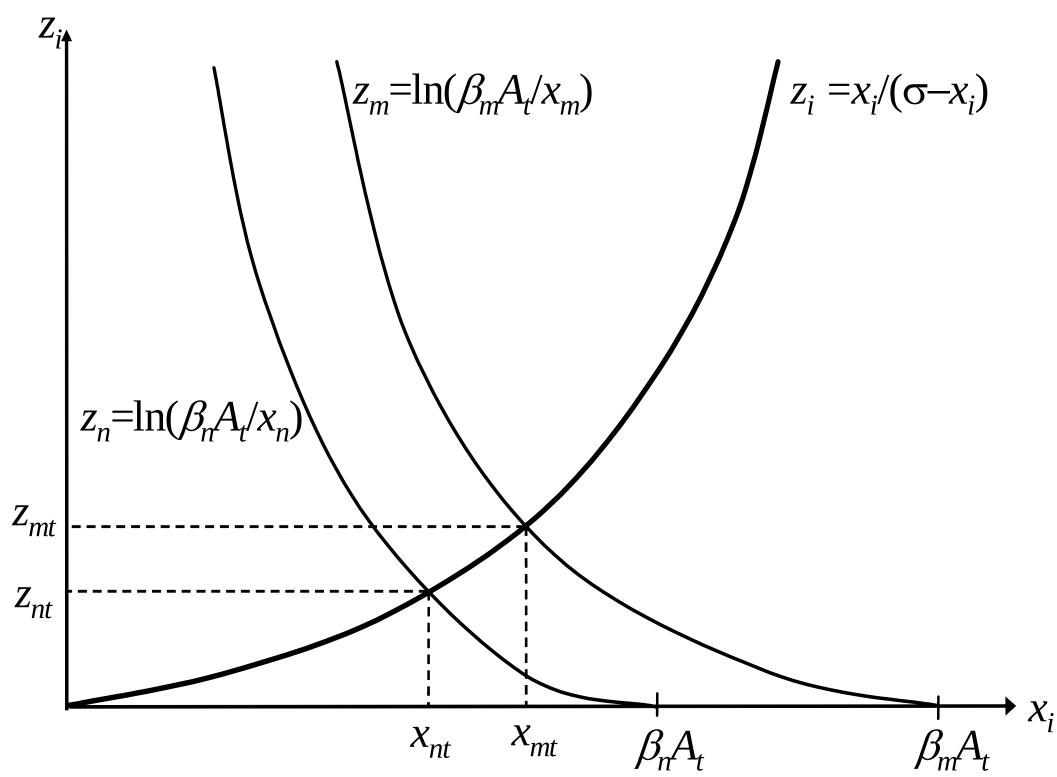


Figure 3



2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
Growth (%)
1
26
51
76
101
126
151
176
201
226
251
276
Years
Contribution of Quantity
Contribution of Quality
GDP Growth

Figure 4



z_i
$z_m=\ln(\beta_m A_t/x_m)$
$z_i = x_i/(\sigma - x_i)$
$z_n=\ln(\beta_n A_t/x_n)$
z_{mt}
z_{nt}
x_i
x_{nt}
x_{mt}
$\beta_n A_t$
$\beta_m A_t$

Figure 5