A Model of Confidence in Financial Intermediation*

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Abstract

We develop a model of financial intermediation in which depositors decide whether to withdraw their funds or keep them deposited They act sequentially and observe the other depositors' actions only if connected by a network. A depositor has confidence in her bank if given the available information she can be sure that sufficient other depositors will not withdraw their funds from the bank, so the bank will not collapse. The main result of this paper shows that confidence arises if a depositor is in a sufficiently large subset of completely connected depositors, a clique. If such a clique exists, the bank will not collapse due to massive withdrawal. The relevance of the result is shown through a bank-run episode in Turkey.

Keywords: financial intermediation, social networks, coordination failures, multiple equilibria

JEL Classification: C70, C90; D85; G21

[Work in progress]

1 Introduction

The understanding of the recent financial turmoil will be an important job for the economists´ community in the years to come. In a speech delivered in 2009 (Bernanke 2009), Ben Bernanke offered an interpretation of the crisis. He stated that besides the crucial role of fundamentals coordination problems arising in form of a classic panic were important in setting off the crisis. He argued that the massive rush of depositors and investors to financial institutions to withdraw their funding was critical in the development of the events. Such a run need not to be based on fundamental reasons, since as he put it, "a panic is possible in

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any situation in which longer-term, illiquid assets are financed by short-term, liquid liabilities, and in which suppliers of short-term funding either lose confidence in the borrower or become worried that other short-term lenders may lose confidence."[1] Runs do not only affect commercial banks, but other financial institutions characterized by the previous funding traits like investment banks, structured investment vehicles, funds (hedge or money market mutual) may be exposed to it as well.

As a consequence, during the financial turbulence one of the main public aims in most of the countries was to maintain the confidence in financial intermediation and to avoid bank runs. In pursuit of this goal, many governments increased the level of coverage of their deposit insurance schemes and provided financial assitance to many banks and financial institutions in trouble. Changes in the deposit insurance schemes intended to inspire confidence toward depositors that other depositors will not withdraw since they were insured, while financial assistance purported to generate trust in the financial institutions. Concretely, Directive 2009/14/EC of the European Union increased the minimum coverage level from EUR 20000 to EUR 50000 and foresees an additional increase to EUR 100000 by 31 December 2010. In the US, the Federal Deposit Insurance Corporation implemented a temporary increase from $100000 to $250000 as a reaction to the perceived loss of confidence in financial institutions. The intervention of the Federal Reserve to save AIG or the financial assistance of the German authorities to Hypo Real Estate are two examples of governmental action to bring stability into a turbulent environment.

This paper studies the issue of confidence among depositors. We say that a depositor has confidence in her bank if given the available information she can be sure that sufficient other depositors will not withdraw their funds from the bank, so the bank will not collapse. Confidence implies that a depositor will not withdraw even if her prior beliefs about the other depositors´ disposition to keep the money in the bank are extremely pessimistic, a charecteristic of crisis times. We look for the conditions on the information structure that generates this sort of confidence. To understand the microeconomic mechanisms that lead to confidence is important because the financial safety net built up by governments may prove to be insufficient to prevent bank runs as shown in the case of Northern Rock, a bank in the UK. This bank suffered massive withdrawals in September of 2007 within days in spite of being covered by deposit insurance.[2] The words of Anne Burke, an ordinary depositor who decided to withdraw from Northern Rock testify nicely why depositors may find it reasonable to withdraw: “It's not that I disbelieve Northern Rock, but everyone is worried and I don't

[1] The importance of coordination problems during crisis has been shown in earlier studies as well. For instance, Calomiris and Mason (2003) find empirical evidence that suggests that bank runs during the Great Depression cannot be explained by changes in the fundamentals alone.

[2] At the time of the run, bank deposits in UK were fully insured only up to 2000 pounds, and above this sum up to 35000 pounds 90 percent of the deposits was insured. See more details about the run on Northern Rock in Shin (2009).

want to be the last one in the queue. If everyone else does it, it becomes the right thing to do."[3] The quote reflects that in spite of the perceived lack of fundamental problems (she did not disbelieve Northern Rock) coordination issues may result in a bank run. In the paper, we build a model without fundamental troubles and we focus on the coordination issues that may arise.

Diamond and Dybvig (1983) provide the seminal model of coordination problems between depositors. They represent the problem as a simultaneous move game in which multiple equilibria emerge; one of these equilibria involves depositors participating in a bank run. Although many researchers have continued to use, and build on this model, descriptions of real-world bank runs (Sprague, 1910; Wicker, 2001) and statistical data (Starr and Yilmaz 2007) make clear that depositors' decisions are not entirely simultaneous, but partially sequential. Thus, many depositors have information about what other depositors have done and use this information when making their decisions (Kelly and O Grada, 2000; Iyer and Puri, 2008). An important contribution of this paper is to introduce the possibility of observing previous actions and to investigate how this modification affects the outcome, more precisely whether the outcome is unique and without bank run.

In the model, depositors decide one after the other without knowing their exact position in an exogenously given random sequence of decision. Before the decision, depositors learn their own type (determined by the liquidity needs they have) and they observe a set of previous actions (but not types). Diamond and Dybvig (1983) corresponds to the case when the set is empty. Observability is modelled using network theory. It is assumed that depositors belong to an underlying social network, so that a link connecting two depositors implies that the depositor who acts later can observe the other depositor's action. Likewise, the depositor who acts earlier knows that her action is being observed. These features allow the connected depositors to play a sequential game while the depositors who are not linked play a simultaneous game. The social network structure determines the information flow among depositors.

In a Diamond-Dybvig setup, it pays off to wait, if sufficient other depositors do so as well.[4] Hence, a depositor will have confidence if the number of those who wait reaches a threshold. The main result of this paper shows that given this threshold and the parameters of the model a depositor without immediate liquidity needs has confidence in her bank if she is in a sufficiently large subset of completely connected depositors, a clique. The high degree of observability implies that the game that the depositors forming the clique play has a unique equilibrium in which only those hit by the liquidity shock withdraw their funds. If the clique is sufficiently large, it means that the bank surely will have funds to pay a high consumption to those who have waited, independently of the actions taken by those not included in the clique. Our result

[3] Quote retrieved from Bloomberg.com:
http://www.bloomberg.com/apps/news?pid=2060.1087&sid=aeypCkzcRlU4

[4] We will use "to keep the money in the bank" and "to wait" in an interchangeable manner.

entails the possibility of partial runs, with only a fraction of depositors participating in it as shown by Ennis and Keister (2009b).

Our finding represents a demanding condition, because a sufficiently large clique may require a substantial share of all depositors. It suggests that confidence understood in the microeconomic terms put forward by this paper is hard to achieve. Nevertheless, by considering agents with deposits of different sizes (contrary to the homogeneous deposit size adopted by Diamond and Dybvig (1983)) our model becomes more relevant as it will be shown through the example of a bank run that occurred in Turkey.

The rest of the paper is organized as follows. In Section 2 we review the literature. In section 3 we present our theoretical framework and the main result. In section 4, a bank-run episode that occurred in Turkey is analyzed in light of our findings. Section 5 concludes.

2 Literature Review

Two strands of work are related to our paper: the theoretical literature on bank runs and the network literature that studies coordination.

The literature on bank runs follows the seminal work of Diamond and Dybvig (1983) and shows the existence of multiple equilibria, one of which involves depositors running the bank. Our goal is to find conditions that ensure a unique equilibrium without bank runs. This aim has been achieved by Green and Lin (2003) who keep the simultaneous framework, but allow for a more complex contract and assume imperfect but inferrable knowledge about the position in the line. We consider the issue of observability, which is missing from most models on bank runs. Observability plays a central role in Chao Gu (2009) but the scope of her paper is very different. She considers that patient depositors withdraw only if they expect the bank to perform poorly, so she focuses on a signal extraction problem, while leaving aside coordination problems.

Regarding coordination and network, Chwe (2000) is the closest paper to ours. While both papers find the minimal structures that lead to coordination and study how structure and strategy are interrelated, Chwe does not consider actions, only willingness in participate in a revolt, the topic of his paper. Moreover, he limits his attention to a model with one type, while we study a model with two types.

3 The Model

Our model builds on the seminal paper by Diamond and Dybvig (1983), but we modify it on several points. Contrary to the simultaneous-move framework proposed by Diamond and Dybvig (1983) we emphasize the role of observability. Since a depositor only may observe actions already taken, we introduce a sequence of decision. Depositors decide consecutively according to their position in the sequence. We do not restrict our analysis to full observability, so we introduce a social network. A link connecting two depositors implies that the depositor who acts later can observe the other depositor's action. Likewise, the depositor who acts earlier knows that her action is being observed.

3.1 The Environment

There are three time periods, indexed by $t = 0, 1, 2$. We consider a finite number ($n > 2$) of agents and $N = \{1, 2, ..., n\}$ denotes the set of agents. Each agent is endowed with 1 unit of the unique good in the economy.

In period 0 each agent is identical, but it is common knowledge that at the beginning of period 1 some of the agents will suffer a liquidity shock and will become impatient (*imp*), while the rest of agents will be patient (*pat*). More precisely, there will be a constant number $p \in [2, n-1]$ of patient agents and the rest (that is, $n - p$) will be impatient. Impatient agents care only about consumption in period 1, whereas patient agents value consumption both at period 1 and 2. An agent has utility $u(c_1, c_2, \theta)$, where $c_1$and c_2 represent first- and second-period consumption, while θ stands for the type of the agent. If $\theta_i = 0$, then the depositor is impatient, $\theta_i = 1$ stands for a patient depositor. Following Ennis and Keister (2009a), we use the following utility function

$$u(c_1, c_2, \theta_i) = u(c_1 + \theta_i c_2). \quad (1)$$

The utility function, $u : R^2_{++} \to R$ is twice continuously differentiable, increasing, strictly concave, satisfies the Inada conditions and the relative risk-aversion coefficient $-cu''(c)/u'(c) > 1$ for every c. The total number of agents (n) and the number of patient agents (p) are common knowledge. Depositors are expected utility maximizers.

Since impatient depositors always withdraw, our analysis considers the decision of patient depositors.

In the economy there is a constant-returns-to-scale technology for transforming the endowment into consumption in the later periods. A unit of the good placed into the technology in period 0 yields one unit of the good if liquidated in period 1 and yields $R > 1$ if liquidated in period 2. The long-term return, R, is constant, so there is no fundamental uncertainty regarding the return. The unique technology determines consumption in autarchy: impatient agents consume 1 at $t = 1$, whereas patient agents consume R at $t = 2$.

Since agents are not restricted to autarchy, they set up a collective arrangement to improve their situation. To insure against individual risk agents pool their resources at $t = 0$ and form a bank.[5] Thus, the agents become the depositors of the bank.

3.2 The bank and the first best allocation

If a planner could observe each depositor's liquidity needs and assign an allocation, then the resulting first-best allocation would solve

$$\begin{aligned} &\max\ (n-p)u(c_1) + pu(c_2) \\ &s.t. \\ &(n-p)c_1 + [pc_2/R] = n \end{aligned} \tag{2}$$

In the formulation of the problem we imposed the optimality condition that the $n-p$ impatient depositors consume only in period 1, whereas the patient depositors consume only in period 2 to earn the return on the deposit.

This problem yields the solution

$$u'(c_1^*) = Ru'(c_2^*), \tag{3}$$

which implies $R > c_2^* > c_1^* > 1$.

The bank formed by the agents aims at implementing the first best. Therefore, the bank offers c_1^* to those who withdraw in period 1. Notice that the amount of consumption given to an impatient depositor in the first-best allocation is greater than her initial endowment. In this sense does the planner's allocation provide liquidity insurance to depositors.

The bank pays immediately to those who want to withdraw (unless it has run out of funds). The bank does not condition payment on information that is unavailable at the moment as, for example, the total number of withdrawals in the period, which has not yet been observed. Hence, the bank respects the sequential service constraint. Notice that $c_1^* > 1$ and the sequential service constraint jointly imply that the bank cannot pay to each depositor c_1^* in period 1. This opens up the possibility of bank runs.

Depositors who have waited in period 1 receive in period 2 a pro-rata share of the funds which were not withdrawn but were augmented by the productive technology. Formally, the period-2 consumption is

$$c_2(\eta) = \max\left\{0, \frac{R(n-(n-\eta)c_1^*)}{\eta}\right\} \tag{4}$$

[5] For simplicity, we do not consider the "pre-deposit game" as Peck and Shell (2003); our approach is the same as that in Diamond and Dybvig (1983). A "pre-deposit game" would not change qualitatively our results.

where $\eta \in [0, p]$ is the number of depositors who wait in period 1.

If $\eta = p$ (that is, only impatient depositors withdraw in period 1), then $c_2(\eta) = c_2^*$. In this case patient depositors will enjoy a higher consumption, than impatient depositors. Nevertheless, if η is too low, then withdrawing in period 1 may have been a better option for a patient depositor who has waited. We can find a threshold value for η such that if the number of those who kept the money in the bank is less than this threshold, then their period-2 consumption will be less than c_1^*.

Lemma 1 *There exists a* $1 \leq \bar{\eta}(n, p, R) \leq p$ *such that*

$$c_2(\bar{\eta} - 1) < c_1^* \text{ for any } \eta \leq \bar{\eta} - 1$$
$$\text{and} \tag{5}$$
$$c_1^* \leq c_2(\bar{\eta}) \text{ for any } \eta \geq \bar{\eta}.$$

The proof of the lemma is given in the Appendix. Note that the threshold $\bar{\eta}$ is uniform for all patient depositors because it regards the aggregate conditions that result in a sufficiently high period-2 consumption.

A patient depositor has confidence if she can be sure that after period 1 the bank has enough funds to pay her a high consumption. Whether this is the case, it is determined by the threshold $\bar{\eta}$. If a patient depositor can be sure that at the end of period 1 besides her there will be $\bar{\eta} - 1$ other patient depositors who have not withdrawn, then she will wait. When a depositor decides, generally she will not know the number of depositors who wait in period 1.

3.3 Network

We assume that depositors are embedded in an underlying social network. A network (Γ) is the set of existing links among the depositors. Two depositors are neighbors if a link connects them. A link is represented by a pair of numbers ij for $i, j \in \{1, 2, ...n\}$, $i < j$. The numbers denote the identity of the depositors. Notice that identity is composed of the type and the links a depositor has. We assume that a depositor knows her links (that is, she knows whom she observes) and she also knows whether her neighbors are linked. This assumption follows Chwe (1999).

Another important concept is the clique. A clique $s \in N$ in the network Γ is a subset of N such that $i, j \in s \rightarrow ij \in \Gamma$. Thus, in a clique everybody is linked to everybody else.

The social network represents information flow. It does not require that neighbors know each other closely, it just says that a depositor in the network can observe her predecessor neighbors' actions. Neither the position in the sequence of decision, nor the type of the neighbor is observed. Hence, if a patient depositor observes a withdrawal, then she knows that before her there was somebody who has withdrawn. The empty network can be interpreted as a simultaneous-move game where depositors have no information about other depositors' actions and it corresponds to Diamond and Dybvig (1983). On the other extreme, the complete network represents a fully sequential setup, meaning that depositors observe each predecessors' actions. Between the two extremes there is a wide range of possible structures.[6]

In this paper, we do not consider how the social network is formed, we take it as given. We suppose that links are independent of types, so depositors of the same type are not more likely to be linked and there is no correlation between types and the number of links. It is also assumed that the bank has no information about the network structure.

3.4 Timing and decision

At period 0 agents deposit their initial endowments in the bank they form. At the beginning of period 1 depositors privately learn their types (patient or impatient) and their links. Then, Nature uses a random process to determine the sequence of decision. Depositors form a random line, but they do not know their position in the line. We assume that Nature calls them to decide (wait or withdraw) one by one. Before decision, depositors may observe their predecessor neighbors' actions, but neither the type nor the position in the sequence of decision is observed. As already stated, those who withdraw receive immediately c_1^* (unless the bank has run out of funds), while the consumption of those who choose to wait is given by equation (4).

Following the literature, we consider only pure-strategy equilibria and we assume that in period 1, depositors are isolated from each other and no trade can occur.

3.5 An example

Before discussing the main result of the paper, we show a small example to illustrate the mechanisms underlying the result.

Imagine that there are three depositors, two of them patient and an impatient. For concreteness, assume that that depositors have the utility function $u(c) = c^{1-\gamma}/(1-\gamma)$ with risk aversion parameter $\gamma = 6$ and $R = 1,05$.[7] Given these parameters, the first-best payoffs are as follows:

$$c_1^* = 1,027;$$
$$c_2^* = c_2(\eta = 2) = 1,036.$$

[6] The number of all possible network structures when there are n nodes is given by 2^n.

[7] We use the same utility function and the same risk aversion parameter as Ennis and Keister (2009b).

These payoffs imply that the threshold is equal to 2. The bank is able to pay c_1^* to the first two withdrawing depositor, if after two withdrawals the last depositor chooses to withdraw, then she receives $c_1^{last} = 0,945$. If there is only one depositor who decides to wait in period 1, then she receives $c_2(\eta = 1) = 0,993$ in period 2. In this small example, a patient depositor has confidence if she can be sure that the other patient depositor waits in period 1 so if she waits, she can have a high consumption in period 2.

First, consider the empty network that corresponds to the Diamond and Dybvig (1983) setup. Suppose that depositor A and C are patient, while B is impatient.

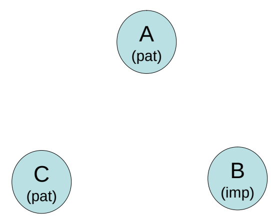

Figure 1: Diamond-Dybvig case

Actions are unobservable due to the lack of links, so strategy depends on beliefs. Regarding A and C the beliefs "the other patient depositor will withdraw" and the strategy "to withdraw" form an equilibrium.[8] Even though the belief "the other patient depositor waits" and the strategy of waiting yields the first best, the fact of having multiple equilibria means that this information structure does not generate confidence.

As a next step, consider the complete network in which the same depositors are linked. Although position is unknown, but since everybody is connected to everybody, it may be inferred based on the number of observed actions. The type sequence remains unknown, for example a patient depositor in the first position does not know if the second depositor to act is patient or impatient. She only knows that both possibilities are equiprobable. Since position is known, we can draw the extensive form of the game as represented in

[8] Given the belief that the other patient depositor withdraws, the expected utility of withdrawal is $\frac{2}{3}u(c_1^*)+\frac{1}{3}u(c_1^{last})$, because withdrawal yields different payoffs depending on the position. The expected utility of waiting is $u(c_2(\eta = 1))$. Calculation shows that the expected utility of withdrawal is higher than that of waiting.

Figure 2.

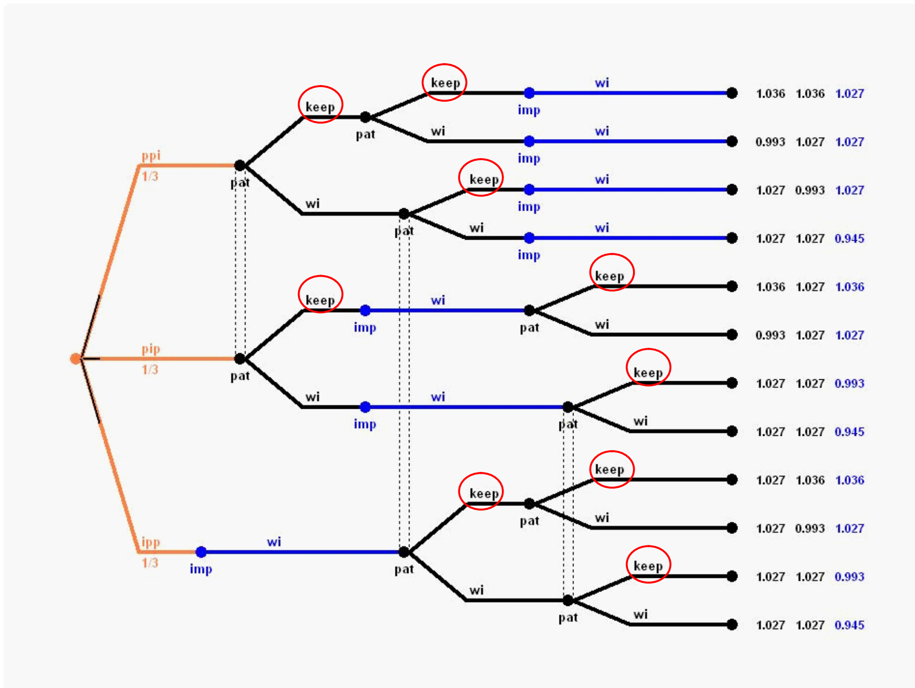

Figure 2: Sequential game

The uncertainty about the type sequence implies that there are several information sets that are not singletons. As a consequence, subgame perfection can be applied only partially because there is just a limited number of proper subgames. Nevertheless, the game has an intuitive unique prediction. The decision of the patient depositors in the last position is clear, since they have to compare only the payoffs their action would imply. It is also clear that a patient depositor upon observing a waiting will wait, because it gives her the highest possible payoff. As a consequence, a patient depositor in position 1 also waits because - since she will be observed - she can induce the other patient depositor to wait as well. Therefore, if a patient depositor in position 2 observes a withdrawal, she concludes that she must observe the impatient depositor's action. This implies that the other patient depositor is in the last position and by choosing to wait the patient depositor in position 2 can induce her to wait. Thus, for patient depositors the best response when observing a withdrawal is to keep the money deposited. When considering together all the strategies it becomes clear that the game - independently of the type sequence - leads to the outcome in which patient depositors do not withdraw. The circles in Figure 2 show the optimal actions and indicate that in each branch the first-best allocation is achieved. Therefore, the complete network generates confidence.

The optimal decision for a patient depositor in the last position is easily defined. She waits upon observing any history. We know also that if a patient depositor observes any history containing a waiting, then her best response is to wait.

Thus, in any equilibrium a patient agent waits when observing the following history

$$(k),(k, wi),(wi, k) \text{ or } (wi, wi). \tag{6}$$

The previous strategies also imply that we must have

$$s_1(pat, \emptyset) = k, \tag{7}$$

because otherwise a patient depositor would deviate unilaterally and receive $u(c_2^*)$, the highest possible payment.

Run happens if at least one of the patient depositors withdraws. When patient depositors at the beginning of the line wait, then later-coming patient depositors will wait as well, so to generate a run we should have the first patient depositor withdraw. Hence, we propose the following strategies:

$$s_1(pat, \varnothing) = wi,$$
$$s_2(pat, (wi)) = wi.$$

If these strategies really form an equilibrium, then depositors at any position should observe histories consisting only of withdrawals. These strategies can be supported by the following belief:

- when observing (wi) : the withdrawing agent was patient, and the impatient type still has to come.

Given the proposed run strategy the probability that a patient depositor has withdrawn in position 1 is $\frac{1}{2}$.

Whether we have a perfect Bayesian equilibrium with run boils down to the question if a deviation at the first position is profitable or not. The profitability of deviation depends on how subsequent depositors interpret the deviation. If a patient depositor at the first position deviates, then subsequent depositors will observe histories which are off the equilibrium path. The concept of weak perfect Bayesian equilibrium (see Mas-Colell *et al.*) does not restrict in any way beliefs off the equilibrium path, so after (wi) a patient agent may believe with probability 1 that the last agent will be the impatient one. Hence, the optimal action is to withdraw and we have a bank run.

We use the concept of perfect Bayesian equilibrium that imposes restrictions on the off-equilibrium beliefs. More concretely, we use assumption B(ii) in section 8.2.3 in Fudenberg-Tirole (1991). Using this restriction, the second agent´s belief about the type of the first agent (who has withdrawn) is the following:

$$\delta_2(\theta_1 = pat \mid (wi)) = \tag{8}$$
$$= \frac{\delta_2(\theta_1 = pat \mid (\emptyset)) * s_1(wi \mid (\emptyset), pat)}{\delta_2(\theta_1 = pat \mid (\emptyset)) * s_1(wi \mid (\emptyset), pat) + \delta_2(\theta_1 = imp \mid (\emptyset)) * s_1(wi \mid (\emptyset), imp)}$$

In the formula $\delta_2(\theta_1 = pat \mid (wi))$ represents the belief of depositor in position 2 about depositor in position 1 being of type patient, when depositor 2 observes the history (wi), and $s_1(wi \mid (\emptyset), pat)$ is the probability that a patient agent plays "wi" upon observing nothing. This probability is zero, because $s_1(wi \mid (\emptyset), pat) = 0$.

Expression (8) incorporates the idea that when observing nothing a patient agent always follows suit. Therefore, when observing (wi) the second agent cannot assign positive probability to the second agent being a patient one. As a consequence, when observing (wi) depositors know that the first agent has been the impatient one, so the strategy for any patient agent should be to keep the money in the bank. That is $s_2(pat, (wi)) = k$. This implies that a patient agent at the first position will choose to keep, since the ensuing game (that is, each game beginning with (k)) leads to the first best.

The complete network leads to the desired outcome, but do we need it or less links are enough to generate confidence? To answer the question consider the following information structure.

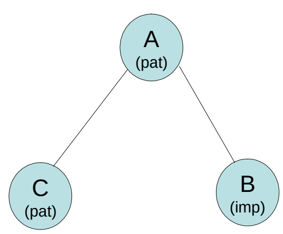

Figure 3: Intermediate case

Notice that since the position is unknown and for two players (C and B) cannot be inferred, we cannot represent the extensive form.[9] There are two cases: the depositor with two links is either one of the patient depositors or the impatient depositor. In any case, A will know her position. If A is patient and in position 1 or 3, then given the payoffs her optimal choice is to wait. If A is patient and in position 2, then the optimal choice upon observing a waiting is to wait, whereas the optimal choice when observing a withdrawal

[9] See chapter 3 in Fudenberg-Tirole (1991).

depends on the strategy of the other patient depositor. For the patient depositor with one link waiting upon observing a waiting should be part of her equilibrium strategy. Nevertheless, the previous arguments do not pin down for her what to do when observing nothing or a withdrawal.

Consider the following behavioral strategy for a patient depositor:

- withdraw when having one link and observing nothing or a withdrawal;
- withdraw, when having two links and observing a withdrawal;
- wait otherwise.

Given this strategy there are situations in which patient depositors withdraw. For instance, this is the case when the first depositor to act is either the impatient depositor or a patient one with one link. Therefore, the information structure in Figure 3 does not generate confidence.

3.6 The main result

We have seen that in the case of two patient and an impatient depositor, the unique information structure that generates confidence requires that all of them be connected. In this subsection we present the condition that ensures confidence for a depositor in a general setup. First we define when an information structure generates confidence.

Definition 1 Γ *represents an information structure that generates confidence for depositor* i *if for any sequence of decision, the unique equilibrium strategy for depositor* i *is to keep her funds deposited.*

Any depositor i will keep her funds deposited if she can be sure that there will be enough patient depositors who do so. Therefore, the information structure should be such that an individual patient depositor have the certainty that - independently of the sequence of decision - when depositors decide consecutively at the end there will be enough depositors who keep their funds deposited.

The main result of the paper is the following:

Proposition 1 Γ *generates confidence for depositor* i *if depositor* i *is in a clique of size* $\bar{\eta} + (n - p)$.

The proof is relegated to the Appendix. We use perfect Bayesian equilibrium as the solution concept.

The proposition says that a sufficiently big clique generates confidence for the patient depositors who are in it. Sufficiently is determined by the number of impatient depositors and the threshold that depends on the parameters of the model. If in a large enough clique the outcome of the game depositors play yields that a sufficient number of patient depositors wait, then patient depositors will wait. The size of the clique guarantees that the decision to wait is optimal independently of the decisions of depositors who are not included in the clique. Even if all of the depositors outside the clique withdraw, the bank will have enough funds to pay a relatively high payoff (that is, $c_2(\eta) > c_1^*$) to the depositors who choose to wait.

The intuition for the need of a clique is illustrated in Figure 3, but now imagine that all the depositors are patient and the threshold is 3.[10] Since there are three patient depositors in this example, payoffs are not the same as those in the example of the previous subsection. For simplicity, suppose that $u(c_2(\eta = 3)) > u(c_1^*) > u(c_2(\eta < 3))$ and $u(c_1^*) > \frac{1}{3}u(c_2(\eta = 3)) + \frac{2}{3}u(c_2(\eta = 1))$. Given these payoffs, can the first best achieved, that is do all depositors wait? Depositor A´s strategy is clear: she waits when observing nothing (that is, when she is in position 1), when observing two waitings or two withdrawals and she withdraws otherwise. Consider the following behavioral strategy for depositors C and B:

- withdraw when observing nothing or a withdrawal;
- wait otherwise.

Depositors C and B are in a symmetric position. Consider the sequence of decision: C, B, A. When C or B is called to decide, she only knows that she is in position 1 or 2. The belief "the other will withdraw" sustains an equilibrium in which both withdraw. The conditions on the payoffs may seem somewhat arbitrary, but notice that the complete network leads to the first best under the same conditions. Thus, even if all the depositors are patient, if they cannot observe each other perfectly, coordination on the good equilibrium may fail.

Regarding the size of the clique, the necessity of having $\bar{\eta} + (n - p)$ is clear. This is the minimal size that ensures that there are $\bar{\eta}$ patient depositors in the clique. But, is it sufficient to ensure the unique outcome? The proof shows that it is and the intuitive argument is as follows. Depositors know their positions in the clique because they can infer it based on the number of previous actions. The optimal choice of a patient depositor when observing $\bar{\eta} - 1$ waitings at any position is to wait. Consider now the history consisting of $\bar{\eta} - 2$ waitings and no withdrawals. A patient depositor observing it knows that there is at least one more patient depositor later in the sequence of decision. Knowing the best response of a patient depositor observing $\bar{\eta} - 1$ waitings, a patient depositor's optimal action is to wait. But then the history beginning with $\bar{\eta} - 2$ waitings followed by a withdrawal reveals that the last depositor must have been an impatient one. Therefore, a patient depositor's best response is to wait, because there must be at least one more patient depositor and this decision induces any subsequent patient depositor to wait as well. As a consequence, the number of withdrawals will be at least $\bar{\eta}$, and that makes waiting the optimal action. We may apply the

[10] If the threshold is 2, then the game is trivial.

same line of reasoning to show that for any history beginning with $\bar{\eta}-2$ waitings any subsequent withdrawal must be due to an impatient depositor. A patient depositor upon observing any such history knows that there is at least one more patient behind her and if she waits, then any subsequent patient depositor will wait as well. Hence, at any equilibrium the strategy for a patient depositor when observing a history which begins with $\bar{\eta}-2$ waitings should be to wait. Otherwise, she would deviate unilaterally. By moving one step further, consider now the history consisting of $\bar{\eta}-3$ waitings and no withdrawals. By the previous result a patient depositor's best response when observing this history is to wait. Thus, observing a history beginning with $\bar{\eta}-3$ waitings and then followed by a withdrawal reveals that the last depositor must have been an impatient one. Therefore, a patient depositor observing this history knows that there are at least two more patient depositors behind her. If she waits, then the resulting history will have $\bar{\eta}-2$ waitings and by the previous arguments a patient depositor would know that there is at least one more patient depositor behind her, and her best response would be to wait. The same reasoning holds for any history beginning with $\bar{\eta}-3$ waitings. At any equilibrium the strategy for a patient depositor when observing a history which begins with $\bar{\eta}-3$ waitings should be to wait. Otherwise, she would deviate unilaterally.

We can continue along the same lines to show that at any equilibrium the strategy for a patient depositor when observing a history which begins with $[0,\bar{\eta}-1]$ waitings should be to wait. The reasoning excludes the possibility of equilibria where patient depositors at the beginning of the sequence withdraw because they believe that subsequent patient depositors will withdraw as well. If they wait, then they can induce those subsequent patient depositors to wait as well. As the game begins, if the first depositor is a patient one, then she anticipates the best responses of subsequent depositors and chooses to wait. Hence, the second depositor can be sure that if she observes a withdrawal, then it is due to an impatient depositor. This line of reasoning leads to the conclusion that patient depositors in the clique will wait and the first best obtains.

The main result has a straightforward corollary.

Corollary 1 *If the information structure generates confidence, then the maximal number of withdrawals is* $n-\bar{\eta}$.

If an information structure generates confidence then each patient depositor who is in the clique will wait. This puts an upper bound on the maximal number of withdrawals. The corollary implies that there may be outcomes with many withdrawals (suggesting that a bank run is occuring), but the bank still may survive. This is contrary to Diamond and Dybvig (1983) who have either all or none of the patient depositors running. Our model allows for intermediate cases with only a fraction of patient depositors participating in the bank run. Ennis and Keister (2009b) call this case a partial run and they show how they may arise in a simultaneous-move model á la Green and Lin (2003) where depositors know their position in the sequence of decision. Our model sheds light on how these partial bank runs may appear in a setup in which observability matters.

3.7 Discussion

The size of the clique required by our main result can be very large, it should involve a substantial proportion of all depositors. It is hard to conceive how an information structure between depositors may arise in which everybody may observe everybody. This suggests that confidence in the sense put forward in this paper is difficult to appear. Therefore, the "fickle nature of confidence" (Reinhart and Rogoff 2009) seems to remain an inherent part of financial intermediation.

The main result is based on the fact that depositors are *ex ante* identical with the same deposit size. Suppose instead that there are depositors of different size and the large depositors control a considerable share of the total amount of deposits. Since period-2 consumption is affected by the amount of money the bank has after $t = 1$, large depositors' decision becomes important. In the next section we present a bank run that occured in Turkey to illustrate that in such a setup our model becomes relevant.

4 A case study: Turkey's Special Finance Houses

Starr and Yilmaz (2007) analyze the dynamics of depositor behavior during a bank-run episode that occurred in Turkey in the first half of 2001, a period of macroeconomic and financial troubles in that country.[11] In February, the largest Special Finance House (SFH) closed its doors abruptly, setting of a series of runs on the other finance houses. SFHs are Islamic banks that comply with prescriptions of the Islamic law, the Shariah. This subsector of the Turkish banking industry was not covered by deposit insurance.

Starr and Yilmaz (2007) use the detailed data provided by Kuwait Turk Evkaf Special Finance House (KTEFH) to study the withdrawal dynamics. The run started on February 12, 2001 after the closure of the largest SFH and finished on April 25, 2001 when depositors regained confidence and net flows into the SFH became positive again. Data show that small depositors (those with less than \$5000 deposited) accounted for 93,3% of the total number withdrawals, but those withdrawals represented only 14,1% of the total amount withdrawn. On the other hand, depositors with accounts exceeding \$25000 were responsible only for 1,3% of all withdrawals, but this small percentage accounted for 62% of the amount withdrawn. To understand how

[11] The presence of macroeconomic distress makes it hard to disentangle whether the bank runs were due to fundamental or coordination issues. The authors argue that coordination problem was important in trigerring the bank runs.

unexpected changes in withdrawals by some types of depositors affect the withdrawals of others the authors classified the depositors in three groups and carried out a VAR analysis.

They find that small depositors are quite responsive to other small depositors' withdrawal. Nevertheless, they are only marginally responsive to shocks coming from medium-size depositors (those with accounts between $5000 and $50000) and are unresponsive to large depositors' (with accounts over $50000) increased withdrawal rates. These results are probably the consequence of the fact that the behavior of large depositors is hard for other depositors to observe. The behavior of medium-size depositors was highly affected by bursts of withdrawals of both the small and medium-size depositors, while the effect of a shock to withdrawals by large depositors was not significant. Large depositors did not react to shocks to withdrawals from small accounts and were only modestly affected when the withdrawal rate of medium-size depositors. Nevertheless, large depositors reacted heavily and quickly to a shock to withdrawals from large accounts.

The run on KTEFH relates to our result in the following way. Large depositors in KTEFH represented only a reduced subset of all depositors, only 0,6 % of all depositors had more than $25000 in her account. These depositors seemed to be able to observe each other (they reacted to the withdrawals of other large depositors) so they were densely connected. Although the large depositors probably did not form a clique, but the information structure was dense enough to avoid massive withdrawals and the bank did not collapse. We do not have enough evidence to claim that the survival of KTEFH was due to the dense network among large depositors, but our model roughly fits the behavior of large depositors in this bank-run episode. The clique-like structure of the large depositors may have been an important factor to avoid a full-fledged bank run.

This real-world example calls the attention to the relevance of large depositors. Our model implies that to generate confidence among this subset of depositors controling a substantial share of the total amount of deposits may be possible due to the limited number of large depositors. This suggests that policymakers should pay attention to large depositors. Existing deposit insurance schemes are geared toward small depositors. If policy is able to foster confidence among large depositors and it is made known to the rest of depositors, then it may prevent bank runs.

5 Conclusion

The recent financial crisis has shown the importance of confidence in financial intermediation. Since empirical evidence points to the relevance of observability among depositors, we use social network to model how confidence may arise. We find that a sufficiently large clique is necessary and sufficient to generate confidence.

Such a clique ensures that all the patient depositors who are part of it will not withdraw their funds. This result also implies that partial bank runs with many patient depositors participating in them are possible.

The main result requires the formation of a possibly huge clique when considering homogeneous depositors regarding the deposit size. This seems to be implausible, but this is not the case if deposit sizes differ. The Turkish bank-run episode shows that dense connectedness of large depositors may curb coordination problems. In light of the importance of large depositors policymakers should perhaps revise deposit insurance schemes that are geared toward small depositors.

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6 Appendix

Lemma 2 *There exists a* $1 \leq \bar{\eta} \leq p$ *such that*

$$\begin{gathered} c_2(\bar{\eta}-1) < c_1^* \textit{ for any } \eta \leq \bar{\eta}-1 \\ \textit{and} \\ c_1^* \leq c_2(\bar{\eta}) \textit{ for any } \eta \geq \bar{\eta}. \end{gathered} \tag{9}$$

There exists a $1 \leq \bar{\eta} \leq p$ such that $c_2(\bar{\eta}-1) < c_1^* \leq c_2(\bar{\eta})$.

Proof. Note that $\left\lfloor \frac{n}{c_1^*} \right\rfloor$, that is the integer part of $\frac{n}{c_1^*}$, is the maximum number of agents to whom the bank is able pay c_1^*. Since $1 < c_1^*$, we have that $\left\lfloor \frac{n}{c_1^*} \right\rfloor < n$. That is, the bank cannot pay in period 1 to all depositors

$1 < c_1^*$, since it has only n units of deposits. Hence, for any $\eta < n - \left\lfloor \frac{n}{c_1^*} \right\rfloor$, $c_2(\eta) = 0$. It just says that if the number of withdrawals is too high, then the bank runs out of funds and cannot pay anything to those who have waited.

On the other hand, $c_2^* = c_2(p)$ and $c_2(x) > c_2(x-1)$ for any $n - \left\lfloor \frac{n}{c_1^*} \right\rfloor < x - 1 < p$, so given

$$c_2(\eta) < c_1^* < c_2^* = c_2(p) \text{ for } \forall\ \eta < n - \left\lfloor \frac{n}{c_1^*} \right\rfloor \tag{10}$$

there is a unique $\bar{\eta}$ such that for any $\bar{\eta} \leq \eta$ we have $c_1^* \leq c_2(\eta)$, whereas for any $\eta < \bar{\eta}$ we have $c_2(\eta) < c_1^*$. ■

6.1 The main result

Proposition 2 *Γ generates confidence for depositor i if depositor i is in a clique of size $\bar{\eta} + (n-p)$.*

Proof. The proof proceeds in three steps. First, we show via a counterexample that the lack of a clique may result in that confidence does not emerge. The second step concerns the necessity of having a clique of the size that the proposition requires. In the last step, it is shown that a clique of this size is sufficient.

Step 1: Consider the example presented in subsection 3.5. The last part of the example shows that in the absence of a clique confidence cannot be generated.

Step 2: The threshold $\bar{\eta}$ implies that the clique should have at least $\bar{\eta}$ patient depositors. Since types are not observable, the minimal size that ensures that there are $\bar{\eta}$ patient depositors in the clique is $\bar{\eta} + (n-p)$.

Step 3: We show that in a sufficiently big clique the unique (sequential) equilibrium implies that patient depositors keep their funds deposited.

Notice that in depositors in the clique are able to infer their position in the sequence of decision within the clique. Henceforth, when we speak about position, we refer to the position in the sequence of decision within the clique and not to the position in the sequence of decision of all depositors.

We denote by a_i depositor i´s action. If she withdraws, then $a_i = 0$ and if she waits $a_i = 1$. Let $h_i = (a_1, a_2, .., a_{i-1})$ denote the history observed by depositor in position i, $i \in \{1, 2, ..., n_c\}$ where n_c denotes the number of depositors in the clique. Let ω_i and η_i represent the number of withdrawals and waitings in the history of depositor in position i.

A pure strategy for depositor i is a map $\mathbf{s}_i : \boldsymbol{\theta}_i \times H_i \rightarrow \{0, 1\}$.

Definition 2 *Depositor i acts truthfully if $a_i = \theta_i$.*

Definition 3 *A history truthful, if it can be unambigously verified that all previous actions have been truthful.*

Denote by $H^{tr}(\eta)$ the set of truthful histories which contain η waitings (and any $\hat{\omega}$ withdrawals). ■

Lemma 3 *Assume that once an element in $H^{tr}(\eta_i \geq \bar{\eta})$ is reached all subsequent depositors will act truthfully, that is, $s_i(\theta_i, h^{tr}(\eta_i \geq \bar{\eta})) = \theta_i$ for $i = \bar{\eta} + \hat{\omega} + 1, ..., n_c$, where $h^{tr}(\eta_i \geq \bar{\eta}) \in H^{tr}(\eta_i \geq \bar{\eta})$. Then, for the set of truthful histories which contain $\bar{\eta} - 1$ waitings (and any $\hat{\omega} \in [0, n_c - \bar{\eta}]$ withdrawals), we have $s_i(\theta_i, h^{tr}(\eta_i \geq \hat{\eta} - 1)) = \theta_i$ for $i = \bar{\eta} + \hat{\omega}, ..., n_c$.*

Proof. The lemma assumes that once a truthful history containing $\bar{\eta}$ waitings and possibly withdrawals is reached, for any possible continuation sequence subsequent patient depositors will wait. Therefore, the only equilibrium strategy when observing a truthful history with $\bar{\eta} - 1$ waitings is to act truthfully. If a patient depositor observes $h^{tr}(\bar{\eta} - 1) \in H^{tr}(\bar{\eta} - 1)$, then by waiting she will cause a history which belongs to $H^{tr}(\bar{\eta})$. By our assumption, all subsequent depositors will be truthful, so the first best obtains yielding the highest obtainable payoff to the patient depositors. Since any truthful history is equivalent to a degenerate belief, given such a history the unique sequential equilibrium strategy is to be truthful, since there is no unilateral profitable deviation. ■

The previous induction step can be used repeatedly.

Corollary 2 *Assume that for the set of truthful histories which contain $\bar{\eta}$ waitings (and any $\hat{\omega}$ withdrawals) we have $s_i(\theta_i, h^{tr}(\eta_i \geq \bar{\eta}) = \theta_i$ for $i = \bar{\eta} + \hat{\omega} + 1, ..., n$. Then, for the set of truthful histories which contain $\acute{\eta} \in [0, \bar{\eta} - 1]$ waitings (and any $\hat{\omega}$ withdrawals), we have $s_i(\theta_i, h^{tr}(\eta_i \geq \acute{\eta})) = \theta_i$ for $i = \acute{\eta} + \hat{\omega} + 1, ..., n$.*

Proof. In the previous lemma we have shown the case when $\acute{\eta} = \bar{\eta} - 1$. What happens if a patient depositor observes a truthful history with $\acute{\eta} = \eta - 2$ and $\hat{\omega}$ withdrawals? By waiting, the resulting history will be a truthful one with $\bar{\eta} - 1$ waitings and $\hat{\omega}$ withdrawals. By our common knowledge assumption all subsequent depositors will know that up to depositor $\bar{\eta} + \hat{\omega} - 1$ all actions have been truthful, so by the previous lemma all subsequent actions will be truthful as well. The resulting first best yields the highest possible payoff to any patient depositor, so there is no unilateral profitable deviation. Hence, given the belief embodied in the history the unique sequential equilibrium strategy is to be truthful. The same argument can be applied iteratively. ■

Consider a patient depositor who observes a history which contains $\bar{\eta} - 1$ waitings and $\omega_i \in [0, n_v - \bar{\eta} - 1]$ withdrawals. Given any such history the only equilibrium strategy is $s_i(pat, h^{tr}(\bar{\eta} - 1)) = 1$ because it leads to the first best which yields the highest obtainable payoff. Therefore, we may apply the corollary to this set of truthful histories.

Lemma 4 *For the set of truthful histories which contain $\acute{\eta} \in [0, \bar{\eta} - 1]$ waitings (and any $\hat{\omega}$ withdrawals), we have $s_i(\theta_i, h^{tr}(\eta_i \geq \acute{\eta})) = \theta_i$ for $i = \acute{\eta} + \hat{\omega} + 1, ..., n$.*

Proof. Apply corollary to $H^{tr}(\bar{\eta}-1)$. ■

The previous lemma determines the best responses for any history that may come up in the game. Moreover, all these histories will be truthful! This is the case, because - for instance - any history starting with waitings is truthful, and since our strategy prescribes truthful action to these histories the resulting histories must be truthful as well. Any withdrawal after histories starting with waiting(s) must be due to impatient depositors. If a history begins with a withdrawal, then it is truthful, because a patient depositor would have waited according to the best responses we have found, so histories starting with withdrawals will be truthful as well.

Proposition 3 *The strategy* $\mathbf{s}_i(\boldsymbol{\theta}_i, \theta_1^{i-1}) = \theta_i$ *and the belief* $\mu(\theta_{i+1}^n \mid \theta_1^i)$ *for all* i *is the unique sequential equilibrium of the game.*

Proof. Consider the history consisting of $\acute{\eta}$ waitings and no withdrawal, where $\acute{\eta} \in [0, \bar{\eta}-1]$. The unique compatible belief is that it is a truthful history, so by the previous lemma $\mathbf{s}_{\acute{\eta}+1}(\theta_{\acute{\eta}+1}, \theta_1^{\acute{\eta}}) = \theta_{\acute{\eta}+1}$. As a consequence, the history starting with $\acute{\eta}$ waitings and followed by a withdrawal reveals that the last depositor must have been an impatient depositor. Therefore, a patient depositor observing this history knows that it is a truthful one, so $\mathbf{s}_{\acute{\eta}+2}(\theta_{\acute{\eta}+2}, \theta_1^{\acute{\eta}+1}) = \theta_{\acute{\eta}+2}$. This argument shows that any history starting with $\acute{\eta} \in [0, \bar{\eta}-1]$ waitings must be a truthful one, so the previous lemma applies to them. Now consider how the game unfolds. If the first depositor is patient, then her belief is $\mu(\theta_2^n \mid pat, \emptyset) = \mu(\theta_2^n \mid pat)$ which corresponds to our definition of a truthful history. The previous lemma ensures that her optimal action is to wait. Thus, the second depositor can be sure to observe a truthful history, so her optimal action is to act truthfully as is the case for each later-coming depositor. depositors at any position can be sure to observe a truthful history to which the unique equilibrium strategy is to be truthful. ■