Networks That Enable Coordination*

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February 15, 2010

Abstract

We show how in a coordination problem with multiple equilibria certain conditions on the social structure determine whether the equilibrium is unique or not. The conditions regard the observability of actions. An agent may observe other agents' actions only if connected by an underlying social network. We identify necessary and sufficient conditions over the structure of the social network for the emergence or avoidance of such events. The coordination problem embodied in our game with binary actions easily extends to diverse socioeconomic situations as investment decisions, strikes, revolts or bank runs.

Keywords: social networks, coordination failures, multiple equilibria

JEL Classification: C70, C90; D85; G21

[Work in progress]

1 Introduction

In many socioeconomic environments an agent's payoff depends on her own action, the actions of others and possibly on some exogenous conditions, e.g. economic fundamentals. In these settings coordination problems arise in a natural way. Coordination on a specific outcome generates a surplus for the agents if a sufficiently high proportion decides to take a given action. A common feature of these problems is that even though the socially efficient decision requires that agents take a specific action, possibly they fail to do so. The intuition behind the coordination failure is that agents fear that the number of other agents taking the desirable action will fall short of the number that would make that action optimal.

*Preliminary draft. Hubert Janos Kiss is grateful for the financial support from the Spanish Ministry of Science and Innovation under research project number ECO2008-00510.

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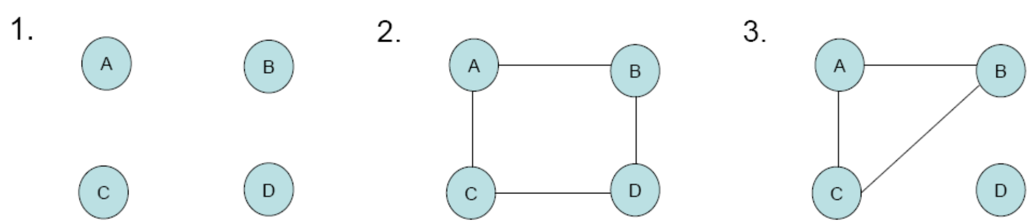
Coordination games typically have multiple equilibria that in many cases may be Pareto-ranked. The basic aim when studying these games is to find conditions or mechanisms that lead to a unique outcome without coordination failure. We pursue the same objective and establish conditions about the social structure that lead to coordination on the Pareto-superior outcome. In the spirit of Chwe (2000) we look for minimal sufficient networks that enable coordination regardless of the beliefs of the agents. In our model, the network allows the agents to observe other agents' actions. More precisely, agent i observes agent j's action if j acted before and they are connected by a link. We find conditions about the network structure that ensure that independently of the order in which agents decide coordination obtains. The knowledge agents have about the network is parametrized. If this knowledge is limited, then agents may observe only the agents they are connected to (henceforth, neighbors), while if knowledge is wider, then they may observe, for instance, the neighbors of their neighbors.

We use as illustration the following classic investment example. A finite number of agents face an investment and choose between investing or not. The investment project is carried out only if sufficient agents decide to invest.[1] In this case it yields a high return, while if the number of investors does not reach the threshold, the project is not executed and those who invested incurr a loss. Socially it is efficient that the project be carried out since those who participate obtain a profit. Moreover, in the first best everybody invests and harvests the profits. We look for conditions that ensure that the project is executed.

To illustrate the problem consider the following example. Suppose that there are four investors and if three of them invest, then the project is carried out and yields a positive net return. Suppose that the network structure is common knowledge and consider the following social structures:



On picture 1 investors are not connected. If they are sufficiently optimistic and believe that the other

[1] Lawrence H. Summers in his Richard T. Ely lecture about international financial crises proposed the following situation: "Now suppose that my fundamental situation were such that everyone would be paid off as long as no more than one-third of the investors chose to withdraw. What would you do then?" (Summers, 2000) Our setup is obtained by substituting "withdraw" for "not invest"

investors invest, then the project will be executed. But in case of pessimism the opposite happens. Therefore, picture 1 represents a social structure that does not lead with certainty to the socially efficient outcome, so this network does not enable coordination. On picture 2 investors are more connected, but still this structure does not enable coordination independently of beliefs. If any of the agents observes two other agents investing, then she invests as well. But if only one other investing agent is observed, then it is not clear what the optimal choice is. If B observes A investing, then she may find it optimal not to invest. It is the case, because even if she invests and it is observed by D, B does not know with certainty if C will invest and upon observing only one other agent investing, D may choose not to invest. In other words, there is no profitable deviation from the strategy that has "do not invest when observing one other agent investing". The network depicted on picture 3 - even though it has less links than the network on picture 2 - enables coordination, because A,B and C will invest. Beliefs for them do not play any role. Independently of the order in which they act, all of them will invest. This is the case because upon observing that two other agents have invested, the dominant strategy for the third is to invest as well. By backward induction, if the best response when observing that somebody has already invested is to invest as well. By applying this backward reasoning, if any of the three investors does not observe any action (so she knows to be the first to act), then her best choice is to invest and induce the other two agents to follow suit. Since the network structure is common knowledge, agent D knows that the other three agents will coordinate, so she invests as well. Notice that if we restricted the knowledge about network structure in a way that agents know only their direct neighbors and the neighbors of the direct neighbors, then still agents A,B and C invest and the project is carried out, but whether agent D invests or not depends on her belief.

In the model, we assume that agents are called to decide in a random order. Actions are taken sequentially and the network determines observability: therefore, if two agents are connected, the posterior one acts after observing her predecessor's action. We say that a network enables coordination if independently of the sequence of decision (that is, the order according to which agents decide) the number of the agents who invest is at least as high as the threshold, in any equilibrium of the game. If all agents decide to invest, then we say that the network is sufficient for obtaining efficiency.

In the investment game we study the case of homogenous agents, who share the same threshold. In this case, given a threshold t, we show that the existence of a subset of t completely connected agents is required for coordination. If agents in the network are able to observe that such a clique exists, all agents would invest, and the first best would be reached.

After that we analyze a set up with two different type of agents, with some agents interested and some other who are not interested in the investment (i.e., the coordination outcome). In such a context we obtain a similar result, that also highlights how the existence of a sufficiently high subset of agents able to observe their mutual actions enables the coordination on the efficient outcome. We apply this result to the classic model of bank runs by Diamond and Dybvig (1983).

Our model may be applied to a wide range of socioeconomic situations characterized by binary actions and interdependence of payoffs. In economics, currency attacks (see Morris and Shin (1998) in which speculators choose whether to attack or not a currency, and bank run situations (see Diamond and Dybvig (1983)) with depositors deciding if to withdraw or not are just two examples that show the importance of coordination issues. From other fields, examples also abound. In strike situations workers are more willing to participate if there are already many others participating. The same argument applies to revolts (see Chwe (2000)) and other threshold problems of sociology (Granovetter 1978)

2 Literature Review

Our paper is related to several strands of literature as we will detail. Global games introduced by Carlsson and van Damme (1993) have been shown to lead to a unique ourcome in problems that previously were thought to have multiple equilibria. In global games, it is assumed that there is a noise technology which produces private signals about the fundamentals for the agents, and this technology is common knowledge among the players. These noisy signals generate beliefs about fundamentals, beliefs about other players' beliefs about fundamentals, and so on. Models using global games arrive at a unique equilibrium because the heterogeneity of signals aligns the beliefs of the agents in a way that fosters coordination on the good outcome. Another important feature of these models is that players do not have any information about other players' action. Our approach is different. We consider a setup with fixed fundamentals, and we assume that it is common knowledge. Thus, we dispense with the possibility of achieving uniqueness through heterogeneous signals. Instead, we allow players to observe other actions. The conditions on the social structure that lead to the unique equilibrium achieve it by setting up a game of complete information which has a unique subgame perfect equilibrium.

The literature on herding that started with Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) is akin to our paper because in this literature previous actions are observed and taken into account. Similarly to global games, private signals about an uncertain fundamental are an integral part of these models. The main question in herding is how the different sources of information (private signals and previous actions) affect the decision of the agents. As already said, we do not consider uncertain fundamentals and our focus is also different, because our aim is to find the conditions that ensure that the right decision is taken.

Costain (2007) presents a setup that allows to test the robustness of global games and herding models. His main result shows that if most agents may observe a few previous actions instead of playing a simultaneous-move game, then in the face of fundamental uncertainty multiplicity of outcomes is restored. Our paper - without considering fundamental uncertainty - asks how the social structure enabling observability of previous actions should be so that agents choose optimally. We show that for the execution of the investment project it is sufficient if enough investors can observe each other mutually, while the first best (that is, all investors investing) requires that everybody be observable.

Coordination problems in an investment framework have been studied by Gale (1995) and Dasgupta (2007). They **consider** a game that is very similar to ours, but they **aim at studying** how the option to delay affects coordination. They find that this option reduces the incidence of coordination failure. In our model, we do not consider the possibility of delay, but the unique equilibrium of our game depends on backward induction arguments. Thus, investors in the beginning will invest because they know that subsequent investors will observe and follow suit.

The threshold model we use are very related to Granovetter (1978) and Chwe (2000). Chwe (2000) is the closest paper to ours regarding its aim. While both papers find the minimal structures that lead to coordination and study how structure and strategy are interrelated, Chwe does not consider actions only willingness in participate in a revolt, the topic of his paper. We consider actions and it allows us to use subgame perfection.

3 Model

3.1 Environment

We model situations of coordination, when agents require that a certain action is taken by others in order to find profitable for herself also taking the action.

Let be $N = \{1, 2, ..., n\}$ a set of agents. These agents have the possibility of taking an action $a = \{0, 1\}$. Let a_i be the action of agent i, and let t_i be the specific threshold of agent i. In order to model the coordination problem, let the utility of agent i be as follows:

$$u_i(a_i = 1, \sum_{j \in N} a_j \geq t_i) > u(a_i = 0) > u(a_i = 1, \sum_{j \in N} a_j < t_i)$$

Note that an agent prefers the action a_i if a sufficient amount of agents also takes that action. So we name $a = 1$ as the"risky action", and $a = 0$ the "safe action". If the decisions are taken simultaneously, there exist several equilibria, depending on how many agents coordinate **on** taking the action. When an agent chooses $a_i = 1$ we say that she takes the "risky action", because it is only profitable if a enough proportion of the other agents take it also.

If agents are homogeneous ($t_i = t, \forall i \in N$) the utility of agents can be described as follows:

$$u_i(a_i = 1, \sum_{j \in N} a_j \geq t) > u(a_i = 0) > u(a_i = 1, \sum_{j \in N} a_j < t)$$

Note that in this case, the game has two equilibria in pure strategies:

$$\begin{aligned} Eq.1 &: \quad a_i = 1, \forall i \in N \\ Eq.2 &: \quad a_i = 0, \forall i \in N \end{aligned}$$

where Eq.1 Pareto-dominates Eq.2. In one of them, all agents decide to take the "safe action". In the other one, agents coordinate on the "risky one".

3.2 Network and Information

We assume that there exists a social network that connects the agents. The network allows agents to observe other agents' actions, thus the one who decides later can base her decision on her neighbors' action who preceded her. A network Γ is the collection of pairs ij such that if ij belongs to Γ means that, under Γ, agents i and j are able to observe **each other's** actions (and, in particular, one of them plays after observing the action of the other). Therefore the network is undirected, meaning that $ij \in \Gamma \rightarrow ji \in \Gamma$. It is also worthy to define a clique. In network theory, a clique $s \in N$ in the network Γ is a subset of N such that $i, j \in s \rightarrow ij \in \Gamma$. We define also a *path* between two nodes i and k under the network Γ as **a** sequence of nodes $\{j_1, j_2, ..., j_{p-1}, j_p\}$ such that $\{ij_1, j_1j_2, ..., j_{p-1}j_p, j_pk\} \in \Gamma$. We define also the geodesic *distance* $d(i, j)$ as the minimal amount of links (the "shortest path") that separates any two nodes i and j, which are joined by a path. If it is not possible to find a path between two nodes, the geodesic distance is equal to infinity.

Since links allow the observability of actions, the existence of links transform the game in a sequential problem. Since possibly only a part of the agents are connected, the network allows for the existence of simultaneous and sequential decisions in a natural way. Therefore, the network determines which kind of game is played.

A key question is: what information do agents have. As we have said, links enable the observation of other actions, but when deciding, an agent may act in one way or another depending on the knowledge it has about the network. Therefore in the model we parametrize also the ability of the agents for observing the network structure. We say that a coordination game is of observational distance $\vec{d}$ if any node knows who are the neighbors of her (indirect) neighbors at a maximum geodesic distance $\vec{d}$. For example, $\vec{d} = 2$ means that any given node knows who are their direct neighbors, who are the neighbors of their neighbors (these nodes are at a distance $d = 2$), and who are the neighbors of these indirect neighbors. If the game is of observational distance $\vec{d} = \infty$, we assume that the network structure is common knowledge.

In such a context, we are interested in the conditions that are required over the network structure in order to obtain the efficient equilibrium. Our aim is to find network structures that allow the coordination of the agents or that imply univocally the coordination. In this sense, we define two type of structures:

Definition 1 *Let G be a game of coordination among a set N of agents who are connected through the network Γ. If the agents have an observational distance $\vec{d}$ we say that Γ is a Network that Allows Coordination (NAC) if in the game of coordination, in any equilibrium, there is a subset of agents in N who play the risky action.*

This means that a NAC generates a subset of agents coordinating on the efficient equilibrium for sure. A stronger version of this notion is the following.

Definition 2 *Let G be a game of coordination among a set N of agents who are connected through the network Γ. If the agents have an observational distance $\vec{d}$ we say that Γ is a Sufficient Network for Efficiency (SNE) if in any equilibrium, efficiency is achieved.*

Note that in our context, any SNE is also a NAC. Under this stronger version of the definition, we find conditions that determine that all agents in the society take the action that generates efficiency. If there are different groups, some of the interested in the coordination problem and other who were not interested, if there exist a SNE, all agents interested in coordination would take the risky action in any equilibrium of the game.

The concept of SNE is very closed to Chwe (2000). We look for structures that implement uniquely the efficient equilibrium, while the concept of sufficient networks in Chwe (2000) describes network structures that generate that all agents revolted (an action similar to our "risky action").

The equilibrium notion which is used varies depending on the type of coordination game subjects are effectively playing. Now we detail the results.

4 Networks that Enable Coordination

4.1 Homogeneous Agents and the Investment Game

We say that the game is played among homogeneous agents if $t_i = t, \forall i \in N = \{1, 2, ..., n\}$. Under this framework, we have a classical Investment Game. Assuming that investing has a uniform cost of c, the previous utilities may be expressed by appropriate normalization as

$$u_i(a_i = 1, \sum_{j \in N} a_j \geq t) = 1 - c;$$

$$u(a_i = 0) = 0;$$

$$u(a_i = 1, \sum_{j \in N} a_j < t) = -c.$$

Assuming that $t \leq n$, It is also easy to check that, under complete information and if the game is strictly sequential, the first best (everyone invests) emerges as the unique Subgame Perfect Equilibrium. It occurs because the last one to decide, if the previous agents have taken the risky action (invested) decides to invest. By backward induction, is easy to find that everyone investing is the unique Subgame Perfect Equilibrium.

How can be described this situation under our network framework? If the game is completely sequential, and everyone have perfect information about all steps of the game, we would say that all the agents are connected among them. We say then that there exists a *complete network*, that is, in such a case, $ij \in \Gamma, \forall i \in N, \forall j \in N$. What is needed about the observational distance? Since all agents are connected to everyone more in the network, in order to know this fact they have just to know who are the neighbors of their direct neighbors. Only with that information, agents know that everybody is connected and they are playing a classical sequential game. Therefore observational distance $\vec{d} = 1$ is required. This result is presented as follows

Remark 1 *In an Investment Game with homogenous agents, if observational distance is $\vec{d} \geq 1$, the complete network Γ is a Sufficient Network for Efficiency (SNE), considering Subgame Perfect Equilibria.*

The objective we have is to find the "minimal conditions" that allow for a NAC or a SNE. We state this result in the following Theorem.

Theorem 1 *In an Investment Game with homogenous agents, if $t \leq n$ and we consider Subgame Perfect Equilibria,*

1. *Γ is a NAC if and only if there exists a clique $s \in \Gamma : \#s \geq t$.*

2. for $\vec{d} \geq 1$, Γ is a SNE if and only if there exists a clique $s \in \Gamma : \#s \geq t$ and $d(i,j) \leq \vec{d} - 1, \forall i \in N, \forall j \in s$

Proof. *See Appendix* ■

The result allows us to predict situations in which we would expect the coordination emerging for sure. It is interesting to note that the existence of a clique is required. This occurs because, if a clique exists, all the agents who form it are able to observe their mutual actions, and this fact is which allows coordination.

Intuitively, when there exists a clique of size t (or larger), there are enough agents observing their mutual actions. The first one to act knows that, and in the subsequent game, enough people will be able to show their actions to the rest of the clique, and therefore all these agents take the risky action. In this sense, if such a clique exists, we know that, at least, members of the clique will take the action, and the network structure is therefore a NAC.

The second part states that if all agents in the network are sufficiently near to the clique, and this allows them to know that the clique exists, then everyone takes the risky action, and the efficient outcome emerges.

It is also interesting to see that if $\vec{d} = 1$, it is required in order to implement the efficient outcome that a complete network exist. Moreover, if $\vec{d} \geq 2$, networks which are SNE are of the core-periphery type. Finally, if $\vec{d} = \infty$, the existence of a clique of size t is enough for obtaining a SNE.

4.2 Agents who have different objectives

Imagine now that the society is also formed by two different groups of agents, which differ in their utilities. We let agents with the previous utility form be in set $N_1 = \{1, 2, ..., n_1\}$, while we introduce another set $N_2 = \{n_1 + 1, n_1 + 2, ..., n_1 + n_2\}$ of agents who are not interested in the coordination. Therefore utilities are now:

$$u_i(a_i = 1, \sum_{j \in N} a_j \geq t) = 1 - c; u(a_i = 0) = 0; u(a_i = 1, \sum_{j \in N} a_j < t) = -c, \forall i \in N_1$$

$$u_i(a = 0) > u_i(a = 1), \forall i \in N_2$$

This means that there is a subset of the society who is not interested in the previous coordination problem, but they simply prefer the non-risky action. What conditions over the network structure ensure that agents interested in the coordination problem take the risky action?

Theorem 2 *In a Coordination Game with two type of agents, if $t \leq n_1$ and we consider Sequential Equilibria,* *1.* Γ *is a NAC if there exists a clique* $s \in \Gamma : \#s \geq t + n_2$.

2. for $\vec{d} \geq 1$, Γ *is a SNE if there exists a clique* $s \in \Gamma : \#s \geq t + n_2$ *and* $d(i,j) \leq \vec{d} - 1, \forall i \in N, \forall j \in s$

Proof. Work in progress ■

This result states that if a sufficient part of the agents are able to observe their mutual actions, the efficient outcome is the unique result even if there are some agents who do not want it. This model can be applied to situations as revolts, when people desire to revolt if an enough proportion of the society do, although there are others who do not want. If sufficient agents are able to observe everyone else, the unique outcome of the game will be that agents effectively coordinate.

Another interpretation of the model is the one of bank runs.

4.2.1 Application to Bank Runs

This model is closed to the classical approach to the bank run problem by Diamond and Dybvig (1983). In Diamond and Dybvig (1983), there is a certain amount of patient and impatient depositor. Before each one knows her type, a bank pools all the resources and invest part of them in a long-run investment project, offering a contract that offers a higher return to the agents who wait. After that, types are privately revealed by the nature, and impatient agents desire to withdraw immediately.

The game is simultaneous, and there are two different equilibria in pure strategies. In one of them, all patient depositors wait until the end of the period and receive the higher payoff, being obtained the efficient result. However, in the other equilibrium, all patient depositors try to withdraw in the first period, and it is optimal given that the others are also doing the same, generating the bankruptcy of the bank.

In our framework, we can say that patient depositors belong to N_1 while impatient depositors belong to N_2. We can name the "risky" action $a = 1$ as "not withdrawing" the money from the bank, while $a = 0$ would be "withdrawing". In this sense, impatient depositors prefer strictly the action $a = 0$ while patient depositors prefer the "risky" action of not withdrawing only if a sufficient proportion of other patient depositors do the same.

In the literature of bank runs that follows Diamond and Dybvig (1983), the issue of uniqueness or multiplicity of equilibria is relevant, specially since Green and Lin (2000), which ensure that, if general contracts are allowed, bank runs because of coordination problems should not arise. The contribution to this debate from Theorem 2 would be that, even if contracts are simple, if there is a sufficiently connected society (i.e., there exists a clique sufficiently large) we obtain that no bank run (the "efficient equilibrium") is the unique outcome.

5 Conclusion

We explore which conditions over the social structure determine univocally one equilibrium (the efficient one) in a classic problem of coordination. As a result, the existence of a clique of agents able to observe their mutual actions is sufficient for enabling the coordination in the efficient outcome. Interestingly, there can be societies more connected, in the sense of having more connections, but which do not obtain the efficient equilibrium. It occurs if there does not exist this subset of agents completely connected.

The model can be applied to some classic problems, determining the conditions under which they have a unique equilibrium prediction. The investment game as well as he case of bank runs can be understood in this set up. Other interesting extension is the analysis of revolts and other threshold problems, as those analyzed by Granovetter (1978). We are working on these lines.

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7 Appendix

Proof. of Theorem 1

Let analyze what occurs in the subgame played by agents belonging to the clique. Let be the agents in the clique

$$s_{clique} = \{c_1, c_2, ..., c_t\}$$

and let

$$\begin{aligned} s^* \left\{s_1^*, ..., s_{c_1}^*, s_{c2}^*, ..., s_{ct}^*, ...\right\} &: \\ U_i\left(s_i^*, s_{-i}^*\right) &\geq U_i\left(s_i, s_{-i}^*\right), \forall s_i \in S_i, \forall i \in N \end{aligned}$$

be an equilibrium.

For the $t-th$ agent in the clique, note that any equilibrium strategy must include that se chooses $a_t = 1$ if observes that their $(t-1)$ predecessors in the clique choose also $a = 1$, since

$$u\left(a_{c_t} = 1, \sum_{j \in N_{-c_t}} a_j \geq t-1\right) = u\left(a_{c_t} = 1, \sum_{j \in N} a_j \geq t\right) > u(a_{tth} = 0)$$

Therefore

$$s_{c_t}^*\left(\sum_{j=c_1}^{c_{t-1}} a_j = t-1\right) = 1, \quad (1)$$

This information must be taken into account by agent $(t-1)\,th$ in the clique. Given (1), for the $(t-1)\,th$ agent in the clique, if she observes that the $t-2$ predecessors in the clique choose $a = 1$

$$u_{c_{t-1}}\left(a_{c_{t-1}} = 1, \sum_{j \in N_{-c_t}} a_j \geq t-2, s_{c_t}^*\right) > u_{c_{t-1}}(a_{c_{t-1}} = 0)$$

Therefore

$$s_{c_{t-1}}^*\left(\sum_{j=c_1}^{c_{t-2}} a_j = t-2\right) = 1$$

Doing this recursively, we find that in any equilibrium s^*

$$\begin{aligned} s_{c_{t-2}}^*\left(\sum_{j=c_1}^{c_{t-3}} a_j = t-3\right) &= 1 \\ &... \\ s_{c_2}^*\left(a_{c_1} = 1\right) &= 1 \\ s_{c_1}^* &= 1 \end{aligned}$$

That is, if there exist a clique of size t, any equilibrium strategy of the game includes that first agent who decides chooses $a_{c_1} = 1$, and it is a equilibrium path of the subgame played by the members of the clique.

Given this fact, if such clique exist and it is known by all the subjects in the game, we find that

$$u_i\left(s_i = 1, s^*_{c_1}, s^*_{c_2}, ..., s^*_{c_t}\right) > u_i\left(s_i = 0, s^*_{c_1}, s^*_{c_2}, ..., s^*_{c_t}\right)$$

in any equilibrium path of the game.

We have shown that, if the network includes a clique of size t, agents in the clique coordinate in the "risky action" and therefore Γ is a NAC. Now, we show that if Γ is a NAC, it includes a clique of size t.

Let P_i be the set of predecessors of i such that i is able to observe their action, and A_{P_i} be the set of actions of subjects in P_i over a certain history. Thus strategy of i is a mapping from A_{P_i} to $\{0,1\}$. Note that, if Γ is a NUE with $t \leq N$, the following strategy:

$$\vec{S} = \left\{ \vec{s}_i = \begin{cases} 0 \text{ if } \sum_{a_j \in A_{P_i}} a_j \leq t-2 \\ 1 \text{ if } \sum_{a_j \in A_{P_i}} a_j > t-2 \end{cases}, \forall i \in N \right\}$$

cannot be part an equilibrium, since in such case we would obtain the pareto-dominated equilibrium. However, for any individual and in any subgame we have that

$$U_i\left(\vec{s}_i, \vec{S}_{-i}\right) \geq U_i\left(s_i, \vec{S}_{-i}\right)$$

in any subgame except in the subgame where

$$\sum_{a_j \in A_{P_i}} a_j > t-2$$

Therefore, if $\vec{S}$ is not an equilibrium, it is because there exist at least an agent i who observes at least $t-1$ predecessors, and therefore in any Γ which is NUE, there exists at least an agent i whith at least $t-1$ predecessors. Let name c_t this agent.

If Γ is NUE, the following strategy profile

$$S' = \left\{ s'_i = \begin{cases} 0 \text{ if } \sum_{a_j \in A_{P_i}} a_j \leq t-3 \\ 1 \text{ if } \sum_{a_j \in A_{P_i}} a_j > t-3 \end{cases}, \forall i \in N, i \neq c_t \right\}$$

can not be an equilibrium profile, since over its equilibrium path the dominated equilibrium would emerge. When would this strategy not be part of an equilibrium? When some agent have incentives to deviate in some subgame.

For an agent j who observes $\sum_{a_k \in A_{P_j}} a_j < t-3$ it would be optimal, since after her decission (and out of the subgames she plays) everyone would choose $a_i = 0$, since in such subgame she maximizes her payoffs with $a_j = 0$, and S' can be an equilibrium.

For an agent j who observes $\sum_{a_k \in A_{P_j}} a_j > t-2$ it would be optimal, since in such subgame she maximizes her payoff with $a_j = 1$, and S' can be an equilibrium.

For an agent j who observes $\sum_{a_k \in A_{P_j}} a_j = t-2$ we have two posibilities:

$P_j \notin Pi, \forall i : j \in P_i$. In such a case, following S' any agent j would maximize her payoff with the strategy s'_j, and S' can be an equilibrium.

$P_j \notin Pi, \forall i : j \in P_i$. In such a case, given that

$$s'_i = 1 \text{ if} \sum_{a_j \in A_{P_i}} a_j > t-2$$

agent j would increase her payoff with the strategy

$$s'_j = \begin{cases} 0 \text{ if } \sum_{a_j \in A_{P_i}} a_j \leq t-4 \\ 1 \text{ if } \sum_{a_j \in A_{P_i}} a_j > t-4 \end{cases}$$

in the subgame where it observes $\sum_{a_j \in A_{P_i}} a_j = t-2$, since his succesor knows also the subgame and in that case they can generate the pareto-dominant equilibrium.

Therefore, in order to avoid that S' is an equilibrium, it is needed that an agent j observes actions of $t-2$ predecessors, who are also predecessors of one subject i who is also a succesor of j.

Applying this reasoning recursively, we find that a network Γ is NAC only if there exists a set of t agents such one of them is predecesor of all the rest, another is succesor of this one and predecesor of al the rest, and so on, being this set on subjects a clique of size t.

In order to show Point 2, we point that, if agents do not know the existence of the clique, they can play the safety action, since they do not know if the risky action will be played or not. All agents will play for sure the "risky" action if and only if each one of them knows that the clique exists. In that case, Γ is a Sufficient Network for Efficiency. ■