Social Housing under Oligopoly*

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Abstract

In this paper it is stressed that the setting up of a system of affordable (or social) housing, similar to the Low Income Housing Tax Credit (LIHTC) program created by the Tax Reform Act of 1986 in the US, may have a "collusive effect" in a housing market where there is imperfect competition among housing developers. That system may decrease the total number of houses built in the market, increase the price of (non-social) houses, reduce social surplus and increase the profits that housing developers obtain from non-social houses. The design of the social housing system and of the scheme to compensate housing developers should take into account these results.

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1 Introduction

Governments intend to provide affordable housing for low- and moderate-income households. The costs of direct government provision of affordable housing are, however, very high. Hence, governments have looked for other mechanisms to guarantee affordable houses for those households. With this objective, they have designed systems where housing developers must provide some affordable or social houses when they obtain a permit to build non-social houses (throughout the paper I maintain the term "social houses" for those affordable houses provided by profit-maximizing housing developers).[1]

In these affordable-housing systems the government regulates the price and the number of social houses, the maximum income level that a household may have to be eligible for a social house and the compensation that each housing developer may obtain from the government for the provision of social houses. The Low Income Housing Tax Credit (LIHTC) program, created in the US by the Tax Reform Act of 1986, is a social housing system with these characteristics.

The LIHTC program establishes the requirements that housing development projects must satisfy to qualify for tax credits and the mechanism to calculate these credits that provide incentives for private sector production of affordable housing for low-income household. The requirements to qualify for tax credits refer to maximum prices (rents) of social houses, minimum periods where social houses must be provided by the housing developer and maximum income levels that allows eligibility for a social house. Each housing developer may select the combination of the maximum income level that limits eligibility for a social house and the percentage of houses devoted to social housing. The selection has to be made from a set of pairs of values of those variables established by the regulator. There are no limits on the prices (rents) that may be charged to households that live in non-social houses.

This work considers a social housing system with the following characteristics: i) housing developers must provide some social houses when they obtain a permit to build non-social houses, ii) the number and price of social houses are decided by the government, and iii) social houses are allocated at random among households with incomes below some income

[1] In some countries the term social houses is used only for affordable houses provided by non-profit organizations or directly by the government.

level fixed by the regulator. This characterization simplifies some aspects of a program like the LIHTC, but it outlines an affordable-housing system close to the LIHTC.

The analysis below focuses on the "collusive effect" that a social housing system may have in a housing market where there is imperfect competition among housing developers. It is proved that, as a consequence of that effect, the setting up of a social housing system may decrease the total number of houses built in the market, induce a price of non-social houses greater than the price of houses without that system, reduce social surplus and increase the profits of housing developers even in situations where the price of social houses is below the unit cost of production.[2]

The traditional view on the impact of social housing accepts that, as social housing is an alternative for some households, it reduces the demand for non-social houses. This reduction decreases the quantity of non-social houses and, when the supply curve of houses is increasing, it also decreases de price of houses. The magnitudes of these effects depend on the elasticities of the supply and demand curves in the housing market and on the parameters that characterize the social housing system. When the supply curve of houses is increasing, the total number of houses built increases with social housing as the decrease in the number of non-social houses is smaller than the number of social houses built.[3] Perfect competition in the housing market is implicitly assumed under this view.

The results obtained in this work require the combination of housing developers with market power and a situation where households eligible for a social house are not only those who cannot afford to buy a house when there is not social housing, but also some households who can afford a house in this latter case.

In the analysis of housing markets it is often implicitly assumed that there is perfect competition in housing supply. However, this is not a correct assumption for an important proportion of the local housing markets. Often these markets can be approximated by districts or cities which are far enough apart for only local supply and demand conditions to affect prices

[2]For an empirical analysis of the effect of the LIHTC program on house building see Malpezzi and Vandell (2002). Baum-Snow and Marion (2009) evaluate the impact of the LIHTC program on property values and on house building.

[3]If the supply curve of houses were horizontal, social housing would not affect the price of non-social houses and the total number of houses built would increase when social houses are priced below non-social houses.

and output, and the number of housing developers in each market is limited. It seems adequate to consider that there may be imperfect competition in those housing markets and, hence, that housing developers may have market power.[4]

In a housing market there may be an important stock of used houses and some of these used houses may be empty. Moreover, the used houses may be a good substitute for the new houses supplied by housing developers. In this work it is considered that there are new households looking for a house in the housing market and that, if there were empty used houses in that market, these empty houses would be only a small part of the demand for houses by those new households.[5] The existence of the empty used houses does not prevent households from having market power.

In this work it is considered that there are more households eligible for a social house than the number of social houses available and that the available social houses are allocated at random among the eligible households.[6] The regulator could eliminate the excess demand for social houses by reducing the maximum income level that qualifies for a social house. But he prefers to maintain that excess demand. A reason for this preference may be that there is asymmetric information on household income between the regulator and each household, as a consequence of some non uniformly distributed fraud in income disclosure among households. Moreover, the regulator may want to make sure that all households with low- and moderate-incomes are eligible for a social house.

It is also assumed in the analysis that this behavior of the regulator causes that some households who could afford to buy a house when there is not social housing obtain a social house. Hence, the regulator considers that some households that would buy a house without social housing must be eligible for a social house as their incomes are not high. As it will be shown below, if only households who cannot afford to buy a house when there is not social housing obtained a social house, the price and number of non-social houses would be the same as the price and number of houses when there is not social housing. In this latter case the profits of housing developers would

[4]See Somerville (1999) for an analysis of builders size and market concentration in housing supply.

[5]The housing market may be expanding due to an increase in population in that market.

[6]Different ways to allocate social houses among eligible households have been used in practice. See, for example, Olsen (2003) and Sinai and Waldfogel (2005).

decrease if social houses were priced below cost and they do not receive a compensation from the government.

The intuition for the results is as follows: imperfect competition among housing developers implies that the number of houses built when there are not social houses is above the joint profit-maximization (monopoly) level. Producers would benefit from a reduction in total production, but they cannot collude and agree on that lower quantity (commitment to collude may not be feasible and collusion is not allowed). Social housing may make up for the lack of ability of producers to coordinate on a lower production level and a higher price in the market of non-social houses. This may occur as it is less valuable to build a greater number of houses, because the expected demand for non-social houses is smaller and it does not have the same slope as the demand for houses without social housing. These changes induced by the setting up of a social housing system may allow oligopolists to increase their profits, even if they must sell social houses at a price below production cost. We may say that social housing induces a "collusive effect" in the imperfectly competitive housing market. The setting up of a social housing system may enhance oligopolistic control over housing development.[7]

The analysis in this work proceeds using a one period (static) housing market model and assuming Cournot competition among housing developers. As in a one period model there is no difference between renting and selling, we may, therefore, consider that social houses are rented or sold. In this context a linear demand for houses suffices to obtain the results. The possibilities of obtaining also these results in a dynamic model where houses are a durable good, or in contexts with other forms of imperfect competition, are discussed in section 5.3.

The reaction of housing developers, with market power, to the setting up of the social housing system must be taken into account when deciding on the design of the social housing system and on the way to compensate housing developers. If the setting up of the social housing system, combined with permits to build non-social houses, increased the expected profits of housing developers, no direct government funding, through tax credits or any other transfer mechanism, would be required for that system.

The paper is organized as follows: Section 2 presents the model used in this work and the equilibrium in the housing market when there are not

[7]In Sinai and Waldfogel (2005) there is an analysis on the effects of some low-income housing subsidies on the housing stock.

social houses. Section 3 analyzes the consequences of the setting up of a social housing system on the number of houses built, the price of non-social houses and the expected profits of housing developers. The effects on social surplus of the social housing system are studied in section 4. Section 5 extends the results obtained to the situation where affordable-houses have lower quality and to a possible implication of the asymmetric information on household income, and discusses the relevance of those results for other contexts. Finally, section 6 contains the conclusions.

2 Model and equilibrium without affordable-housing projects

Consider that the housing development industry is oligopolistic, with n housing developers that compete *à la Cournot* ($n \geq 2$). All sites for housing development are homogeneous and each housing developer builds houses only on one particular site, different for each housing developer.[8] Hence, the number of sites available for housing development is given and equal to n. The reason may be that the opportunity cost of land is very high in all other sites or that housing development is only allowed in the n sites considered.

All houses offered in the market are identical from the households point of view. Housing congestion in sites is assumed away (capacity limits of sites are such that they are not attained in this housing market).[9] The unit cost of production of houses, represented by c, is constant. Moreover, housing developers are neutral to risk.

There are many price-taking households. Each household demands a house. The inverse demand function for houses is: $p = a - bQ$, where Q represents the quantity of houses in the market, $b > 0$ and $a > c > 0$. Hence, there are $\frac{a-p}{b}$ households that are willing to pay at least p for a house. Households and housing developers know the demand function for houses and the cost of production.

In the housing market there may be used houses previously built. In this case, let $p = a - bQ$ be the inverse demand function for new houses and Q the

[8] Assume that the extension of a site is big enough to allow several-story buildings to be built on it. Hence, a site is bigger than a lot, and it may be seen as closer to a subdivision.

[9] The possibility that each housing developer had several sites where he might build houses, considering that there are n housing developers and there is not congestion in any site, is discussed in section 5.3.

quantity of new houses in the market. Assume that new houses and houses built in the past are perfect substitutes. A used house might be sold (rented) at a price equal to the price of a new house: if there were U used houses it would be $a = a_o - bU$, with a_o constant. If there were some empty used houses, more used houses would be sold (rented) by their owners when the price of houses were higher. In this case it would be $a = a_o - bU(p)$, where $U(p)$ is the supply function of used houses and $U'(p) > 0$.[10]

When there is not a social housing system each active housing developer i, $i = 1, ..., n$, will solve the following problem:

$$\max_{q_i}(a - bQ - c)q_i,$$

where q_i is the number of houses built by housing developer i and $Q = \sum_{i=1}^{n} q_i$. The first order condition of this problem is:

$$a - bQ - c - bq_i = 0.$$

Adding up the n first order conditions we get:

$$q_i^* = \frac{a-c}{b(n+1)}.$$

Hence,

$$p^* = a - bQ^* = a - \frac{n(a-c)}{n+1} = \frac{a+nc}{n+1},$$

and the total profits of housing developers are:

$$\pi^* = \frac{n(a-c)^2}{b(n+1)^2}.$$

3 Affordable-housing projects

Let us consider a situation where the regulator establishes that housing developers must sell (or rent) D social houses ($\frac{D}{n}$ houses each housing developer) at a price equal to r. Moreover, each housing developer is free to decide the number of non-social houses to build on his site, besides the $\frac{D}{n}$

[10] If the stock of empty houses were big enough, the inverse demand curve of new houses might be significatively more horizontal than the inverse demand curve of (new and used) houses (inverse demand curve more horizontal: $\left|\frac{dp}{dq}\right|$ not greater for any p and $\left|\frac{dp}{dq}\right|$ smaller for some p).

social houses required by the regulator. There may be a zone within each site where social houses are built. It is considered in this and the following sections that social houses and non-social houses have the same quality. The situation where social houses are of inferior quality is discussed in section 5.

The regulator wants to allocate social houses to households with low incomes, or at least with incomes that are not high. In this work it is considered that there may be asymmetric information on household income between the regulator and each household, due to a non uniformly distributed fraud in income disclosure among households. Moreover, the regulator may want to make sure that all households with incomes that are not high are eligible for a social house. As a consequence, the households eligible for social houses are not only those households with the D lowest reported incomes. The households eligible for a social house are those households with the T lowest reported incomes, with $T > D$.[11]

Let us consider that the valuation of housing services increases with income. Hence, there is an m, with $r < m < a$, such that only households with willingness to pay smaller than m for the services of a house are eligible for a social house. We may assume without loss of generality that all households with willingness to pay for a house smaller than m are eligible for a social house.[12] As there are more than D households eligible for a social house we have $\frac{m-r}{b} > D$.[13]

In the analysis it is considered that it is $r < p^* = \frac{a+nc}{n+1} < m$. This implies that some households that would buy a house without social housing are eligible for social houses. The regulator considers that some households that would buy a house without social housing must be eligible for a social house as their incomes are not high.

The available social houses are allocated at random among the eligible households. The probability of obtaining a social house for an eligible household is, therefore, equal to $\frac{bD}{m-r}$.

[11] Hence, the social housing policy considered in this work is a policy of the kind considered in Nichols and Zeckhauser (1982): a targeting policy. However, sometimes the social housing policy is more a policy of the type suggested by Akerlof (1978): a tagging policy, that classifies households according to characteristics over which they have no control (disability or age, for instance).

[12] The analysis would be analogous if we considered that for any x such that $r < x < m$, the number of households eligible for a social house were $g(x) < \frac{m-x}{b}$, with $g(x)$ continuous and $g'(x) < 0$. This possibility will be discussed in section 5.

[13] Therefore, the value of m is in the interval $(r + bD, a)$. Households with willingness to pay for a house smaller than r will not ask for a social house.

The allocation at random of social houses among the eligible households implies that the demand for non-social houses will depend on the resultant allocation. In this work it is considered that the decision on house building is a long or middle-term decision and, hence, that housing developers have to choose the number of non-social houses to build in a context where they are uncertain about the demand for non-social houses. As housing developers are neutral to risk, they will only take into account the expected price of non-social houses that would be obtained for each number of non-social houses built.

Let us proceed to obtain the expected inverse demand function for the services of non-social houses. Let H denote the total number of non-social houses built and h_i the number of houses built by housing developer i. Note that $H < \frac{a-r}{b} - D$ is required to have non-social houses that are more expensive than social houses. With the notation of section 2 it will be $Q = H + D$ and $q_i = h_i + \frac{D}{n}$.

If $0 < H \leq \frac{a-m}{b}$ the price of non-social houses would be $a - bH$ (given by the inverse demand function without social housing). The reason is that, in this case, there would not be households eligible for a social house that value the services of a non-social house in, at least, $a-bH$ ($a-bH \geq a-b\frac{a-m}{b} = m$).

When $\frac{a-m}{b} < H < \frac{a-r}{b} - D$ households with willingness to pay for a house between $a - bH$ and m are eligible for a social house and some of them may obtain a social house. The price of non-social houses for each number of non-social houses built will, thus, depend on the result of the allocation process of social houses. As these houses are allocated at random among the eligible households the price of non-social houses for H between $\frac{a-m}{b}$ and $\frac{a-r}{b} - D$ might take many different values and, in general, will be smaller than $a - bH$. That price will be a random variable: $\tilde{p}$. However, as housing developers are neutral to risk they will take into account only the expected price of non-social houses, $E(\tilde{p}(H))$, for each level H of non-social houses.

The expected price of non-social houses when $\frac{a-m}{b} < H < \frac{a-r}{b} - D$ is equal to the price of non-social houses that is obtained when social houses happen to be distributed among the eligible households in a particular way. This particular distribution corresponds to the case where, for any group of eligible households, the proportion of those households that obtain a social house is equal to the probability that any eligible household has of obtaining a social house ($\frac{bD}{m-r}$).

With that particular distribution of social houses among the eligible households, the number of non-social houses sold at a price p of non-social houses such that $m > p > r$ would be $\frac{a-p}{b} - \frac{m-p}{b}(\frac{bD}{m-r})$.[14] Hence, if H non-social houses were built when housing developers are uncertain about the allocation of social houses, with $\frac{a-m}{b} < H < \frac{a-r}{b} - D$, the expected price of non-social houses, $E(\tilde{p}(H))$, would be such that

$$H = \frac{a - E(\tilde{p}(H))}{b} - \frac{m - E(\tilde{p}(H))}{b}(\frac{bD}{m-r}),$$

that is,

$$E(\tilde{p}(H)) = \frac{a(m-r) - mbD}{m-r-bD} - \frac{b(m-r)}{m-r-bD}H.$$

Note that $a > m > r + bD \Rightarrow \frac{a(m-r)-mbD}{m-r-bD} > a$ and $\frac{b(m-r)}{m-r-bD} > b$.

Therefore, the expected inverse demand function for non-social houses when a social housing system has been established is

$$E(\tilde{p}(H)) = \begin{cases} \frac{a(m-r)-mbD}{m-r-bD} - \frac{b(m-r)}{m-r-bD}H & \text{for } \frac{a-m}{b} \leq H < \frac{a-r}{b} - D, \\ a - bH & \text{for } H \leq \frac{a-m}{b}. \end{cases}$$

We also have that the expected inverse demand function for houses by those households that obtain a social house is $m - \frac{m-r}{D}z$, for $0 \leq z \leq D$.

As $r < p^* = \frac{a+nc}{n+1} < m$, we know from the analysis in section 2 that the price of non-social houses, when there is social housing, will be at most m. The equilibrium in the market of non-social houses may correspond to a corner solution, with the expected price of non-social houses equal to m, or to an interior solution, with the expected price of non-social houses in the interval (r, m). To obtain this interior solution we proceed as in section 2, using $\frac{a(m-r)-mbD}{m-r-bD}$ instead of a and $\frac{b(m-r)}{m-r-bD}$ instead of b, and assuming that the situation is such that the n housing developers remain active.[15] Then, we get[16]

$$h_i^* = \left[\frac{a-c}{b} - \frac{(m-c)D}{m-r}\right]\frac{1}{n+1} < q_i^* \tag{1}$$

[14] Note that there are $\frac{a-p}{b} - \frac{a-m}{b} = \frac{m-p}{b}$ eligible households that are willing to pay between p and m for a house.

[15] We have to add the term $(r-c)\frac{D}{n}$ to the total profits of each firm, but the (interior) solution in the market of non-social houses does not depend on this additional term, as long as the n firms remain active.

[16] Note that $r+bD < m < a$ implies $(a-c)(m-r)-(m-c)bD \geq (a-c)(m-r-bD) > 0$ and $h_i^* > 0$.

and the expected price of non-social houses will be[17]

$$E(\tilde{p})^* = \left[a + nc + \frac{(a-m)bD}{m-r-bD}\right]\frac{1}{n+1} > p^*. \tag{2}$$

The interior solution given by (1) and (2) requires $E(\tilde{p})^* < m$ ($\Leftrightarrow H^* > \frac{a-m}{b}$). If $\left[a + nc + \frac{(a-m)bD}{m-r-bD}\right]\frac{1}{n+1} \geq m$ the corner solution $p^c = m$ and $h_i^c = \frac{a-m}{bn}$ will be obtained.

From (1) we obtain

$$H^* + D = \left[\frac{a-c}{b} - \frac{(m-c)D}{m-r}\right]\frac{n}{n+1} + D = \frac{(a-c)n}{b(n+1)} + \frac{m+nc-(n+1)r}{(m-r)(n+1)}D.$$

It cannot be guaranteed that the setting up of a social housing system as the one considered in this paper will increase the total number of houses built, even considering a linear demand function for houses (the number of houses built will increase in that case if and only if $m + nc > (n+1)r$).

Therefore, we have proved:

Proposition 1 *The setting up of a social housing system may increase the price of non-social houses and it may reduce the total number of houses built in the market.*

If the total number of houses built decreased when the social housing system is established, some households that would have got a house without social housing would not have neither a social house nor a non-social house in the new situation. If the price of non-social houses were greater than the price of houses without social housing, the social housing system would provide affordable houses to some households but the price that other households would have to pay for housing services would increase. Moreover, even if the total number of houses built increased with the social housing system, the price of a non-social house might be greater than the price of a house when there is not social housing.

The variation in expected profits of housing developers with the setting up of the social housing system is

$$E(\pi(D,r,m))^* - \pi^* = (E(\tilde{p})^* - c)H^* + (r-c)D - \pi^*$$

[17] The price of non-social houses for some allocations at random of social houses may be below p^*.

$$= nD\frac{a(m-r)(a-2m)+c(m-r-bD)(2m-c)+bDm^2}{(m-r-bD)(m-r)(n+1)^2}+(r-c)D. \quad (3)$$

Therefore, the setting up of a social housing system may increase the expected profits of housing developers (for instance, when $r \geq c$ and $a \geq 2m$ it is $E(\pi(D,r,m))^* - \pi^* > 0$). Moreover, from (3) we have that the expected profits of housing developers may increase even if $r < c$, that is, even in situations where they have to sell social houses at a price below production cost. We, thus, have:

Proposition 2 *The setting up of a social housing system may increase the expected profits of housing developers, even if the price of social houses were below the unit cost of production.*

Housing developers would be willing to participate in a social housing system that increases their expected profits. No direct government funding (tax credits or any other transfer mechanism) would be required to make that social housing system acceptable to housing developers.[18] Housing developers may even try to convince the regulator to design a social housing system profitable for them.[19] However, under that system the buyers of non-social houses would pay for the increase in profits of housing developers. Moreover, if social houses were sold at a price below production cost, there would be a cross subsidy from the buyers of non-social houses to households that obtain a social house.[20]

The results in Propositions 1 and 2 could be unexpected as it may be thought that the setting up of a social housing system implies that non-social houses face more competition. The explanation for those results lies on the assumption of imperfect competition in housing development. Cournot oligopolists are producing above the monopoly level when there are not social houses. The setting up of a social housing system may help housing developers to reduce the number of non-social houses built, as it may make

[18]Nevertheless, as it will be pointed out in the following section, some government expenses may be required to control the correct implementation of the social housing system.

[19]This possibility would depend on the ability of housing developers to capture the regulator of the social housing system. See Laffont and Tirole (1993) for a general analysis of the problem of capture of the regulator by economic agents.

[20]In Baum-Snow and Marion (2009) it is pointed out that there is an excess of applications for the LIHTC program. This may indicate the profitability of the program for chosen housing developers.

less valuable to build a greater number of non-social houses. We may say that social housing induces a “collusive effect” in the imperfectly competitive housing market.

The expected demand for non-social houses is smaller and it does not have the same slope as the demand for houses without social housing. These are the changes, induced by the setting up of a social housing system, that may allow oligopolists to increase their profits. The setting up of a social housing system may enhance oligopolistic control over housing development.

From (1) we have that in the linear case the quantity of non social houses increases with m and decreases with D and with r. From (2) we have that the expected price of non social houses decreases with m and increases with D and with r. From (3) it may be shown that the expected profits of housing developers may increase with D even if social houses have to be sold at a price below the unit cost of production.

These results for the linear case are a consequence of the change in the slope of the expected inverse demand function for non-social houses, for rental prices between m and r, when the social housing system is established. The absolute value of this slope $(\frac{b(m-r)}{m-r-bD})$ decreases with m and it increases with D, with r and with the setting up of a social housing system. The intuition for the results is as follows: When D increases the reduction in the expected demand for the services of non-social houses for prices lower than m is greater and, hence, a lower (rental or sale) price of those houses is less valuable for housing developers. When r increases or when m decreases the number of consumers eligible for social houses that are willing to pay the price of a social house decreases. Hence, in these latter cases the probability that an eligible household obtains a social house increases and, thus, the reduction in the expected demand for non-social houses for prices between r and m is also greater.

Therefore, we have:

Corollary 3 *An increase in the number of social houses may increase the price of non-social houses and the expected profits of housing developers, even if social houses had to be sold at a price below the unit cost of production.*

The result in Corollary 3 could be unexpected as it may be thought that an increase in the number of social houses implies that non-social houses face more competition. The explanation is, again that an increase in the number of social houses may help housing developers to reduce the number of non-social houses built, as it may make less valuable to build a greater number of non-social houses.

4 Social surplus

Let us discuss in this section the effects of the setting up of a social housing system on social surplus and on the surplus of householders and of housing developers. Without social housing, social surplus (SS) is defined as householders surplus (HS) plus housing developers surplus (π^*). As

$$HS = \left[\int_0^{\frac{n(a-c)}{b(n+1)}} (a - bQ)dQ - \frac{a+nc}{n+1}(\frac{n(a-c)}{b(n+1)}) \right] = \frac{1}{2}n^2 \frac{(a-c)^2}{b\,(n+1)^2}$$

and $\pi^* = \frac{n(a-c)^2}{b(n+1)^2}$, we have $SS = \frac{1}{2}n\,(a-c)^2\,\frac{n+2}{b(n+1)^2}$.

Expected social surplus with social housing (ESS) is defined as expected householders surplus (EHS) plus expected housing developers surplus ($E(\pi(D,r,m))^*$) minus the cost of implementation of the social housing system (CI). With a social housing system let us denote by EHS_1 and EHS_2, respectively, the expected surplus of the buyers of non social houses and the expected surplus of households that obtain a social house (hence, $EHS = EHS_1 + EHS_2$).

We know that $E(\pi(D,r,m))^* = (E(\tilde{p})^* - c)H^* + (r-c)D$. We also have:[21]

$$\begin{aligned} EHS_1 &= \int_0^{\left[\frac{a-c}{b} - \frac{(m-c)D}{m-r}\right]\frac{n}{n+1}} (\frac{a(m-r) - mbD}{m-r-bD} - \frac{b(m-r)}{m-r-bD}H)dH - E(\tilde{p})^* H^* \\ &= \frac{1}{2}n^2 \frac{\left((a-c)(m-r) - bD(m-c)\right)^2)}{b\,(m-r)\,(m-r-bD)\,(n+1)^2}, \end{aligned}$$

and

$$EHS_2 = \int_0^D (m - \frac{m-r}{D}x)dx - rD = \frac{(m-r)D}{2}$$

When some households that would buy a house without social housing are eligible for social houses and the setting up of the social housing system reduces the total number of houses built in the market, the expected social

[21] Note that, when $D = 0$, $EHS_1 = \frac{1}{2}n^2\frac{(a-c)^2}{b(n+1)^2} = HS$, and $(E(\tilde{p})^* - c)H^* = \frac{n(a-c)^2}{b(n+1)^2} = \pi^*$.

surplus diminishes, and this occurs even if $CI = 0$. In this case $EHS_2 > 0$ and the expected profits of housing developers may increase. However,

$$EHS_1 + EHS_2 + (E(\tilde{p})^* - c)H^* + (r - c)D < HS + \pi^*,$$

as less houses are built and the households that buy a house without social housing are those households that are willing to pay more for a house.

When some households that would buy a house without social housing are eligible for social houses and the setting up of the social housing system increases the total number of houses built in the market, the expected social surplus may also decrease. This decrease may occur even if $CI = 0$. The reason is that with social housing the price of non-social houses may be higher, even if the total number of houses built increased. In this case some households that would buy a house without social housing do not buy a non-social house, and they may not obtain a social house. The total surplus lost by those households, that do not obtain a house, may not be compensated with the additional total surplus generated with the sales (or rentals) of the social houses. Nevertheless, $EHS_2 > 0$ and the expected profits of housing developers may increase.

We thus have:

Proposition 4 *The setting up of a social housing system may reduce the expected social surplus, even if there were no costs of implementation of that system.*

The number of houses built would increase for sure with the setting up of a social housing system if only households who cannot afford a house without social housing obtained a social house. In this case the quantity and price of non-social houses are not affected by the social housing system ($EHS_1 = HS$ and $(E(\tilde{p})^* - c)H^* = \pi^*$). The expected surplus of householders that obtain a social house would increase (as $EHS_2 > 0$), and the change in the profits of housing developers would be equal to $(r - c)D$. Hence, the variation in social surplus would be $EHS_2 + (r - c)D - CI$. If CI were big enough, social surplus might also decrease with this social housing system even if $(r - c)D \geq 0$.

If the expected social surplus increased with the setting up of a social housing system, there might be a decrease in the expected surplus of each buyer of a non-social house and a decrease in the expected profits of housing developers. In this case the winners with the social housing system would be those households that obtain a social house.

Finally, if social housing induced a decrease in the price of non-social houses and if CI were not big, the setting up of a social housing system might increase expected social surplus, the expected surplus of each buyer of a non-social house, the surplus of households that obtained a social house and the profits of housing developers. The profits of housing developers might increase in this case only if the price of social houses were greater than the unit cost of production of houses, as there would be a decrease in the profits that housing developers would obtain from the sale of non-social houses.

5 Extensions

5.1 Asymmetric information on household income and eligibility

It has been considered in section 3 that all households with willingness to pay for a house smaller than m are eligible for a social house. This may not be the case, as the households eligible for a social house are those households with the lowest reported incomes, but the valuation of housing services increases with income. The possible difference between income and reported income may cause the non-eligibility for social houses of some households with willingness to pay for a house below m, but close to m. We may consider that, for any x such that $r < x < m$, the number of households eligible for a social house is $g(x) < \frac{m-x}{b}$, with $g(x)$ continuous and $g'(x) < 0$. In this case the analysis would be analogous to that developed in the previous sections. The probability of obtaining a social house for an eligible household would be $\frac{D}{g(r)}$, and the number of non-social houses sold at a price p of non-social houses such that $m > p > r$ would be $\frac{a-p}{b} - \frac{m-p}{b}(\frac{D}{g(r)})$.

5.2 Lower quality of affordable-houses

The quality of social houses is often smaller than the quality of non-social houses. To generalize the analysis in the previous sections, let us consider that the quality of a non-social house is 1 and the quality of a social house is α, with $0 < \alpha \leq 1$. The value of α is given by minimum requirements on the characteristics of social houses established by the regulator, while the quality of non-social houses is the usual quality of non regulated houses in that housing market.[22] Consider also that the constant unit cost of production of houses of quality s is $c(s)$, with $c(s) > 0$, $c'(s) > 0$ and $c(1) = c$. Moreover, assume that, for each household, the willingness to pay for the services of a house of quality s is equal to her willingness to pay for the services of a non-social house multiplied by $f(s)$, with $f(s) > 0$, $f'(s) > 0$ and $f(1) = 1$. In this context, the analysis follows as in the previous sections if social houses of quality α must be sold at a price $f(\alpha)r$, we use $c(\alpha)$ instead of c for those houses, and, furthermore, $f(\alpha)$ and r are such that any eligible household prefers to obtain a social house at a price $f(\alpha)r$ rather than a non-social house at the resultant market price of non-social houses.[23]

There may be, however, situations where some eligible households prefer to get a non-social house at the resultant market price of non-social houses rather than a social house at a price $f(\alpha)r$. When there are houses with two different quality levels, we may consider that households select the type of house to buy (or rent) as in a vertical differentiation context.[24] In that context households' preferences may be described as follows: a household, identified by j, enjoys (indirect) utility $U(j) = js - p$ when consuming a product of quality s sold at a price p (j represents the taste for quality of that household).[25] A household who does not buy (rent) a house obtains a utility equal to 0. In this context we have $f(s) = s$.

If $p_n \in \tilde{p}$ happens to be the price of non-social houses and social houses are sold at a price αr we may have $\frac{p_n - \alpha r}{1-\alpha} < m$. In this case, households with willingness to pay between $\frac{p_n - \alpha r}{1-\alpha}$ and m prefer to buy a non-social house while households with willingness to pay between r and $\frac{p_n - \alpha r}{1-\alpha}$ prefer to obtain a social house. Hence, it may occur that some eligible households prefer to get a non-social house at the resultant market price of non-social houses rather than a social house at a price αr.[26]

[22] Let us consider that the quality of social houses does not affect, or it has a very small effect, on the quality of non-social houses.

[23] When the price of a social house is equal to $f(\alpha)r$, the set of eligible households that are willing to pay that price for a social house does not depend on α.

[24] Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) settled the basis for many later analyses of quality differentiation.

[25] This structure of preferences is very usual in the literature on vertical differentiation, following Mussa and Rosen (1978). See Peitz (1995) for the construction of a direct utility function that has as its counterpart an indirect utility function as the one proposed.

[26] The problem that housing developers will solve in this case will not be the same as the one presented in section 3. Housing developers will now realize that the preference for a social house of some eligible households depends on the resultant price of non-social houses.

As $\frac{p_n-\alpha r}{1-\alpha}$ increases with p_n, we have that the higher the price of non-social houses the greater the set of eligible households that would prefer a social house. As an increase in the number of eligible households that prefer a social house reduces the probability of obtaining a social house for an eligible household, we have that, when p_n increases, the expected demand for non-social houses will change its slope and it will move closer to the demand for houses without social housing. This change in the expected demand for non-social houses may make less attractive a reduction in the number of non-social houses for housing developers (an increase in the expected price of non-social houses), for a given α. The effect will depend on the variation of the expected demand for non-social houses with p_n.

The effects of a reduction in the quality of social houses, and in their corresponding price, depend on the variation of $\frac{p_n-\alpha r}{1-\alpha}$ with α (note that p_n may also change with α). If $\frac{p_n-\alpha r}{1-\alpha}$ increased with α, a reduction in the quality of social houses might cause the maximum willingness to pay for a house among those eligible households that prefer a social house be lower than the price of houses without social housing. If this were the case we know from section 4 that the quantity and price of non-social houses would not be affected by the social housing system. A reduction in the quality of social houses could also avoid the "collusive effect" of social housing, discussed in previous sections, when the maximum willingness to pay for a house among those eligible households that prefer a social house is still higher than the price of houses without social housing. Nevertheless, this possibility depends on the variation induced in the expected demand for non-social houses. Moreover, the reduction in the quality of social houses would serve also to overcome in part the asymmetric information on income between the regulator and the households, if it induced the selection of a non-social house by those eligible households with higher incomes.

5.3 Relevance for other contexts

The analysis and results included in the previous sections would be the same if each housing developer had several sites where he might build houses and he decided the number of social houses and of non-social houses to build on each site, in a situation where congestion in each site were assumed away and the required number of social houses had to be provided ($\frac{D}{n}$ social houses each housing developer). To have n housing developers in this context it may be considered that entry of an additional housing developer is not profitable or that it is not allowed.

In this work it has been used a one period (static) housing market model. In a dynamic model where houses are a durable good the "collusive effect" induced by a social housing system might also exist, as the expected demand for the services of non-social houses in each period would be smaller and it would not have the same slope as the demand for the services of houses without social housing. That dynamic model would allow for simultaneous consideration of renting and selling as differentiated alternatives.[27]

The results, obtained for the case of Cournot competition, might also apply to other forms of imperfect competition among housing developers. The reason is, again, that the relationship between the expected demand for non-social houses and the demand for houses without social housing does not depend on the type of imperfect competition considered.

6 Conclusion

This work has considered a housing market where there is imperfect competition in the supply of houses and a social housing system where housing developers must provide some social houses when they obtain a permit to build non-social houses. The regulator determines the number of social houses to build, their price and the maximum income level that a household may have to be eligible for a social house. Social houses are allocated at random among eligible households, and there are households who are eligible for a social house and that would be able to afford a house if there were not a social housing system. The number of housing developers is given to the regulator. In the introduction of the paper it has been pointed out how close is the social housing system considered to the Low Income Housing Tax Credit (LIHTC) program, created by the Tax Reform Act of 1986 in the US.

In this paper it has been shown that, when there is imperfect competition in the housing market, a social housing system may have a "collusive effect" in that market. As a consequence of this effect, the setting up of the social housing system may reduce the total number of houses built, increase the

[27] A dynamic model might also allow the consideration of housing as a consumption good and as an investment asset: see, for example, Brueckner (1997), Dusanski and Wilson (1993) and Dusanski and Koç (2007).

price of non-social houses, reduce social surplus and increase the profits that housing developers obtain from non-social houses. When the setting up of a social housing system substitutes for the inability of housing developers to collude and enhances oligopolistic control over housing development, households that do not obtain a social house will face a higher price for a (non-social) house.

Hence, the reaction of housing developers with market power to the setting up of the social housing system must be taken into account when deciding on the design of the social housing system and of the scheme to compensate housing developers. If the profits of housing developers increased with social housing, direct government funding of the social housing system (through tax credits or any other transfer mechanism) would not be required.

A reduction in the quality of social houses could avoid the "collusive effect" of social housing, but this depends on the variation induced in the expected demand for non-social houses. That reduction in the quality of social houses would also overcome in part the asymmetric information on income between the regulator and the households, if it induced the selection of a non-social house by those eligible households with higher incomes.

A reduction in the asymmetry of information on income between the regulator and each household (for instance, through an increase in fiscal inspection) may make the "collusive effect" discussed in this work less likely. If that asymmetry of information were smaller, there would be less eligible households and there would also be less eligible households that would buy a house without social housing. The effects, on the price of non-social houses and on the total number of houses built, of a reduction in this asymmetry of information with respect to income depend on the variation induced in the expected demand for non-social houses by that increase in fiscal inspection.

If the social housing system were designed in such a way that only households with very low incomes were eligible for a social house, there would not be eligible households who can afford to buy a house when there is not social housing. In this case the price and number of non-social houses would be the same as the price and number of houses when there is not social housing. That social housing system would not induce a "collusive effect", but there might still be some low- and moderate income households that do not obtain a house.

7 References

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