

Growth, sectoral composition, and the evolution of income levels*

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Abstract

This paper asserts that the endowments of production factors cause cross-country differences in GDP per capita by generating disparities in the sectoral composition. We characterize the dynamic equilibrium of a two-sector endogenous growth model with many consumption goods that are subject to minimum consumption requirements. In this model, economies with the same fundamentals but different endowments of capitals will end up growing at a common rate, although the long run level and sectoral composition of GDP will be different. Because the total factor productivity in multisector models depends on sectoral structure, these differences in capital endowments will also generate sustained differences in the total factor productivities.

JEL classification codes: O30, O40, O41.

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1. Introduction

New growth theory has provided increasing evidence suggesting that the accumulation of production factors alone cannot explain the observed cross-country differences in GDP per capita (see, for instance, McGrattan and Schmitz, 1999; and Parente and Prescott, 2004). Authors like Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) argue that differences in GDP per capita are mainly explained by differences in total factor productivity (TFP, henceforth). Simultaneously, another branch of development literature explains international differences in the growth rates of GDP as the result of differences in the sectoral composition of GDP (see Echevarria, 1997; and Laitner, 2000). Recently, Caselli (2005), Cordoba and Ripoll (2004), and Chanda and Dalgaard (2005) unify these two lines of research by showing that changes in the sectoral composition contribute not only to output growth, but also to productivity growth without any true technological change. By using multisector growth models as the basis of growth accounting exercises, these works demonstrate that the aggregate level of TFP can be decomposed into a contribution from sectoral composition and a contribution from the level of technology. Since the empirical evidence shows that there exist meaningful differences in the sectoral composition across countries, the composition effect can then explain a large part of the differences in aggregate TFP levels across countries.

In order to account for the causes of the cross-country variation in output per capita, we then need theories that help us to explain the sustained differences in the sectoral composition of output across countries. Recent literature offers some explanations based on supply-side factors like differences in the aggregate productivity across sectors and the existence of barriers to allocate inputs to high productivity sectors.¹ In this paper, we however offer a complementary explanation based on the same demand-side argument used by literature to explain the structural change: the income elasticities of demand differ across consumption goods.² By using a growth model with non-homothetic preferences, we show that the stationary sectoral composition depends on the endowments of production factors. This result is in stark contrast with those derived from the neoclassical (either exogenous or endogenous) growth models, which predict convergence on sectoral composition across countries with the same fundamentals even when they start with different endowments. However, we will show that these countries can converge to different sectoral compositions when the non-homotheticity of preferences makes the sectoral composition of consumption and the sectoral allocation of production factors depend on the income level even in the long-run. As TFP depends on the sectoral composition, we will then conclude that the contribution of production factors to explain GDP is larger when TFP is endogenous.

We obtain our results from a baseline model based on the following fact: economies experiment meaningful changes in the structure of the production activity along the process of economic development. On the one hand, empirical evidence has shown that there is a relationship between the level and the sectoral composition of GDP. Baumol and Wolf (1989), Chenery and Syrquin (1975) and Kuznets (1971), among others, show

¹Caselli (2005) documents the main points of these theories.

²See, for instance, Echevarria (1997), Laitner (2000), Kongsamunt et al. (2001) or Foellmi and Zweimuller (2004).

that the process of development is related to the process of structural change. On the other hand, as Chari et al. (1997) point out, “the recent literature emphasizes that a broad measure of capital is needed to account for at least some of the regularities in the data.” In particular, the process of development is related to the growth of human capital, which explains the existence of a strong accumulation of human capital along the development process. Galor (2005) and Galor and Moav (2004) have shown the link between human capital accumulation and GDP growth. Therefore, according to the data, the process of development is linked to structural change and to the accumulation of human capital.

In this paper we consider a growth model that takes into account the dynamic relationship between human capital accumulation and structural change along the transition adjustment and, moreover, it is also consistent with the Kaldor facts regarding the long-run regularities in economic growth. More specifically, we extend the two-sector model of endogenous growth with constant returns to scale and with physical and human capital accumulation, that was introduced by Uzawa (1965) and Lucas (1988). Apart from the absence of external effects, the main departure from Lucas (1988) is in the modeling of preferences. We consider that consumers derive utility from the consumption of two heterogeneous goods. Moreover, preferences are assumed to be nonhomothetic to capture different income elasticities of demand for these two goods at any finite level of income. Hence, we impose that these differences in income elasticities of demands even hold along the balanced growth paths (BGP, henceforth). For that purpose, we introduce minimum consumption requirements that grow over time at exogenous rates. In particular, we assume that individuals use the consumption level of the most advanced economies as a reference with respect which their own consumption is compared to.³ Finally, for the sake of simplicity, and without loss of generality, we assume that there are only two production sectors and that each of them produces a commodity that can be devoted either to consumption or to increase one of the capital stocks.

These key assumptions on preferences yields important changes in the growth patterns predicted by the standard two-sector growth model.⁴ As in the standard model (see, for instance, Caballé and Santos, 1993), there is a continuum of BGPs and, moreover, the initial conditions on the two capital stocks determine the BGP the economy converges to. However, in contrast with the standard two-sector growth model, the BGPs differ in their ratios of physical to human capital and in their sectoral compositions when the following conditions hold: (i) individuals derive utility from the consumption of the two heterogeneous goods; (ii) the income elasticities of demand

³The basic assumption of this paper is the nonhomotheticity of preferences for any finite level of income. We conjecture that this property may also be obtained under some form of non-linear endogenous aspiration in consumption or an expansion in the quality of goods. However, in this paper, we consider the idea of the international demonstration effect developed by Nurkse (1953) in order to simplify the analysis and to focus the exposition on the derived behavior of sectoral structure. This author extended Dusenberry’s (1949) notion of demonstration effect to explain the low propensity to save of the developing economies during the first decades of the twentieth century. The aforementioned author asserted that, after having reached some level of output, the emerging economies try to catch up the consumption patterns of the developed economies.

⁴By standard growth model we will mean a growth model with a unique consumption good and with homothetic preferences.

differ across these consumption goods; and (iii) the technologies used by the two sectors exhibit different capital intensities. Thus, our model predicts that economies with the same fundamentals but different endowments of human and physical capital will converge to a common level of the relative price and to the same growth rate, although the long-run ratio of physical to human capital, the GDP to capital ratio and the sectoral structure will be different.

An important economic implication of these results is that, according to our theory, economies with different endowments will converge to different sectoral capital allocations and different sectoral compositions of consumption and GDP. The non-convergence to a common long-run sectoral composition has interesting consequences for the conclusions derived from the exercises of output decomposition. In fact, as was mentioned before, TFP in a multisector growth model depends on the sectoral structure, which in our model is endogenous and depends on capital endowments. Our results then imply that the levels of physical and human capital are a source of sustained differences in TFPs across economies. Thus, we assert that, because of the differences in the income elasticities for consumption goods, capital accumulation also affects the level of GDP by means of the induced changes in the sectoral composition and TFP. We can then conclude that, under the assumption of non homothetic preferences, capital endowments force a particular sectoral composition that limits the value of the aggregate production that can be attained.

Therefore, according to our model, the empirical studies of development accounting, by assuming an exogenous TFP, obtain biased measures of the contribution of capital endowments to explain the observed cross-country differences in GDP. In this paper, we show numerically the contribution of capital accumulation to explain differences in the values of TFP both at the BGP and along the transition. We show that this contribution of capital may be larger when economies are assumed to be out of the BGP because the process of structural change occurs along the transition. In particular, we show that two economies with different initial levels of physical and human capital exhibit significantly different sectoral structures along the transition when there is a negative relationship between the accumulation of the two capital stocks along the equilibrium path. In this case, the contribution of capital endowments to explain differences in GDP across economies is larger along the transition than at the BGP.

The plan of the paper is as follows. Section 2 presents the model. Section 3 characterizes the steady-state equilibrium. In Section 4, we study the contribution of capital stocks to explain differences in GDP per capita across countries by means of their effects on sectoral composition. Section 5 characterizes the implications for development accounting of taking into account the adjustment process during the transition towards the BGP. Section 6 concludes the paper and presents some possible extensions to the present research. All the proofs and lengthy computations are in the Appendix.

2. The economy

Consider a large number of closed economies that can differ only in their inputs endowments. In each economy, there are two sectors and two types of capital k and h , that we denote physical and human capital, respectively. One sector produces a commodity Y according to the technology $Y = A (sk)^\alpha (uh)^{1-\alpha} = Auhz_Y^\alpha$, where s and

u are the shares of physical and human capital allocated to this sector, respectively, and $z_Y = \frac{sk}{uh}$ is the capital ratio in this sector. The commodity Y can be either consumed or added to the stock of physical capital. The law of motion of the physical capital stock is thus given by

$$\dot{k} = A(sk)^\alpha (uh)^{1-\alpha} - c - \delta k, \quad (2.1)$$

where c is the amount of Y devoted to consumption, and $\delta \in (0, 1)$ is the depreciation rate of the physical capital stock. The other sector produces a commodity H by means of the production function $H = \gamma [(1-s)k]^\beta [(1-u)h]^{1-\beta} = \gamma(1-u)hz_H^\beta$, where $z_H = \frac{(1-s)k}{(1-u)h}$. This commodity can also be devoted either to consumption or to increase the stock of human capital. The evolution of the human capital stock is thus given by

$$\dot{h} = \gamma [(1-s)k]^\beta [(1-u)h]^{1-\beta} - x - \eta h, \quad (2.2)$$

where x denotes the amount of H devoted to consumption, and $\eta \in (0, 1)$ is the depreciation rate of the human capital stock. Because the two sectors produce final goods, we define the GDP as follows:

$$Q = Y + pH, \quad (2.3)$$

where p is the relative price of good H in terms of good Y .

The economy is populated by an infinitely lived representative agent characterized by the following utility function:

$$U(c, x) = \frac{[(c - \bar{c})^\theta x^{1-\theta}]^{1-\sigma}}{1 - \sigma},$$

where $\theta \in [0, 1]$ is the share parameter for good c in the composite consumption good, $\sigma > 0$ is the constant elasticity of marginal utility with respect to this composite consumption good, and \bar{c} is a minimum consumption requirement. An important feature of preferences is that they exhibit asymmetric consumption requirements across goods. In particular, we normalize the minimum consumption requirement on good x to zero, which implies that the income elasticity of demand is less than one for good c , and greater than one for good x .⁵ We also consider that the minimum consumption requirement \bar{c} is a fraction of the consumption level achieved by the most developed economy in the world. In other words, individuals use the consumption level of the most advanced economy as a reference with respect which their own consumption is compared to. Individuals in the studied economy then aspire to catch up the standard of living reached by the reference economy. Our economy and the reference economy only differ in their capital endowments and their level of development and, moreover, the reference economy is not subject to minimum consumption requirements ($\bar{c} = 0$) because it is the richest economy in the world. For simplicity, we assume that the

⁵If we consider that good x is also subject to a minimum consumption requirement \bar{x} , then the income elasticities of demands are different when $(1-\theta)\bar{c} \neq \theta p\bar{x}$. In particular, if $(1-\theta)\bar{c} > \theta p\bar{x}$, then the income elasticity of demand is less than one for good c , and greater than one for good x . Obviously, our assumption of $\bar{c} = 0$ satisfies this case.

reference economy is in a BGP all the time, where consumption is growing at rate g^L . Thus, the minimum consumption requirement evolves according to

$$\bar{c} = c_0 e^{g^L t}, \quad (2.4)$$

where c_0 is a given fraction of the consumption level of the reference economy in the initial period. Finally, note that the introduction of the minimum consumption using this additive functional form implies that the constraint $c > \bar{c}$ must hold for all t .

The representative agent maximizes the discounted sum of utilities

$$\int_0^\infty e^{-\rho t} U(c, x) dt,$$

subject to (2.1) and (2.2), where $\rho > 0$ is the discount rate. Let μ_1 and μ_2 be the shadow prices of k and h , respectively. Appendix A provides the first order conditions of this maximization problem, and derives the system of dynamic equations that fully characterizes the equilibrium paths by following the standard procedure used in the two-sector models of endogenous growth (see, for example, Bond et al., 1996). In the remainder of this section, we only provide the equations that define the equilibrium dynamics. First, since by definition $p = \frac{\mu_2}{\mu_1}$, we get the equation that drives the growth of the relative price

$$\frac{\dot{p}}{p} = -(1 - \beta) \gamma z_H^\beta + \beta \gamma z_H^{\beta-1} p + \eta - \delta, \quad (2.5)$$

with

$$z_H = \phi \left[\frac{\beta(1 - \alpha)}{\alpha(1 - \beta)} \right] p^{\frac{1}{\alpha - \beta}}, \quad (2.6)$$

where

$$\phi = \left(\frac{\gamma}{A} \right)^{\frac{1}{\alpha - \beta}} \left(\frac{1 - \beta}{1 - \alpha} \right)^{\frac{1 - \beta}{\alpha - \beta}} \left(\frac{\beta}{\alpha} \right)^{\frac{\beta}{\alpha - \beta}}.$$

As follows from (2.5), the growth of the price is driven by the standard non-arbitrage condition that states that the returns on physical and human capital must coincide. Note that (2.5) is a function of p alone and only depends on technology parameters. In contrast, the value of the relative price is driven by the marginal rate of substitution, i.e.,

$$p = \left(\frac{1 - \theta}{\theta} \right) \left(\frac{c - \bar{c}}{x} \right), \quad (2.7)$$

which shows that the relative price depends on the parameter θ and on the composition of consumption.

We now proceed to characterize the growth rate of consumption expenditure. In this economy with two consumption goods, we define consumption expenditure as $w = c + px$. Moreover, we denote the fraction of consumption expenditures devoted to the purchase of good c by w_c , where

$$w_c = \frac{1}{1 + \left(\frac{1 - \theta}{\theta} \right) \left(1 - \frac{\bar{c}}{c} \right)}, \quad (2.8)$$

as follows from (2.7). Note that w_c provides a measure of the composition of consumption expenditures, which depends on θ and on the ratio $\frac{\bar{c}}{c}$. It follows that the composition of consumption changes along the development process because of the minimum consumption requirement. In fact, the consumption requirement makes the utility function be non homothetic, which implies that the composition of consumption depends on the level of income. In Appendix A, we obtain that

$$\frac{\dot{w}}{w} = g^L + \left(\frac{w - \bar{c}}{\sigma w} \right) \left[\beta \gamma p z_H^{\beta-1} - \delta - \rho - \sigma g^L - (1 - \theta)(1 - \sigma) \frac{\dot{p}}{p} \right]. \quad (2.9)$$

As follows from (2.9), the existence of two consumption goods implies that the convergence is not only driven by the diminishing returns to scale but also by the change in the relative price. Moreover, we also observe that the intertemporal elasticity of substitution (IES, henceforth) is given by $\chi = \frac{w - \bar{c}}{\sigma w}$. Note that during the transition, and unlike the case of homothetic preferences, the IES is not constant.

Finally, we characterize the growth rate of the two capital stocks. For that purpose, we use (2.7) and the definition of w to rewrite the ratios $\frac{c}{k}$ and $\frac{x}{h}$ as functions of p , w , k and h . Given these functions, we get in Appendix A that

$$\frac{\dot{k}}{k} = A \left(\frac{uh}{k} \right) z_Y^\alpha - \delta - \frac{\theta w + (1 - \theta) \bar{c}}{k}, \quad (2.10)$$

and

$$\frac{\dot{h}}{h} = \gamma (1 - u) z_H^\beta - \eta - (1 - \theta) \left(\frac{w - \bar{c}}{ph} \right), \quad (2.11)$$

with

$$z_Y = \phi p^{\frac{1}{\alpha-\beta}}, \quad (2.12)$$

and

$$u = \frac{z - z_H}{z_Y - z_H}, \quad (2.13)$$

where z denotes the aggregate ratio from physical to human capital, i.e., $z = \frac{k}{h}$.

We can now define the dynamic equilibrium as a set of paths $\{w, p, k, h, \bar{c}\}$ that, given the initial levels of the two capital stocks k_0 and h_0 and of the initial consumption requirement \bar{c}_0 , solves the system of differential equations formed by (2.4), (2.5), (2.9), (2.10), and (2.11), together with (2.6), (2.12), (2.13) and the usual transversality conditions

$$\lim_{t \rightarrow \infty} \mu_1 k = 0, \quad (2.14)$$

and

$$\lim_{t \rightarrow \infty} \mu_2 h = 0. \quad (2.15)$$

Note that the equilibrium will be characterized by three state variables, k , h and \bar{c} , and two control variables, w and p . Because there are three state variables, the transition will be driven not only by the imbalances between the two capital stocks, as occurs in the standard two-sector growth model, but also by the initial levels of the capital stocks.

3. The balanced growth path

A steady-state equilibrium or BGP in our economy is an equilibrium path along which both capital stocks, both consumption goods and consumption expenditures grow at a constant rate, and capital allocation between sectors, relative prices and the ratio from aggregate output to physical capital are constant. This section lays down the properties of a BGP and the conditions for its existence. First, we prove that the BGPs of any economy exhibit the same relative price and the same growth rate as the reference economy because both economies are defined by the same fundamentals. By setting $\bar{c} = 0$, and using (2.5) and (2.9), we obtain that the consumption expenditure in the reference economy grows at the rate

$$g^L = \frac{\beta \gamma z_H^{\beta-1} p^L - \delta - \rho}{\sigma}, \quad (3.1)$$

where the relative price p^L is the unique solution to

$$-(1 - \beta) \gamma z_H^\beta + \beta \gamma z_H^{\beta-1} p^L + \eta - \delta = 0. \quad (3.2)$$

Note that variable w grows at a constant rate only if the price level p^L is constant. The existence of a p^L follows by continuity of (3.2) and by noticing that the left hand side of this equation changes its sign as the price rises from zero.

By considering $\bar{c} > 0$, we can now characterize the properties of the BGPs of any less developed economy.

Proposition 3.1. *Consider an economy with $\bar{c} > 0$. If p^* is the relative price along a BGP, then $p^* = p^L$. Moreover, along a BGP the two capital stocks and consumption expenditure grow at the same constant growth rate $g^* = g^L$.*

We have shown the existence and uniqueness of a long-run price level and growth rate. Obviously, this does not imply the existence of a BGP, but it implies that if a BGP exists then the price level and growth rates will be unique. Note that these long-run values of the relative price and growth rate neither depend on the weight of consumption goods in the utility function, θ , nor on the initial consumption requirement, \bar{c}_0 . Thus, the assumptions made on preferences do not affect the long-run value of these two variables that, as in the standard two-sector growth model, only depends on technology. We show next that the long-run level of the variables depends on the assumptions made on preferences. For that purpose, we normalize the variables w , k , and h as follows

$$\hat{w} = w e^{-g^* t}, \quad (3.3)$$

$$\hat{k} = k e^{-g^* t}, \quad (3.4)$$

and

$$\hat{h} = h e^{-g^* t}. \quad (3.5)$$

Note that the normalized variables \hat{w} , \hat{k} and \hat{h} will remain constant along a BGP, and let \hat{w}^* , \hat{k}^* , and \hat{h}^* denote the respective steady-state values of these variables. The following proposition characterizes a steady-state equilibrium in terms of the normalized variables \hat{w} , \hat{k} and \hat{h} .

Proposition 3.2. Given \widehat{k}^* , a BGP is a set $\{g^*, p^*, \widehat{h}^*, \widehat{w}^*\}$ that satisfies $(1 - \sigma)g^* < \rho$ and

$$\widehat{k}^* \geq k^c = \left(\frac{\bar{c}_0}{bf}\right) \left\{ b + \left(\frac{1-\theta}{\theta}\right) \left[\frac{Az_H z_Y^\alpha}{p(z_Y - z_H)} \right] \right\},$$

and solves (3.2), (3.1), and

$$\widehat{h}^* = m + n\widehat{k}^*, \quad (3.6)$$

$$\widehat{w}^* = l + j\widehat{k}^*, \quad (3.7)$$

where

$$\begin{aligned} m &= \left(\frac{1-\theta}{\theta}\right) \left(\frac{\bar{c}_0}{bp^*}\right), \\ n &= -\left(\frac{1}{b}\right) \left\{ \left(\frac{1-\theta}{\theta p^*}\right) \left[\left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) \right] + \frac{\gamma z_H^\beta}{z_Y - z_H} \right\}, \\ l &= -\left(\frac{1}{\theta}\right) \left[m \left(\frac{Az_H z_Y^\alpha}{z_Y - z_H}\right) + (1-\theta)\bar{c}_0 \right], \\ j &= \left(\frac{1}{\theta}\right) \left[(1 - nz_H) \left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) \right], \end{aligned}$$

and

$$b = (\eta + g^*) - z_H \left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) \left(\frac{1-\theta}{\theta p^*}\right) - \frac{\gamma z_H^\beta z_Y}{z_Y - z_H}.$$

Moreover, the following statements hold:

- (i) the slopes n and j are positive;
- (ii) if $\bar{c}_0 > 0$, $\theta \in (0, 1)$ and $\alpha \neq \beta$, then $m > 0$ when $\alpha < \beta$, whereas $m < 0$ when $\alpha > \beta$. Otherwise, $m = 0$.

The previous result states the conditions on the fundamentals for the existence of an interior BGP. On the one hand, the condition $(1 - \sigma)g^* < \rho$ guarantees that the transversality conditions hold and the objective function in the representative agent's problem takes a bounded value. On the other hand, the condition $\widehat{k}^* \geq k^c$ ensures that the value of the physical capital stock at the BGP satisfies $\widehat{h}^* > 0$ and $\widehat{c}^* > \bar{c}_0$. From now on, we will assume that these two conditions hold.

Proposition 3.2 also shows that the set of steady-state values of \widehat{w} , \widehat{k} and \widehat{h} is a linear manifold of dimension one. This means that there is a continuum of BGPs, which we will index by \widehat{k}^* . Along this manifold there is a positive relationship between \widehat{w}^* and \widehat{k}^* , and between \widehat{h}^* and \widehat{k}^* . However, the ratios $\frac{\widehat{h}^*}{\widehat{k}^*}$ and $\frac{\widehat{w}^*}{\widehat{k}^*}$ can either change or remain constant from one BGP to another. To see this, note that the linear manifold of BGPs does not emanate from the origin when the independent terms in (3.6) and (3.7) are different from zero. This is an important difference with respect to the standard two-sector growth model, where the set of BGPs forms a linear manifold emanating from the origin.⁶ This difference will yield the main results of the paper. The next corollary provides conditions for this difference to hold.

⁶See Caballé and Santos (1993) for an analysis of the BGP in the standard two-sector growth model.

Corollary 3.3. *The manifold of BGPs does not emanate from the origin if and only if the following statements hold:*

- (i) *individuals derive utility from the consumption of the two goods, i.e. $\theta \in (0, 1)$;*
- (ii) *the minimum consumption requirement is strictly positive, i.e., $\bar{c}_0 > 0$; and*
- (iii) *the capital intensity is different across sectors, i.e., $\alpha \neq \beta$.*

The main implication of the previous result for the purpose of this paper is that the sectoral composition can change along the set of BGPs, which is in stark contrast with the standard two-sector model of endogenous growth. In order to show this conclusion, we first proceed to characterize in some detail the relationship between the values of \hat{h}^* and \hat{k}^* derived from Proposition 3.2. Note that the value \hat{h}^* depends positively on \hat{k}^* because the function (3.6) has a positive slope. However, the stationary ratio between both capital stocks, that we will denote by z^* , can increase, decrease or remain constant after a positive shock in \hat{k}^* . By using Proposition 3.2, we next characterize this dependence of z^* on \hat{k}^* .

Proposition 3.4. *Assume that $\bar{c}_0 > 0$ and $\theta \in (0, 1)$. If $\alpha > \beta$ the ratio z^* is a decreasing function of \hat{k}^* , whereas z^* is an increasing function of \hat{k}^* when $\alpha < \beta$. The ratio z^* does not depend on \hat{k}^* if $\bar{c}_0 = 0$, $\theta = 1$, or $\alpha = \beta$.*

From the previous result, we conclude that different BGPs can exhibit different physical to human capital ratios, although these ratios will be constant in each BGP. The consumption requirement forces the economies to devote more resources to produce the commodity Y when the normalized stock of capital \hat{k}^* at a BGP and, thus, GDP are small. Hence, the ratio between the output of sector Y and the output of sector X will be larger in those economies with a smaller value of \hat{k}^* and decrease as this value rises. As a consequence of the composition of GDP derived from small values of \hat{k}^* , the required stock of physical capital in this case is larger (smaller) than the required stock of human capital if sector Y is more (less) intensive in physical capital than sector X , i.e. when $\alpha > (<) \beta$. Therefore, if $\alpha > (<) \beta$ then the ratio z^* is large (small) in poor economies, where the minimum consumption requirement forces agents to devote most resources to produce the commodity Y . This explains the dependence of this ratio z^* on the normalized stock of physical capital \hat{k}^* established by Proposition 3.4. Observe also that if $\alpha = \beta$ the factor intensity is the same in the two sectors and, thus, the relative requirements of both capital stocks do not change as \hat{k}^* rises. This means that the ratio z^* is constant when $\alpha = \beta$. Finally, if either $\theta = 1$ (there is a unique consumption good) or $\bar{c}_0 = 0$, a rise in the normalized stock of physical capital does not change the composition of consumption and, thus, it does not change the capital ratio z^* . Therefore, it follows that only when $\theta \in (0, 1)$, $\bar{c}_0 > 0$, and $\alpha \neq \beta$, the ratio z^* changes with the normalized stock of physical capital \hat{k}^* .

We next characterize the dependence of the stationary values of u and w_c on \hat{k}^* . For that purpose, we use the definition of u and w_c and the results in Proposition 3.2.

Proposition 3.5. *Let u^* and w_c^* be the steady-state values of u and w_c , respectively.*

- (i) *If $\bar{c}_0 > 0$, $\theta \in (0, 1)$ and $\alpha \neq \beta$, then u^* is a decreasing function of \hat{k}^* . Otherwise, u^* does not depend on \hat{k}^* .*
- (ii) *If $\bar{c}_0 > 0$ and $\theta \in (0, 1)$, then w_c^* is a decreasing function of \hat{k}^* . Otherwise, w_c^* does not depend on \hat{k}^* .*

The previous result implies that the composition of consumption and the sectoral structure at the BGP also depend on the value of \widehat{k}^* . In particular, if the conditions in Corollary 3.3 hold, then these two variables will change from one BGP to another. This will be crucial to understand the mechanics that underlines the endogeneity of TFP in our model. This result is a consequence of the introduction of a minimum consumption requirement and of heterogeneous consumption goods. However, the result does not depend on the relative factor intensity ranking. The intuition is as follows. In economies with a low normalized stock of physical capital at the BGP (poor economies), the minimum consumption requirement forces agents to devote a large amount of resources to produce commodity Y and to consume good c . Thus, the consumption requirement leads both u^* and w_c^* to be large in these economies and to be decreasing in \widehat{k}^* . Hence, the long-run sectoral composition of consumption and GDP depends on the normalized stock of physical capital.

Observe that the long-run sectoral composition does not depend on the actual level of capital k but on the normalized level of capital \widehat{k}^* . This implies that two economies will exhibit a different sectoral composition of GDP for a given level of capital stock k if they attain this level at different periods. The economy that reaches a given level of capital stock later is farther away from the consumption reference, so that a larger fraction of GDP must be devoted to satisfy the larger minimum consumption requirement at that moment. Therefore, in our model the level and the composition of GDP is not directly determined by capital stocks, but by the relationship between these stocks and the minimum consumption requirement.

At this point, it is convenient to analyze the stability of the set of BGPs in order to show how the initial conditions determine the BGP. The standard duality between Rybczynski and Stolper-Sumuelson effects determines the stability property.⁷ Thus, this property neither depends on the factor intensity ranking nor on the assumptions made on preferences.

Proposition 3.6. *Every point in the manifold of BGPs is saddle path stable, which means that there is a unique equilibrium path that converges to each BGP.*

We conclude that two economies with the same fundamentals, but different initial endowments of capital stocks, will end up with the same relative prices and growing at the same rate, although the physical to human capital ratio and the sectoral composition remain being different. Therefore, this model predicts differences across countries in the long-run sectoral composition. Note that this result is not present in the standard two-sector growth model, in which the economies share the same long-run sectoral composition. In this sense, the version of the two-sector growth model we consider is a theory of both economic growth and sectoral composition because their long-run values are endogenously determined.

Echevarria (1997), Rebelo (1991), Steger (2006), among many others, also consider growth models with endogenous sectoral composition that changes as the economy develops along its transitional path. However, in these papers the sectoral composition is constant along the set of BGPs. Thus, although economies converge to different

⁷The role of the factor intensity ranking in the transitional dynamics of multi-sector growth models is extensively presented in Bond, et al. (1996).

BGPs, they exhibit the same long-run sectoral composition. By the contrary, in our model the economies can converge into BGPs with different long-run sectoral composition depending on the initial conditions. Once in a BGP, each economy grows at a constant rate, and its sectoral composition remains constant. The dependence of the long-run sectoral composition on the initial endowments of capitals is a result that emerges from the fact that the income elasticities of demand also differ across consumption goods along the BGP. Since the income elasticity for good c is smaller than for good x , agents are forced to devote a permanently growing amount of resources to produce good Y , which sets a limit to structural change.

4. Output decomposition

The non-convergence to a common sectoral composition will generate cross-country disparities in TFP without any difference in technology levels. In fact, the endogeneity of sectoral composition makes TFP depend on capital stocks. To see this, we use (2.1), (2.2) and (2.3) to decompose GDP as follows

$$Q = A \underbrace{\left(\frac{s}{u}\right)^\alpha \left[\frac{u(\alpha - \beta) + 1 - \alpha}{1 - \beta}\right]}_{TFP} k^\alpha h^{1-\alpha}. \quad (4.1)$$

This decomposition of GDP between production factors and TFP shows that the later depends on the sectoral allocations of capital stocks. Therefore, as in any multisector growth model, the level of TFP depends on sectoral structure. However, while in the standard two-sector growth model the long-run values of these capital shares are equal across countries with the same fundamentals, in our model they do depend on the value of the capital stock \hat{k}^* . Therefore, in our version of the two-sector growth model, TFP is endogenous in the sense that it depends on the capital stocks. In particular, in poor economies the value of u is larger and the TFP will be lower than in richer economies. Note that this result has interesting consequences on development accounting. By taking TFP as exogenous, several authors have concluded that differences in capital stocks cannot explain the observed disparities in the levels of GDP per capita (see, for instance, Hall and Jones, 1999). According to our model, taking TFP as exogenous introduces a bias in the results from the accounting analysis because the differences in capital stocks also imply differences in TFP.⁸ In other words, our model implies that the contribution of capital to explain GDP differences is underestimated when TFP is assumed to be exogenous.

We will now show both analytically and numerically that the differences in capital stocks yield larger differences in GDP levels when TFP is endogenous. For that purpose,

⁸Klenow and Rodriguez-Clare (1997), and Hall and Jones (1999) rewrite (4.1) as $Q = TFP^{\frac{1}{1-\alpha}} \left(\frac{k}{Q}\right)^{\frac{\alpha}{1-\alpha}} h$. They use this transformation because they want to take into account that the impact of the difference in technology between economies is larger than the one measured by TFP, as it also affects the accumulation of capital. We do not have this problem since we do not consider differences in technologies across economies. In contrast, we assume that economies exhibit different TFP values only because they have different initial capital stocks. This means that, in this paper, in order to capture the actual effect of differences in capital stocks we must take into account that TFP is endogenous.

in this section we will focus on the set of BGPs. Note that using (4.1) to explain GDP differences requires a measure of human capital, which is a difficult variable to be measured. To avoid this problem, we use the long-run relationship between physical and human capital implied by (3.6) to rewrite the long-run value of GDP as a function of the long-run value of the normalized stock of physical capital \widehat{k}^* . By combining (3.6) and (4.1), and using Proposition 3.2, the following result characterizes the relationship between the long-run values of GDP and of physical capital implied by the model.

Proposition 4.1. *Let us define the normalized level of GDP as $\widehat{Q} = Qe^{-g^*t}$ and its steady-state value as \widehat{Q}^* . The value of \widehat{Q}^* is the following linear function of the steady-state value of \widehat{k}^* :*

$$\widehat{Q}^* = \widetilde{b} + \widetilde{a}\widehat{k}^*, \quad (4.2)$$

where

$$\widetilde{b} = (1 - \alpha) mAz_Y^\alpha,$$

and

$$\widetilde{a} = \alpha Az_Y^{\alpha-1} + (1 - \alpha) nAz_Y^\alpha.$$

Moreover, the following statements hold: (i) $\widetilde{a} > 0$; and (ii) if $\theta \in (0, 1)$, $\bar{c}_0 > 0$, and $\alpha > (<) \beta$ then $\widetilde{b} < (>) 0$, whereas $\widetilde{b} = 0$ otherwise.

As is usual, in our model the ratio from GDP to physical capital is constant at a steady-state equilibrium. However, this ratio may change along the set of BGPs. In fact, when either $\theta = 1$ or $\bar{c}_0 = 0$, our version of the two-sector constant returns to scale growth model coincides with the standard two-sector growth model. In this case, the relation between the steady-state values of GDP and of physical capital stock is the same for all BGPs under the assumption of constant returns to scale. This result also arises when $\alpha = \beta$ because in this case there is a unique production technology, so that the model coincides with a one sector constant returns to scale growth model.⁹ On the contrary, if $\theta \in (0, 1)$, $\bar{c}_0 > 0$ and $\alpha \neq \beta$, then the GDP to physical capital ratio is not constant along the set of BGPs since \widetilde{b} is different from zero. In this case, this ratio is increasing (decreasing) in the normalized stock of physical capital because $\widetilde{b} < (>) 0$ when $\alpha > (<) \beta$. This implies that economies with twice as much level of \widehat{k}^* exhibit more (less) than twice as much level of \widehat{Q}^* when $\alpha > (<) \beta$. The intuition is as follows. The minimum consumption requirement makes poor economies devote a relatively large fraction of resources to sector Y. This implies that in these economies the physical to human capital ratio is larger (smaller) when sector Y is more (less) intensive in physical capital than sector X, i.e. when $\alpha > (<) \beta$. Thus, if $\alpha > (<) \beta$, then the ratio of GDP to capital is initially large (small) in poor economies and this ratio decreases (increases) as the economy develops.

We have shown that capital stocks in our model have an indirect effect on TFP and GDP by changing the sectoral structure. In what follows, we use the output decomposition in (4.1) to illustrate by means of numerical simulations how differences in the steady-state level of the normalized stock of physical capital yield differences in GDP. The stock of physical capital will affect GDP through three channels: (i)

⁹Note that if $\alpha = \beta$ then $TFP = A$, so that the sectoral structure does not affect the level of TFP.

the direct contribution as a production factor, that we denote by C_k ; (ii) the indirect contribution derived from the induced change in the human capital stock, that we denote by C_h ; and (iii) the indirect contribution derived from the induced change in TFP, that we denote by C_{TFP} . For our purpose, we consider two economies that only differ in their values of \widehat{k}^* . The parameter values are chosen such that the economy with the larger stock of physical capital (*rich economy*) replicates some facts of US economy. We first set arbitrarily the values of \widehat{k}^* and A equal to unity. We then proceed to choose the other parameters as follows: we set $\alpha = 0.42$ from Perli and Sakellaris (1998); we fix $\delta = 5.6\%$ to obtain that the annual investment in physical capital amounts to 7.6% of its stock and, moreover, we assume that $\eta = \delta$; the value of the preference parameter σ is equal to 2, which implies that the IES would be 0.5 if there were no minimum consumption requirements; the value of γ is such that the net interest rate equals to 5.2%; the value of ρ is such that $g^* = 2\%$; the value of \bar{c}_0 is such that $\chi = 0.21$; and θ is such that $w_c = 0.6$. Finally, we take alternative values for the technological parameter β to illustrate how differences in the capital intensities across sectors alters the results from the accounting exercises. In particular, we assume three different values: $\beta = 0.32$, $\beta = 0.15$ and $\beta = 0$. Once the parameters have been calibrated, we fix the value of \widehat{k}^* for the *poor economy* such that the value of w_c in this economy is equal to 0.95.¹⁰

[Insert Table 1]

Table 1 shows the results from the proposed accounting exercise. As was mentioned, in Table 1 the rich and poor economies exhibit the same fundamentals except for the value of \widehat{k}^* . It follows that these economies have a common long-run growth rate, interest rate and relative price. However, the levels of the other variables, including the sectoral composition, are different. In fact, the differences in \widehat{k}^* yield a sectoral adjustment that is made in terms of both the sectoral composition of consumption and the sectoral composition of GDP. The differences in the sectoral composition occur because in the poor economy the minimum consumption requirement is stronger and, thus, affects at a larger extend the composition of consumption. In Table 1, these stronger consumption requirements are shown in the ratio $\frac{\bar{c}_0}{c^*}$, which is larger in the poor economy. Note that these stronger consumption requirement results into a lower IES and a larger value of w_c^* . This different composition of consumption affects the sectoral structure, which is measured by u^* . The effect of these differences in sectoral structures is measured by C_{TFP} , which is clearly higher when β is smaller. This means that the effect of sectoral structure is larger when the difference between the technologies is larger, i.e., when the difference between α and β is larger. However, as can be checked from Table 1, the large contribution of physical capital through the TFP when α and β are very different is obtained at the cost of having an unreasonably low labor share in sector Y . This implies that this contribution C_{TFP} would be smaller if the value of β is set such that u^* takes empirically plausible values.

We also observe that while there is a strong difference between the poor and rich economies in terms of w_c^* , the difference in terms of u^* is small. This dissimilar response

¹⁰Our definition of poor economy then includes those economies whose consumption requirement forces to allocate a large amount of resources to consume good Y . This happens when the fraction w_c is large.

of w_c^* and u^* to the difference in the values of \widehat{k}^* is explained as follows. A larger w_c^* implies that the ratio between the production of sectors Y and X must also be larger. This can be satisfied either by rising the amount of human capital devoted to sector Y or by reducing the consumption of the good produced in sector X in order to raise the stock of human capital, which results into a higher production in sector Y . Note that only the first effect implies changes in the sectoral structure that rises TFP (see equation (4.1)). However, as follows from the previous numerical examples, it seems that the second effect is more important than the first one because large differences in w_c^* translates into a large value of C_h and a small value of C_{TFP} (it is particularly small when we consider the case $\beta = 0.32$). These results are obtained under the assumption that both economies are in their BGPs. In the following section, we will show that if we instead assume that the poor economy is in its transition to the BGP, then the contribution of physical capital through the TFP may be larger.

5. The transitional dynamics

In this section, we show that the contribution of capital to TFP may be larger when we consider that economies are out of their BGPs. For that purpose, we first linearly approximate the policy functions around the set of BGPs, and we limit our analysis to the plausible case with $\alpha > \beta$. In Appendix C, we characterize the linear approximation of the policy functions in this case. From this approximation, we first observe that \widehat{k}^* , which we have used to index the set of BGPs, is a function of the initial values of both capital stocks. This means that, in contrast with the standard two-sector model of endogenous growth (i.e., as $\bar{c}_0 = 0$ and $\theta = 1$), it is not only the initial value of the physical to human capital ratio, but also the levels of both capital stocks what determine the BGP to which the economy converges when our assumptions on preferences hold (i.e., when $\theta \in (0, 1)$ and $\bar{c}_0 > 0$). Therefore, we have proved that economies with different initial endowments of capitals will converge to BGPs with different values of physical to human capital ratio and of sectoral structure if $\theta \in (0, 1)$, $\bar{c}_0 > 0$ and $\alpha \neq \beta$, even though these economies have a common initial capital ratio.

In what follows we use the linear approximation to the policy functions to compare, by means of the numerical examples used in the previous section, two economies that are only different in their initial stock of physical capital. We assume that the rich economy (i.e. the economy with a higher stock of physical capital) is in its BGP, whereas the poor economy is in the transition to the BGP. In particular, we assume that the value of the normalized stock of physical capital in the poor economy is equal to the 90% of its value at the BGP. Table 2 shows the results of this simulation exercise. As follows from the comparison between tables 1 and 2, when we assume that both economies are in the BGP, the differences in sectoral structure given by u^* are small, whereas the differences in the composition of consumption given by w_c^* are large. In contrast, if we assume that the poor economy is outside of the BGP, there are larger differences in terms of u .¹¹ Obviously, this means that the contribution of physical capital to the differences in TFP is larger when we assume that the poor economy is outside of the BGP. The endogeneity of TFP then rises meaningfully the ability of capital endowments to explain

¹¹The dynamic adjustment in u is driven by the capital ratio z and the relative prices.

GDP differences when the process of dynamic adjustment to the BGP is considered.

[Insert Table 2]

The difference between the results of Table 2 and those in Table 1 arises from the fact that TFP depends on the sectoral structure and not on the composition of consumption. Note that w_c^* is a decreasing function of \widehat{k}^* , as follows from Proposition 3.5, whereas u^* is an increasing function of z^* as follows from equation (2.13). Because along the manifold of BGPs the relationship between the two capital stocks is positive, the difference in \widehat{k}^* between BGPs is larger than the differences in the values of z^* , as it can be seen from Figure 1. This means that the difference between two economies at the BGP is larger in terms of consumption composition than in terms of the sectoral structure. This explains the numerical results obtained in Table 1, that show a large difference in w_c^* between the two economies and a small difference in u^* . In contrast, as shown in Figure 1, along the transition the capital ratio experiments a larger variation than the normalized stock of physical capital if the policy function relating the two capital stocks is downward sloping. It then follows that in this case economies adjust their sectoral structures along the transition in a larger extent than their consumption compositions. This explains the results in Tables 1 and 2, where the differences in TFP across economies with different endowments of capitals are larger along the transition than at the BGP.

[Insert Figure 1]

As we have mentioned, the results in Table 2 depend crucially on the negative sign of the policy function relating the two capital stocks along the transition to the BGP. This downward-sloping policy function ensures that the capital ratio z and the sectoral allocations of capitals u and s change more rapidly along the transition than the normalized stock of physical capital \widehat{k} , which is the mechanism generating the larger differences in TFP across economies reported by Table 2. Obviously, these results would be the opposite if the policy function relating the two capital stocks were upward-sloping. The sign of the slope of the policy functions then determines the nature of the transition the economy follows, i.e., it determines the patterns of development and of structural change, as well as the long-run level of TFP. In order to illustrate this fact, let us assume that an economy is initially in the BGP and a sudden injection of physical capital occurs. If the slope of the policy function relating the two capital stocks along the transition is negative, the economy will converge after this shock to a new BGP with a larger stock of human capital and a smaller physical to human capital ratio than those corresponding to the initial BGP. In this case, the structural change induced by the increase in physical capital has a positive effect on the long-run level of TFP since the later depends negatively on the capital ratio (see equation (4.1) and the definitions of u and s .) In contrast, if the policy function relating the two capital stocks is upward-sloping, then the aforementioned shock in the stock of physical capital leads the economy to another BGP with a smaller stock of human capital and a larger capital ratio than the initial one. In this case, the injection of physical capital changes the sectoral structure in a way that reduces the long-run level of TFP. Clearly, the

opposite conclusions would be derived from studying the effects of a negative shock in the stock of physical capital.¹²

6. Concluding remarks and extensions

In this paper we have analyzed the dynamic equilibrium of an extended version of the two-sector, constant returns to scale and endogenous growth model, in which both sectors produce consumption and investment goods. We have shown that the introduction of a second consumption good modifies the patterns of growth both in the long run and during the transition if we assume that preferences are non homothetic. Under these assumptions, two economies with the same fundamentals but different initial endowments will converge to a BGP with the same relative prices and growth rates, although the capital ratio, the output-capital ratio and the sectoral composition will be different. Given that in this model the aggregate TFP depends on the sectoral structure, this TFP is then endogenous because a rise in the capital stock affects it by altering the sectoral structure.

The theoretical results in this paper extend the debate on development accounting initiated by Mankiw et al. (1992). These authors show that the accumulation of capital explains most of GDP differences between rich and poor economies. Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) argue that this analysis is incomplete because differences in technology yield differences in the accumulation of capitals. When this is taken into account, it follows that differences in GDP are mainly explained by differences in technology that result into differences in TFP. Our contribution shows that the previous analysis is also incomplete when the TFP is endogenously determined. In our paper, TFP is endogenous because sectoral structure depends on the capital stocks. This means that a rise in the stock of capital changes the sectoral structure and the TFP. We show that the contribution of the capital stocks to explain GDP differences is larger when this endogeneity is taken into account. This suggests that an appropriate analysis of the contributions of technology and of the stock of capital to explain GDP differences should take into account this interaction between the capital stock and TFP.

We should point out that our results do not reduce the role of technology in explaining international differences in GDP. On the contrary, we interpret that they reinforce its contribution because in our model an increase in the technological level of one sector has the following effects in TFP (see equation (4.1)): (i) a direct effect because this technology level is a primary component of our decomposition of TFP; (ii) an indirect effect because technology directly determines the sectoral structure; and (iii) another indirect effect because technology indirectly affects the sectoral structure by means of the induced changes in capital accumulation. The first two effects have already been computed in the development accounting exercises. However, to the best of our knowledge, the third effect has not been considered before. Note that our results

¹²In the working paper version of this paper (See Alonso-Carrera and Raurich, 2006), we show that the slope of the policy function depends on the initial conditions on the two capital stocks. This implies that two economies with the same fundamentals but different initial endowments of capitals may diverge along the transition to their BGPs, which reinforces our arguments about the international comparison of sectoral composition.

then reconcile the two sides of the debate on development accounting. On the one hand, technology has a crucial role in explaining the observed disparities in GDP across countries. On the other hand, part of the contribution of technology to explain these international differences comes from its interaction with capital accumulation.

Our model may also have other applications to explain some macroeconomic facts, that should be incorporated to the research agenda. For instance, it provides a possible explanation to the following puzzle on the share of labor income on GDP: cross-section data show that richer countries have a larger labor income share, whereas time-series data show that the labor income share remains constant along the development process of each country. Because labor income shares depend on the sectoral structure, our model may help us to understand this puzzle. According to our results, the labor income share is constant at a BGP, whereas this share differs from one BGP to another. In particular, richer countries have a higher share of labor in sector X , which has a higher labor income share. This explains that the labor income share is larger in richer economies.

We have mentioned that there are different forces driving the growth of GDP in this economy. This means that the growth rate may exhibit a non-monotonic behavior along the development process. Therefore, the analysis of convergence seems a promising line of research, that may show up the Kuznets' facts concerning the relation between sectoral composition of GDP and development. However, this convergence analysis would require a generalization of our stylized model and a correct identification of the sectors. Obviously, this is possible only if more sectors are introduced into the analysis. Another line of research is the study of the effects of fiscal policy in the environment proposed in this paper. In our model, fiscal policy also affects economic development by means of modifying sectoral composition. In this environment, it seems interesting to study the level effects of fiscal policy. In fact, some policies, that may not have effects on long-run growth, may modify the sectoral composition and, thus, the level of GDP and TFP. As an example, note that consumption taxes, by modifying the composition of consumption, would affect the long-run level and composition of GDP.

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Appendix

A. The Equilibrium path

The first order conditions of the representative agent's maximization problem are:

$$\left[\frac{\theta(1-\sigma)U}{c-\bar{c}} \right] e^{-\rho t} = \mu_1, \quad (\text{A.1})$$

$$\left[\frac{(1-\theta)(1-\sigma)U}{x} \right] e^{-\rho t} = \mu_2, \quad (\text{A.2})$$

$$\left(\frac{\alpha Y}{s} \right) \mu_1 = \left(\frac{\beta H}{1-s} \right) \mu_2, \quad (\text{A.3})$$

$$\left[\frac{(1-\alpha)Y}{u} \right] \mu_1 = \left[\frac{(1-\beta)H}{1-u} \right] \mu_2, \quad (\text{A.4})$$

$$\left(\frac{\alpha Y}{k} - \delta \right) + \left(\frac{\beta H}{k} \right) \left(\frac{\dot{\mu}_2}{\mu_2} \right) = -\frac{\dot{\mu}_1}{\mu_1}, \quad (\text{A.5})$$

$$\left[\frac{(1-\alpha)Y}{h} \right] \left(\frac{\dot{\mu}_1}{\mu_1} \right) + \left[\frac{(1-\beta)H}{h} - \eta \right] = -\frac{\dot{\mu}_2}{\mu_2}. \quad (\text{A.6})$$

We now proceed to obtain the system of dynamic equations that characterizes the equilibrium. First, combining (A.3) and (A.4), we obtain the expressions (2.12) and (2.6). Moreover, using the definitions of the ratios z , z_Y , and z_H , we obtain the expression of u given by (2.13) and

$$s = \left(\frac{z_Y}{z} \right) \left(\frac{z - z_H}{z_Y - z_H} \right). \quad (\text{A.7})$$

Second, from the definition of p , and using equations (A.3), (A.4), (A.5) and (A.6), we get the equation that drives the growth of prices given by (2.5). Moreover, note that (A.1) and (A.2) imply that (2.7) holds. Third, log-differentiating the definition of w with respect to time, we get that the growth rate of this variable is

$$\frac{\dot{w}}{w} = (1 - w_c) \left(\frac{\dot{p}}{p} + \frac{\dot{x}}{x} \right) + w_c \left(\frac{\dot{c}}{c} \right), \quad (\text{A.8})$$

where w_c is obtained using (2.7) and is defined in (2.8). Differentiating with respect to time (A.1), (A.2) and (2.7), we obtain the growth rate of c and x . Using these growth rates, equation (A.8) yields the growth rate of w given by (2.9). Finally, we use (2.7) to rewrite $\frac{c}{k}$ and $\frac{x}{h}$ as functions of p , w , k and h . Given these functions, we obtain from (2.1) and (2.2) the growth rates of k and h as are given by (2.10) and (2.11), respectively.

B. The steady-state equilibria

B.1. Proof of Proposition 3.1

Denote by g^* the rate of growth of human capital in a BGP. Since the ratio from aggregate output to physical capital are constant, it easy to prove that the growth rate of k in a BGP must be also equal to g^* . More precisely, we can write that

$$\frac{Q}{k} = \left(\frac{h}{k}\right) \left[Au z_Y^\alpha + \gamma(1-u)p z_H^\beta \right].$$

Using (2.6), we eliminate p from the previous equation, so that

$$\frac{Q}{k} = \left(\frac{h}{k}\right) \left[Au z_Y^\alpha + \gamma \psi^{\beta-\alpha} (1-u) z_H^\alpha \right],$$

where

$$\psi = \phi \left[\frac{\beta(1-\alpha)}{\alpha(1-\beta)} \right].$$

Since $z_Y = \frac{sk}{uh}$ and $z_H = \frac{(1-s)k}{(1-u)h}$, we obtain

$$\frac{Q}{k} = \left(\frac{k}{h}\right)^{\alpha-1} \left[Au^{1-\alpha} s^\alpha + \gamma \psi^{\beta-\alpha} (1-u)^{1-\alpha} (1-s)^\alpha \right].$$

Since in a BGP the factor shares are constant, the ratio $\frac{Q}{k}$ will be constant if and only if the ratio $\frac{k}{h}$ is so. Furthermore, the behavior of factor shares and of the ratio $\frac{k}{h}$ in a BGP implies that the variables z_Y and z_H are also constant in such a equilibrium. This property, jointly with the equations (2.12) and (2.6), also implies that the relative price p remains constant in a BGP and satisfies (3.2). Finally, from (2.10) and (2.11), we see that c , x and w must also grow at rate g^* .

By using the definition of g^L in (3.1), we derive from (2.9) that the growth rate of w in a BGP satisfies $g^* = g^L$. This ensures that the *IES* is constant in the long run even for finite values of consumption, which guarantees the existence of BGPs. ■

B.2. Proof of Proposition 3.2

We first use the definition of the normalized variables to rewrite the equations (2.9), (2.10), and (2.11) as follows:

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = A \left(\frac{u\widehat{h}}{\widehat{k}} \right) z_Y^\alpha - \frac{\theta\widehat{w} + (1-\theta)c_0}{\widehat{k}} - (\delta + g^*), \quad (\text{B.1})$$

$$\frac{\dot{\widehat{h}}}{\widehat{h}} = \gamma(1-u) z_H^\beta - (\eta + g^*) - (1-\theta) \left(\frac{\widehat{w} - \widehat{c}_0}{p\widehat{h}} \right), \quad (\text{B.2})$$

$$\frac{\dot{\widehat{w}}}{\widehat{w}} = \left(\frac{\widehat{w} - \widehat{c}_0}{\sigma\widehat{w}} \right) \left[\beta\gamma p z_H^{\beta-1} - \delta - \rho - \sigma g^* - (1-\theta)(1-\sigma) \left(\frac{\dot{p}}{p} \right) \right]. \quad (\text{B.3})$$

Given the initial conditions k_0 , h_0 , and \bar{c}_0 , we define an equilibrium path of the normalized variables $\left\{p, \hat{k}, \hat{h}, \hat{w}\right\}_{t=0}^{\infty}$ as a path that solves the system of differential equations composed of (2.5), (B.1), (B.2), and (B.3), together with (2.12), (2.6), (2.13), (3.1), (2.14) and (2.15).

From the previous dynamic system we can obtain the stationary values of the normalized variables \hat{k}^* , \hat{h}^* , \hat{w}^* . In a BGP $\dot{\hat{k}} = \dot{\hat{h}} = 0$, and then (B.1) and (B.2) can be rewritten as

$$A \left(\frac{u^* \hat{h}^*}{\hat{k}^*} \right) z_Y^\alpha - \frac{\theta \hat{w}^* + (1-\theta) \bar{c}_0}{\hat{k}^*} - \delta - g^* = 0, \quad (\text{B.4})$$

$$\gamma (1-u^*) z_H^\beta - (\eta + g^*) - (1-\theta) \left(\frac{\hat{w}^* - \bar{c}_0}{p^* \hat{h}^*} \right) = 0. \quad (\text{B.5})$$

Let us define $z^* = \frac{\hat{k}^*}{\hat{h}^*}$ and $q^* = \frac{\bar{c}_0}{\hat{h}^*}$. Using the definition of \hat{w} and (B.4), we obtain

$$\frac{\bar{c}_0}{\hat{h}^*} = (z^* - z_H) \left(\frac{A z_Y^\alpha}{z_Y - z_H} \right) - (\delta + g^*) z^*. \quad (\text{B.6})$$

Combining (B.6) with (B.5), we obtain

$$\hat{c}^* = c(z^*) = \frac{\bar{c}_0}{1 - \frac{\frac{\gamma z_H^\beta (z_Y - z^*)}{z_Y - z_H} - (\eta + g^*)}{\left(\frac{1-\theta}{\theta p^*} \right) \left[(z^* - z_H) \left(\frac{A z_Y^\alpha}{z_Y - z_H} \right) - (\delta + g^*) z^* \right]}}. \quad (\text{B.7})$$

By introducing (B.7) in (B.6), and after some algebra we get the following equation:

$$a z^* + b = 0, \quad (\text{B.8})$$

where

$$a = \left(\frac{1-\theta}{\theta p^*} \right) \left[\left(\frac{A z_Y^\alpha}{z_Y - z_H} \right) - (\delta + g^*) \right] + \frac{\gamma z_H^\beta}{z_Y - z_H} - \left(\frac{1-\theta}{\theta p^*} \right) \left(\frac{\bar{c}_0}{\hat{h}^*} \right), \quad (\text{B.9})$$

and

$$b = (\eta + g^*) - z_H \left(\frac{A z_Y^\alpha}{z_Y - z_H} \right) \left(\frac{1-\theta}{\theta p^*} \right) - \frac{\gamma z_H^\beta z_Y}{z_Y - z_H}. \quad (\text{B.10})$$

The following lemma proves that there exists a positive root of equation (B.8). This root is a function of \hat{k}^* and it is given by

$$z^* = \tilde{z}(\hat{k}^*) = -\frac{b}{a}. \quad (\text{B.11})$$

Lemma B.1. *If $\alpha > \beta$ then $a > 0$ and $b < 0$, whereas $a < 0$ and $b > 0$ when $\alpha < \beta$.*

Proof. On the one hand, (B.4) implies that

$$\left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) = \left(\frac{z_H}{z}\right) \left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) + \frac{\widehat{c}^*}{\widehat{k}^*}, \quad (\text{B.12})$$

and a simplifies as follows

$$a = \left(\frac{1 - \theta}{\theta p^*}\right) \left[\left(\frac{z_H}{z^*}\right) \left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) + \left(\frac{\widehat{c}^* - \bar{c}_0}{\widehat{k}^*}\right) \right] + \frac{\gamma z_H^\beta}{z_Y - z_H},$$

which is positive when $\alpha > \beta$ (i.e., $z_Y > z_H$) and $\widehat{c} - \bar{c}_0 > 0$. In this case, b is negative since the equation (B.2) requires that in the BGP the following inequality is satisfied:

$$\frac{\gamma z_H^\beta z_Y}{z_Y - z_H} > \eta + g^*.$$

On the other hand, when $\alpha < \beta$ (i.e., $z_Y < z_H$) one can directly see from (B.10) and (B.9) that $b > 0$ and $a < 0$, respectively. ■

We proceed to characterize the values of \widehat{h}^* , \widehat{c}^* and \widehat{w}^* as functions of \widehat{k}^* . First, note that $\widehat{h}^* = \frac{\widehat{k}^*}{z^*}$, so that from (B.11) we get

$$\widehat{h}^* = \widetilde{h}(\widehat{k}^*) = m + n\widehat{k}^*, \quad (\text{B.13})$$

where

$$m = \left(\frac{1 - \theta}{\theta}\right) \left(\frac{\bar{c}_0}{bp^*}\right), \quad (\text{B.14})$$

and

$$n = -\left(\frac{1}{b}\right) \left\{ \left(\frac{1 - \theta}{\theta p^*}\right) \left[\left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) \right] + \frac{\gamma z_H^\beta}{z_Y - z_H} \right\}. \quad (\text{B.15})$$

Lemma B.2. *The function (B.13) satisfies the following properties: (i) $n > 0$; and (ii) if $\alpha > \beta$ then $m \leq 0$, whereas $m \geq 0$ when $\alpha < \beta$.*

Proof. The part (i) follows from Lemma B.1, the condition (B.12) and from the fact that the sign of $z_Y - z_H$ coincides with that of $\alpha - \beta$. The part (ii) is directly proved by noting that the sign of m coincides with that of b . ■

Second, given the stationary value of z^* , and by using (B.6), we get

$$\frac{\widehat{c}^*}{\widehat{h}^*} = (z^* - z_H) \left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) z^*. \quad (\text{B.16})$$

Thus, using (B.16) and (B.13), we get

$$\widehat{c}^* = \widetilde{c}(\widehat{k}^*) = d + f\widehat{k}^*, \quad (\text{B.17})$$

where

$$d = -m \left(\frac{Az_H z_Y^\alpha}{z_Y - z_H}\right), \quad (\text{B.18})$$

and

$$f = (1 - nz_H) \left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*). \quad (\text{B.19})$$

Lemma B.3. *The function (B.17) satisfies that $d > 0$ and $f > 0$.*

Proof. First, from Lemma B.2 and from the fact that the sign of $z_Y - z_H$ coincides with that of $\alpha - \beta$, it can be shown that $d > 0$. Second, observe that

$$f = \underbrace{\left(\frac{Az_Y^\alpha}{z_Y - z_H} \right)}_{\xi} - (\delta + g^*) - nz_H \left(\frac{Az_Y^\alpha}{z_Y - z_H} \right),$$

and using (B.15) and (B.10), we obtain

$$bf = \xi \left[(\eta + g^*) - \frac{\gamma z_H^\beta z_Y}{z_Y - z_H} \right] + \left(\frac{\gamma z_H^\beta}{z_Y - z_H} \right) z_H \left(\frac{Az_Y^\alpha}{z_Y - z_H} \right). \quad (\text{B.20})$$

Observe that (B.2) implies that

$$(\eta + g^*) - \frac{\gamma z_Y z_H^\beta}{z_Y - z_H} < -\gamma z_H^\beta \left(\frac{z}{z_Y - z_H} \right). \quad (\text{B.21})$$

Using the previous inequality, we can already show the sign of f . On the one hand, if $z_Y - z_H > 0$, then (B.20) and (B.21) implies that

$$bf < z_H \left(\frac{\gamma z_H^\beta}{z_Y - z_H} \right) \left(\frac{Az_Y^\alpha}{z_Y - z_H} \right),$$

so that $f > 0$ because in that case $b < 0$. On the other hand, if $z_Y - z_H < 0$, then we get from (B.20) and (B.21) that

$$bf > -\xi \gamma z_H^\beta \left(\frac{z}{z_Y - z_H} \right) + z_H \left(\frac{\gamma z_H^\beta}{z_Y - z_H} \right) \left(\frac{Az_Y^\alpha}{z_Y - z_H} \right).$$

Thus, using the definition of u^* and ξ , the previous inequality can be written as

$$bf > \left(\frac{\gamma z_H^\beta}{z_Y - z_H} \right) [-u^* Az_Y^\alpha + z^* (\delta + g^*)].$$

Therefore, given that (B.4) implies that $z^* (\delta + g^*) - u^* Az_Y^\alpha < 0$, we obtain that in this case $f > 0$ because $b > 0$ as $z_Y - z_H < 0$. ■

Finally, we use the definition of \widehat{w}^* and (B.17) to obtain

$$\widehat{w}^* = l + j\widehat{k}^* \quad (\text{B.22})$$

where

$$l = \frac{d - (1 - \theta)\bar{c}_0}{\theta},$$

and

$$j = \frac{f}{\theta}.$$

From Lemma A.3 is obvious that $j > 0$. However, the sign of l can not be characterized analytically.

To close the proof of Proposition 3.2, we must show that a stationary solution $\{g^*, p^*, \hat{h}^*, \hat{w}^*\}$ of the system of equations (2.5), (B.1), (B.2) and (B.3) for a given $\hat{k}^* > 0$ is a BGP. First, this stationary solution must satisfy that $\hat{h}^* > 0$ and $\hat{c}^* > \bar{c}_0$. Thus, we must impose constraints on the value of \hat{k}^* . On the one hand, by using (3.6), it can be shown that $\hat{h}^* \geq 0$ if and only if $\hat{k}^* \geq k^h$ where

$$k^h = -\frac{m}{n}. \quad (\text{B.23})$$

On the other hand, by using (B.17) it can be shown that $\hat{c}^* \geq \bar{c}_0$ if and only if $\hat{k}^* \geq k^c$ where

$$k^c = \left(\frac{\bar{c}_0}{bf}\right) \left\{ b + \left(\frac{1-\theta}{\theta}\right) \left[\frac{Az_H z_Y^\alpha}{p(z_Y - z_H)} \right] \right\}. \quad (\text{B.24})$$

Moreover, it can be shown that $k^c \geq \max\{0, k^h\}$. First, after some algebra we get that $k^c > k^h$ when

$$1 > -\left(\frac{1-\theta}{\theta}\right) \left(\frac{1}{bp^*}\right) \left[\frac{\left(\frac{Az_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*)}{n} \right].$$

Using the definition of n in (B.15), the previous inequality implies that

$$-\left(\frac{1}{b}\right) \left(\frac{\gamma z_H^\beta}{z_Y - z_H}\right) > 0,$$

which is satisfied as $b(z_Y - z_H) < 0$. Second, using (B.10), we get

$$k^c = \left(\frac{\bar{c}_0}{bf}\right) \left[\underbrace{(\eta + g^*) - \frac{\gamma z_H^\beta z_Y}{z_Y - z_H}}_{\vartheta} \right].$$

Note that if $z_Y > (<) z_H$ then $b < (>) 0$ and (B.2) implies that $\vartheta < (>) 0$. Now, it is obvious that $k^c > 0$. Therefore, $\hat{h}^* > 0$ and $\hat{c}^* > \bar{c}_0$ if and only if $\hat{k}^* \geq k^c$.

We must also prove that the transversality conditions (2.14) and (2.15) are satisfied at steady-state equilibrium. Note that using (A.1) and (2.7), we get that the transversality condition (2.14) is satisfied when

$$\lim_{t \rightarrow \infty} -\rho - \sigma \left(\frac{\dot{c} - \bar{c}g^*}{c - \bar{c}} \right) + (1-\theta)(1-\sigma) \frac{\dot{p}}{p} + \frac{\dot{k}}{k} < 0,$$

which holds if $(1-\sigma)g^* < \rho$. Following a similar procedure, it can be shown that the transversality condition (2.15) is also satisfied when $(1-\sigma)g^* < \rho$.

The condition $(1-\sigma)g^* < \rho$ also implies that the utility function is bounded. To see this, note that the utility function is bounded when $\lim_{t \rightarrow \infty} e^{-\rho t} U(c - \bar{c}, x) = 0$. By using, (2.7), we get that the previous limit holds if

$$\lim_{t \rightarrow \infty} -\rho + (1-\sigma) \left(\frac{\dot{c} - \bar{c}g^*}{c - \bar{c}} \right) < 0,$$

which is satisfied if $-\rho + (1 - \sigma)g^* < 0$. Note that, even though $c \rightarrow \bar{c}$, the utility function is bounded when this condition is satisfied. ■

B.3. Proof of Proposition 3.5

On the one hand, from the definition of u^* in equation (2.13), we get that this variable only depends on z^* and p^* . Since p^* does not depend on \hat{k}^* , we get directly from Proposition 3.4 the relationship between u^* and \hat{k}^* . On the other hand, the proof of Proposition 3.2 states that \hat{c}^* rises with \hat{k}^* . Moreover, from (2.8) we observe that the fraction of consumption expenditures w_c decreases with \hat{c}^* if $\bar{c}_0 > 0$ and $\theta \in (0, 1)$. Therefore, the dependence of w_c^* on \hat{k}^* follows directly from the two previous relationships. ■

C. The dynamic equilibrium

C.1. Proof of Proposition 3.6

Using (2.5), (B.1), (B.2), and (B.3), we obtain the following Jacobian matrix evaluated at a BGP:

$$J = \begin{pmatrix} a_{11} & a_{12} & -1 & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & a_{34} \\ 0 & 0 & 0 & a_{44} \end{pmatrix},$$

where

$$\begin{aligned} a_{11} &\equiv \frac{\partial \hat{k}}{\partial \hat{k}} = \frac{Az_Y^\alpha}{z_Y - z_H} - (\delta + g^*), \\ a_{12} &\equiv \frac{\partial \hat{k}}{\partial \hat{h}} = -\frac{Az_H z_Y^\alpha}{z_Y - z_H}, \\ a_{14} &\equiv \frac{\partial \hat{k}}{\partial p} = \left[\frac{A\hat{h}^* z_Y^\alpha}{(\alpha - \beta)p^*} \right] \left(\alpha u^* - \frac{z^*}{z_Y - z_H} \right), \\ a_{21} &\equiv \frac{\partial \hat{h}}{\partial \hat{k}} = -\frac{\gamma z_H^\beta}{z_Y - z_H}, \\ a_{22} &\equiv \frac{\partial \hat{h}}{\partial \hat{h}} = \left(\frac{1 - \theta}{\theta} \right) \left(\frac{\hat{c}^* - \bar{c}_0}{p^* \hat{h}^*} \right) - \frac{\gamma z^* z_H^\beta}{z_Y - z_H}, \\ a_{23} &\equiv \frac{\partial \hat{h}}{\partial \hat{c}} = -\left(\frac{1 - \theta}{\theta} \right) \left(\frac{1}{p^*} \right), \end{aligned}$$

$$\begin{aligned}
a_{24} &\equiv \frac{\partial \hat{h}}{\partial p} = \left[\frac{\gamma \hat{h}^* z_H^\beta}{(\alpha - \beta) p^*} \right] \left[\beta(1 - u^*) - \frac{z^*}{z_Y - z_H} \right] + \left(\frac{1 - \theta}{\theta} \right) \left(\frac{\hat{c}^* - \bar{c}_0}{p^2} \right), \\
a_{34} &\equiv \frac{\partial \hat{c}}{\partial p} = \left(\frac{\hat{c}^* - \bar{c}_0}{\sigma} \right) \left[\beta \gamma z_H^{\beta-1} + \frac{(\beta - 1) \beta \gamma z_H^{\beta-1}}{(\alpha - \beta)} - (1 - \theta)(1 - \sigma) \frac{a_{44}}{p^*} \right], \\
a_{44} &\equiv \frac{\partial \hat{p}}{\partial p} = - \left(\frac{\beta \gamma p^* z_H^{\beta-1}}{\alpha - \beta} \right) \left[1 - \alpha + (1 - \beta) \left(\frac{z_H}{p^*} \right) \right].
\end{aligned}$$

It is immediate to see that the eigenvalues λ_i , $i = 1, 2, 3, 4$, of J are $\lambda_1 = 0$, $\lambda_2 = a_{44}$ and the two roots λ_3 and λ_4 are the solution of the following equation:

$$Q(\lambda) = \lambda^2 - \lambda(a_{11} + a_{22}) + a_{11}a_{22} - a_{12}a_{21} = 0.$$

Note that $\lambda_2 < (>) 0$ if $\alpha > (<) \beta$. In order to obtain the sign of λ_3 and λ_4 , we characterize the elements of the polynomial $Q(\lambda)$. First, using (B.2), and rearranging terms, we get

$$\begin{aligned}
a_{11}a_{22} - a_{12}a_{21} &= \\
&= \left[\frac{Az_Y^\alpha}{z_Y - z_H} - (\delta + g^*) \right] \left(\frac{\gamma z_Y z_H^\beta}{z_Y - z_H} - \eta - g^* \right) \\
&\quad + \left[\frac{Az_Y^\alpha z_H}{z_Y - z_H} - (\delta + g^*) z \right] \left(\frac{\gamma z_H^\beta}{z_Y - z_H} \right).
\end{aligned}$$

By manipulating the previous equation by using (B.10), (B.15) and (B.19), we obtain $a_{11}a_{22} - a_{12}a_{21} = -bf$, which is positive when $z_Y > z_H$ and negative otherwise. Next, we consider the other element of $Q(\lambda)$. After some manipulation, where we basically replace $\delta + g^*$ from (B.6), we get

$$a_{11} + a_{22} = \frac{Az_Y^\alpha z_H}{(z_Y - z_H) z^*} + \frac{\hat{c}^*}{\hat{k}^*} + \left(\frac{1 - \theta}{\theta} \right) \left(\frac{\hat{c}^* - \bar{c}_0}{p^* \hat{h}^*} \right) + \frac{\gamma z_H^\beta z^*}{z_Y - z_H},$$

which is positive when $z_Y > z_H$. It follows that one of the roots of the polynomial, for example λ_3 is always positive and the other one, λ_4 , is positive if $\alpha > \beta$ and negative otherwise. Note that, regardless of the relation between α and β , there is a unique negative root, which means that every BGP in the manifold is saddle path stable. ■

C.2. Linear approximation of the policy functions

We proceed to approximate the policy functions around the set of BGPs. Because there are two control variables and two positive roots, using Proposition 3.6 we get the following linear approximation of the equilibrium saddle path:

$$E(t) = V + A + B e^{\hat{\lambda} t}. \quad (\text{C.1})$$

where $E(t) = (\hat{k}(t), \hat{h}(t), \hat{c}(t), p(t))$, $\hat{\lambda}$ is the negative eigenvalue of J , $V = (V_k, V_h, V_c, V_p)$ is a vector of constant terms, $A = (A_k, A_h, A_c, A_p)$ is the eigenvector

associated to the null root and $B = (B_k, B_h, B_c, B_p)$ is the eigenvector associated to $\hat{\lambda}$. We proceed to find some properties of the vectors V , A and B . First, the relationship between the elements of the eigenvector A follows from relationship $JA = 0$. Solving this system of ordinary equations, we get that $A_p = 0$, $A_h = a_h A_k$, and $A_c = a_c A_k$, where

$$\begin{aligned} a_h &= -\frac{a_{21} + a_{23}a_{11}}{a_{22} + a_{23}a_{12}}, \\ a_c &= \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22} + a_{23}a_{12}}. \end{aligned}$$

By using (B.2) and (B.15), we obtain that $a_h = n$ and $a_c = f$.

Second, the relation between the elements of the eigenvector B follows from the system of equations $(J - \lambda)B = 0$, where I is the identity matrix. Assume that $\alpha > \beta$, then $\hat{\lambda} = a_{44}$. Thus, we obtain from the previous matrix relationship that $B_h = b_h B_p$, $B_k = b_k B_p$, and $B_c = b_c B_p$, where

$$\begin{aligned} b_h &= \frac{a_{21} \left(a_{14} - \frac{a_{34}}{a_{44}} \right) - (a_{11} - a_{44}) \left(\frac{a_{23}a_{34}}{a_{44}} + a_{24} \right)}{(a_{22} - a_{44})(a_{11} - a_{44}) - a_{21}a_{12}}, \\ b_k &= \frac{a_{12} \left(a_{24} + a_{23} \frac{a_{34}}{a_{44}} \right) - (a_{22} - a_{44}) \left(a_{14} - \frac{a_{34}}{a_{44}} \right)}{(a_{22} - a_{44})(a_{11} - a_{44}) - a_{21}a_{12}}, \\ b_c &= \frac{a_{34}}{a_{44}}. \end{aligned}$$

Finally, because $\hat{\lambda} < 0$ the set of paths given by (C.1) converges to the following stationary solution:

$$\begin{aligned} k^* &= V_k + A_k, \\ h^* &= V_h + nA_k, \\ c^* &= V_c + fA_k, \\ p^* &= V_p. \end{aligned}$$

This stationary solution corresponds to the manifold of BGPs obtained in Section 3 when $V_k = 0$, $V_h = m$, $V_c = d$, $V_p = p^*$ and $A_k = \hat{k}^*$.

Therefore, the linear approximation of the policy functions is given by

$$\begin{aligned} \hat{k}(t) &= \hat{k}^* + b_k B_p e^{\hat{\lambda}t}, \\ \hat{h}(t) &= m + n\hat{k}^* + b_h B_p e^{\hat{\lambda}t}, \\ \hat{c}(t) &= d + f\hat{k}^* + b_c B_p e^{\hat{\lambda}t}, \\ p(t) &= p^* + B_p e^{\hat{\lambda}t}. \end{aligned}$$

Finally, using the initial conditions $\hat{k}(0)$ and $\hat{h}(0)$, we get

$$\begin{aligned} \hat{k}^* &= \frac{b_h \hat{k}(0) - b_k \hat{h}(0) + b_k m}{b_h - nb_k}, \\ B_p &= \frac{\hat{h}(0) - n\hat{k}(0) - m}{b_h - nb_k}. \end{aligned}$$

Table 1. Comparison at BGP¹³

| The case with $\beta = 0.32$ ¹⁴ | | | |
|--|--------------------------------|--|--------------------------------|
| <u>Rich Economy</u> | <u>Poor Economy</u> | <u>Comparison</u> | <u>Accounting¹⁵</u> |
| $\widehat{k}^R = 1$ | $\widehat{k}^P = 0.63$ | $\frac{\widehat{k}^R}{\widehat{k}^P} = 1.58$ | $C_k = 39.5\%$ |
| $\widehat{h}^R = 0.12$ | $\widehat{h}^P = 0.073$ | $\frac{\widehat{h}^R}{\widehat{h}^P} = 1.66$ | $C_h = 60\%$ |
| $TFP^R = 1.007$ | $TFP^P = 1.004$ | $\frac{TFP^R}{TFP^P} = 1.002$ | $C_{TFP} = 0.5\%$ |
| $\widehat{Q}^R = 0.29$ | $\widehat{Q}^P = 0.18$ | $\frac{\widehat{Q}^R}{\widehat{Q}^P} = 1.63$ | |
| $w_c^R = 0.6$ | $w_c^P = 0.95$ | | |
| $u^R = 0.4$ | $u^P = 0.5$ | | |
| $\frac{\bar{c}}{c^R} = 0.96$ | $\frac{\bar{c}}{c^P} = 0.99$ | | |
| $\chi^R = 0.21$ | $\chi^P = 0.026$ | | |
| The case with $\beta = 0.15$ ¹⁶ | | | |
| <u>Rich Economy</u> | <u>Poor Economy</u> | <u>Comparison</u> | <u>Accounting</u> |
| $\widehat{k}^R = 1$ | $\widehat{k}^P = 0.6986$ | $\frac{\widehat{k}^R}{\widehat{k}^P} = 1.4314$ | $C_k = 30.68\%$ |
| $\widehat{h}^R = 0.2152$ | $\widehat{h}^P = 0.1257$ | $\frac{\widehat{h}^R}{\widehat{h}^P} = 1.7123$ | $C_h = 63.53\%$ |
| $TFP^R = 1.0768$ | $TFP^P = 1.0466$ | $\frac{TFP^R}{TFP^P} = 1.0288$ | $C_{TFP} = 5.78\%$ |
| $\widehat{Q}^R = 0.4418$ | $\widehat{Q}^P = 0.2704$ | $\frac{\widehat{Q}^R}{\widehat{Q}^P} = 1.6339$ | |
| $w_c^R = 0.6$ | $w_c^P = 0.95$ | | |
| $u^R = 0.2685$ | $u^P = 0.384$ | | |
| $\frac{\bar{c}}{c^R} = 0.9667$ | $\frac{\bar{c}}{c^P} = 0.9974$ | | |
| $\chi^R = 0.21$ | $\chi^P = 0.026$ | | |
| The case with $\beta = 0$ ¹⁷ | | | |
| <u>Rich Economy</u> | <u>Poor Economy</u> | <u>Comparison</u> | <u>Accounting</u> |
| $\widehat{k}^R = 1$ | $\widehat{k}^P = 0.97$ | $\frac{\widehat{k}^R}{\widehat{k}^P} = 1.03$ | $C_k = 3\%$ |
| $\widehat{h}^R = 0.58$ | $\widehat{h}^P = 0.33$ | $\frac{\widehat{h}^R}{\widehat{h}^P} = 1.76$ | $C_h = 67\%$ |
| $TFP^R = 1.35$ | $TFP^P = 1.17$ | $\frac{TFP^R}{TFP^P} = 1.16$ | $C_{TFP} = 30\%$ |
| $\widehat{Q}^R = 0.99$ | $\widehat{Q}^P = 0.6$ | $\frac{\widehat{Q}^R}{\widehat{Q}^P} = 1.63$ | |
| $\frac{\bar{c}}{c^R} = 0.96$ | $\frac{\bar{c}}{c^P} = 0.99$ | | |
| $w_c^R = 0.6$ | $w_c^P = 0.95$ | | |
| $u^R = 0.17$ | $u^P = 0.3$ | | |
| $\chi^R = 0.21$ | $\chi^P = 0.026$ | | |

¹³The common parameters are $\alpha = 0.42$, $A = 1$, $\eta = \delta = 0.056$, $\sigma = 2$, $\rho = 0.012$ and $\theta = 0.0476$.

¹⁴The case is defined by parameters are $\beta = 0.32$, $\gamma = 0.0862$ and $\bar{\tau}_0 = 0.0511$.

¹⁵The contributions of the different factors to explain GDP differences is obtained from (4.1) as follows: $C_{TFP} = \ln(TFP^R/TFP^P)/\ln(Q^R/Q^P)$, $C_h = (1 - \alpha) \left[\ln(\widehat{h}^R/\widehat{h}^P) / \ln(Q^R/Q^P) \right]$, and $C_k = \alpha \left[\ln(\widehat{k}^R/\widehat{k}^P) / \ln(Q^R/Q^P) \right]$, where the superscripts R and P indicate the rich and poor economies, respectively.

¹⁶The case is defined by $\beta = 0.15$, $\gamma = 0.1105$ and $\bar{\tau}_0 = 0.0759$.

¹⁷The case is defined by $\beta = 0$, $\alpha = 0.42$, $\gamma = 0.112$ and $\bar{\tau}_0 = 0.18433$.

| Table 2. Comparison along the transition¹⁸ | | | |
|--|----------------------------------|--|---------------------|
| The case with $\beta = 0.32$ ¹⁹ | | | |
| <u>Rich Economy</u> | <u>Poor Economy</u> | <u>Comparison</u> | <u>Accounting</u> |
| $\widehat{k}^R = 1$ | $\widehat{k}_0^P = 0.5682$ | $\frac{\widehat{k}^R}{\widehat{k}_0^P} = 1.76$ | $C_k = 47.05\%$ |
| $\widehat{h}^R = 0.12$ | $\widehat{h}_0^P = 0.0777$ | $\frac{\widehat{h}^R}{\widehat{h}_0^P} = 1.567$ | $C_h = 51.67\%$ |
| $TFP^R = 1.007$ | $TFP_0^P = 1.003$ | $\frac{TFP^R}{TFP_0^P} = 1.0065$ | $C_{TFP} = 1.28\%$ |
| $\widehat{Q}^R = 0.29$ | $\widehat{Q}_0^P = 0.1792$ | $\frac{\widehat{Q}^R}{\widehat{Q}_0^P} = 1.6564$ | |
| $u^R = 0.4$ | $u_0^P = 0.8507$ | $\frac{u^R}{u_0^P} = 0.4642$ | |
| $w_c^R = 0.6$ | $w_0^P = 0.9511$ | $\frac{w_c^R}{w_0^P} = 0.6312$ | |
| $\frac{\bar{c}}{c^R} = 0.96$ | $\frac{\bar{c}}{c_0^P} = 0.9974$ | | |
| The case with $\beta = 0.15$ ²⁰ | | | |
| <u>Rich Economy</u> | <u>Poor Economy</u> | <u>Comparison</u> | <u>Accounting</u> |
| $\widehat{k}^R = 1$ | $\widehat{k}_0^P = 0.6287$ | $\frac{\widehat{k}^R}{\widehat{k}_0^P} = 1.5905$ | $C_k = 37.04\%$ |
| $\widehat{h}^R = 0.2152$ | $\widehat{h}_0^P = 0.106$ | $\frac{\widehat{h}^R}{\widehat{h}_0^P} = 1.6484$ | $C_h = 55.10\%$ |
| $TFP^R = 1.0768$ | $TFP_0^P = 1.0331$ | $\frac{TFP^R}{TFP_0^P} = 1.0423$ | $C_{TFP} = 7.86\%$ |
| $\widehat{Q}^R = 0.4418$ | $\widehat{Q}_0^P = 0.261$ | $\frac{\widehat{Q}^R}{\widehat{Q}_0^P} = 1.6925$ | |
| $u^R = 0.2685$ | $u_0^P = 0.458$ | $\frac{u^R}{u_0^P} = 0.5861$ | |
| $w_c^R = 0.6$ | $w_0^P = 0.9526$ | $\frac{w_c^R}{w_0^P} = 0.6297$ | |
| $\frac{\bar{c}}{c^R} = 0.9667$ | $\frac{\bar{c}}{c_0^P} = 0.9975$ | | |
| The case with $\beta = 0$ ²¹ | | | |
| <u>Rich Economy</u> | <u>Poor Economy</u> | <u>Comparison</u> | <u>Accounting</u> |
| $\widehat{k}^R = 1$ | $\widehat{k}_0^P = 0.8724$ | $\frac{\widehat{k}^R}{\widehat{k}_0^P} = 1.1463$ | $C_k = 10.48\%$ |
| $\widehat{h}^R = 0.58$ | $\widehat{h}_0^P = 0.336$ | $\frac{\widehat{h}^R}{\widehat{h}_0^P} = 1.7270$ | $C_h = 57.91\%$ |
| $TFP^R = 1.35$ | $TFP_0^P = 1.1401$ | $\frac{TFP^R}{TFP_0^P} = 1.1888$ | $C_{TFP} = 31.61\%$ |
| $\widehat{Q}^R = 0.99$ | $\widehat{Q}_0^P = 0.5719$ | $\frac{\widehat{Q}^R}{\widehat{Q}_0^P} = 1.7284$ | |
| $u^R = 0.17$ | $u_0^P = 0.336$ | $\frac{u^R}{u_0^P} = 0.5252$ | |
| $w_c^R = 0.6$ | $w_0^P = 0.9544$ | $\frac{w_c^R}{w_0^P} = 0.6288$ | |
| $\frac{\bar{c}}{c^R} = 0.96$ | $\frac{\bar{c}}{c_0^P} = 0.9976$ | | |

¹⁸The common parameters are $\alpha = 0.42$, $A = 1$, $\eta = \delta = 0.056$, $\sigma = 2$, $\rho = 0.012$ and $\theta = 0.0476$.

¹⁹The case is defined by $\beta = 0.32$, $\gamma = 0.0862$ and $\bar{c}_0 = 0.0511$.

²⁰The case is defined by $\beta = 0.15$, $\alpha = 0.42$, $\gamma = 0.1105$ and $\bar{c}_0 = 0.0759$.

²¹The case is defined by $\beta = 0$, $\gamma = 0.112$ and $\bar{c}_0 = 0.18433$.

Figures

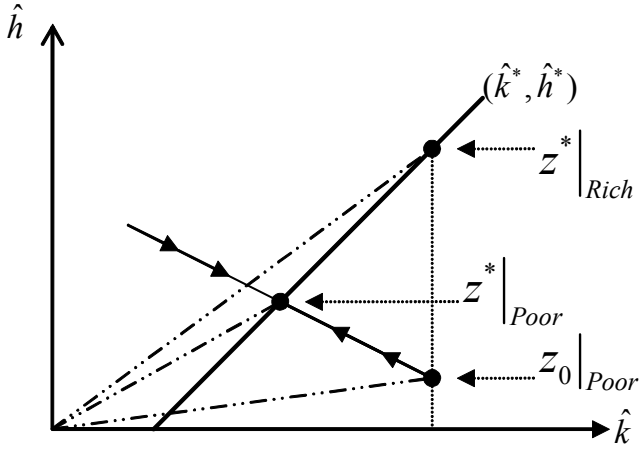


Figure 1