Modeling the Immigration Shock

Michele Boldrin and Ana Montes
WUStL, FEDEA and CEPR; Universidad de Murcia

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Abstract

In this paper we develop an overlapping generation model in the presence of an exogenous immigration shock. We use the model to study the impact of an unexpected immigration flow on the following features in the receiving country (i) the welfare of the different generations, (ii) the distribution of income among different factors of production, and (iii) the intergenerational arrangements such as public education and pensions. The framework is useful to understand same of the most relevant aspects of the recent Spanish experience. In particular, we show that the Spanish trade deficit may have played a beneficial role, in terms of consumption and overall utility, to both the native households and the immigrant ones.

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† Corresponding Author: Ana Montes. Email: anmontes@um.es

1. Introduction

Our interest lies in the following: what are the intergenerational economic effects of a large immigration flow? How does it affect the welfare of the different generations in the receiving country, both those alive and those not yet born? In particular, how does immigration impact on intergenerational arrangements such as public education and pensions? To begin answering these questions we develop a simple theoretical framework with overlapping generations that live for three periods, accumulate human capital in the first, work in the second and retire in the third living off the return from their investments. The latter include both physical capital and the resources they lent the young people to invest in human capital.

We model the immigration shock as an increase in the size of the middle-age generation engendering, among other things, a reduction in the average human capital of the labor force. The immigrants are, in other words, new middle age workers somewhat less skilled than the native ones. The shock lasts one period, after which the economy moves along its new growth path with a larger number of, now heterogeneous, middle age workers. We assume that the children of the immigrants perfectly integrate, hence after one period they accumulate as much per-capita human capital as the offsprings of the native workers. One should note that, in the context of our model, one period lasts about 25-30 years.

Because we are interested in figuring out how households would insure against the "immigration risk", if they could, we assume financial markets are sequentially complete in the baseline model. Because there are always two possible states of the world next period - one with and one without immigration shock - there are two financial assets agents buy from and sell to each other in every period. One asset pays one unit of consumption only when there is an immigration shock while the other pays a unit of consumption only when there is no immigration shock. Through these two assets - accessible to all individuals living in the country - young and middle age people insure themselves from the impact of an immigration shock. In particular: young people, who will be middle age and working next period, would like to insure against the negative impact that the arrival of the middle age immigrants may have on their wages, which they do by purchasing insurance from the currently middle age people. The latter - who are saving for retirement - can use the extra payoff they would receive from their capital investment if, next period, the immigration shock were realized, to provide the young people with such insurance. Old people do not trade in assets as we assume that they must die without debt and the bequest motive is absent.

The buying and selling of insurance takes place at the same time and through the same instruments that middle age and young use to lend/borrow to/from each other. More precisely: middle age individuals invest in physical capital (by purchasing assets issued by the competitive firms carrying out production next period) and in human capital (by purchasing assets issued by the young agents to finance their own education). Because, when there is immigration, the capital invested in the firms pays off more, it compensates for the lower payoff from the human capital investment accruing to the middle age. This assures that, in the benchmark complete markets economy, both young and middle age people implement as much consumption smoothing as it is feasible - consumption taking place when, respectively, they are middle age and old.

This does not imply perfect consumption smoothing, nor that some ex-ante notion of efficiency is satisfied at the equilibrium of our benchmark model. This is because agents cannot insure beforehand against the risk of being born in a period of high immigration. Young agents born in a period with a positive immigration shock are worse off than they would be otherwise, as they must compete with the offsprings of the immigrants both to borrow funds for investing in human capital this period, and in supplying labor to the market next period. This type of risk cannot be insured away. It would be if parents were altruistic and internalized, via bequests, the future welfare of their children. We assume, instead, that parents are selfish and do not leave anything to their children, hence the latter must bear the cost of being born in the "wrong" period. The extension to the case in which a bequest motive leads parents to purchase insurance for the future generations is an interesting venue for future research.

The key channels through which immigration affects welfare in this economy is that it increases labor supply in the face of a predetermined stock of physical capital. This lowers wages and increases the return on physical capital, shifting income from one generation to another. In this sense, factor prices move around because, in the benchmark model, we have assumed zero capital mobility. If there were perfect capital mobility, capital would flow into the country from outside on the footsteps of immigrant labor, and the capital intensity ratio would remain unchanged. In this case factor prices would be unaffected by immigration, which would amount to nothing more than an increase in the size of the economy. Under constant returns to scale in production, which we assume, this does not affect the welfare of the native agents. Notice, in particular, that when capital flows instantaneously into the country, it does not matter if the human capital of the immigrants is higher, lower or equal to that of the native. Because the K/H ratio remains constant, so does the wage per unit of human capital, hence the salaries of the native do not change at all.

Nevertheless, if there are frictions in the international financial markets and capital adjustment is not instantaneous, i.e. it takes time for the capital stock of the country to be built up to restore the initial K/H ratio, then immigration causes a redistribution between generations, as outlined above. The latter observation suggests that, the larger is the trade deficit following an immigration shock, the quicker will be the adjustment toward the old K/H ratio, hence the smaller the redistribution away from native workers and toward native owners of capital.

We ask next whether government policies can be used to substitute for the credit and insurance markets of the baseline model when these, as it is often the case in reality, are either absent or largely incomplete. To do this we build on previous results presented in Boldrin and Montes (2005) - which answered the question in the affirmative for the case of no immigration shock - adapting their framework to the particular circumstances at hand. In the present case we show that pension payments and social security contributions must be negatively indexed to the size of the immigration flow, while educational expenditures and the issuance of public debt financing it should be positively correlated. Intuitively, this is because social security contributions play the role that the repayment of debt plus interest, - by the currently middle age generation to those that lend them money to invest in human capital - plays in the model with sequentially complete financial markets. The pension payments are nothing but these contributions as received by the old people: they correspond to the payoff from the securities that were traded to finance the human capital investment of the young generation in the previous period. Likewise, the educational investment (financed via the issuance of bonds) corresponds to the issuance of the same securities in this period, hence it should increase as the size of the young generation is larger than expected.

Other authors (Shiller (1999), Bohn (1998,1999)) have stressed the positive role of an unfunded social security system as an instrument to efficiently reallocate the economic impact of aggregate shocks across different generations. They argued that, if the returns to capital and wages are imperfectly correlated and driven by an aggregate shock, an unfunded social security system that endows retired households with a claim to labor income may serve as such risk sharing tool between generations. Krueger and Kubler (2005) point out that the potentially positive intergenerational risk sharing role of social security needs to be traded off against the standard crowding-out effect that unfunded social security has on private savings and thus capital formation. In a realistically calibrated economy with stochastic production, they find that the intergenerational risk sharing role of unfunded social security system is dominated in its importance by the adverse effect on physical capital accumulation arising from the introduction of such a system. Sanchez-Marcos and Sanchez (2004) confirm the findings of Krueger and Kubler (2005) for the case of demographic uncertainty.

An important difference with respect to our economy is that, in all of these papers, the authors abstract from accumulation of human capital and, therefore, from the negative effect that missing credit markets have on education. As we show in section 4 (and in more detail in Boldrin and Montes (2005)), when credit markets for education are absent, even in the presence of government financed education there is "too much" investment in physical capital respect to the complete market allocation. This is because of public education allows the working generation to invest in the human capital of the future generations, but it does not allow the former investors to collect the market return from their beneficiaries. This will, generally, lead to an inefficiency: investment in physical capital is too high and there is less intergenerational consumption smoothing than under the complete market allocation. In this sense, the introduction of a PAYGO system, in which social security contributions correspond to the capitalized value of education services received, is a tool for "efficiently" crowding-out physical capital.

The rest of the paper proceeds as follows. In Section 2 we describe the benchmark model, in Section 3 we show the effects of the absence of credit and insurance markets; in Section 4 we look at the efficient welfare state, in the presence of immigration, in a close economy with incomplete markets; in Section 5 we look at an open economy with incomplete financial markets but with public education and pensions. Section 6 concludes with some practical considerations about the Spanish experience.

2. The basic model

We use an OLG model with a representative agent in each generation, who lives for three periods - youth, middle age and old. There is aggregate uncertainty caused by an immigration flow that may increase the size of the middle age group, thereby affecting the total supply of labor, the wage rate, the return on capital, aggregate output and the size of the future generations.

The population structure, in period t, is (N_t^y, N_t^m, N_t^o), with $N_t^m = (1+z_t)N_{t-1}^y$ and $N_t^y = (1+n)\, N_t^m$, where $-1 < n$ and z_t is the realization of the immigration shock in period t. We use superscripts, y, m and o to denote, respectively, young, middle-age and old people. For simplicity, we assume that the shock z follows a two-state Markow process, with state space $Z = \{\overline{z}, 0\}$, $\overline{z} > 0$. The notation $\pi(z_{t+1}|z_t)$ denotes the probability of $z_{t+1} \in Z$, given z_t.

In each period $t = 0, 1, ...$ a new generation $N_t^y = (1+n)\, N_t^m$ is born, with a per-capita endowment of basic knowledge, h_t^y, which is an input in the production of future human capital, according to $h_{t+1}^m = h(d_t, h_t^y)$. With d_t we denote the physical resources invested in education, which the young must acquire as specified below. The function $h(d, h^y)$ is a constant returns to scale neoclassical production function. During the second period of life, individuals work and decide how much of their income to consume, how much to save, and how to allocate the latter among different financial instruments. When old, they have no decisions to make: they consume all their income, and then die. We assume agents draw utility from consumption when middle age and old. We also assume immigrants enter the country with a fraction $0 < \gamma < 1$ of the human capital level of the natives and with zero capital or financial assets. Neither consumption when young, nor leisure, nor the welfare of descendants affect lifetime utility.

Initial conditions are: K_0, for the capital stock, (N_0^y, N_0^m, N_0^o) for the population, h_0^m for the level of per-capita human capital of the middle age, $A_{-1}^y(0)$, $A_{-1}^m(0)$ for the portfolios of middle age and old people, respectively, and $A_{-1}^f(0)$ for the representative firm carrying out aggregate production, which owns K_0. Finally, we assume that there are no immigrants in the first period, hence the initial N_0^m middle age people all have the same human capital level and the same portfolio of financial assets, as do the initial N_0^o old people.

The preferences of an individual born in period $t-1$ are

$$E_{t-1}\left\{u(c_t^m(z_t)) + \delta E_t\left[u(c_{t+1}^o(z_{t+1}))\right]\right\},$$

where δ is the period discount factor and E the expectation operator. The function $u : \Re_+ \longrightarrow \Re$ is assumed to be strictly increasing, strictly concave and C^2.

2.1. Market structure

Normalize to one the price of output in the initial period, in which the state is $z = 0$; write $p_t(z)$ for the price of consumption in period t and state $z \in Z$ in all subsequent periods. We assume sequentially complete financial markets, *i.e.* that - given the current state z_t and the set Z of possible future states - for all $z \in Z$ there exists a competitive market in which contingent claims $A_t(z)$ are traded, with payoff, in units of next period consumption, $b[A_t(z), z_{t+1}] = 1$ if

$z_{t+1} = z$, and zero otherwise. We assume agents cannot die in debt, i.e. we impose $A_t^m(z) \geq 0$ for all t and z. Let $q(z, z_t)$ be the price, in units of consumption at t, of asset $A(z)$ in period t and state z_t. Notice that here, to save notation, the symbol $A_t(z)$ indicates also the number of units of that asset traded in a given period.

2.2. Firms

There is a representative firm, which in each period uses human and physical capital to produce output according to $Y_t = F(K_t, H_t)$, where aggregate human capital is $H_t = (1 + z_t\gamma) h_t^m N_{t-1}^y$, $0 < \gamma < 1$ and $F(K, H)$ is a constant returns to scale neoclassical production function. Firms last one period and own the physical capital, which they finance by issuing state-contingent securities. More specifically, in each period t the representative firm issues securities $A_t^f(z)$ at a price $q(z, z_t)$, for $z \in \{\overline{z}, 0\}$, with the proceedings of which they purchase K_{t+1}, used for production next period. In period $t+1$, after the realization of the shock, the firm hires workers, carries out production, pays off wages, honors its financial liabilities and then dissolves.

Let $w_t(z)$ be the nominal wage in period t and state $z \in Z$. Write $w(z_t)/p(z_t) = \omega(z_t)$ and $\varphi(z_t) = p(z_t)F_K(K_t, H(z_t))$. The problem of the firm is

$$\max_{A_t^f(z), H_{t+1}} E_t \left\{ p_{t+1}(z) \left[F(K_{t+1}, H_{t+1}(z)) - \omega_{t+1}(z) H_{t+1}(z) - A_t^f(z) \right] \right\}$$

subject to,

$$K_{t+1} = \sum_{z \in Z} q(z, z_t) A_t^f(z).$$

The first order conditions for H and for $A^f(z)$ are

$$\omega_{t+1}(z) = F_H(K_{t+1}, H_{t+1}(z)), \text{ and} \quad (1.a)$$

$$q(z, z_t) = \frac{\pi(z|z_t) p_{t+1}(z)}{\sum_{z \in Z} \pi(z|z_t) \varphi_{t+1}(z)} \text{ for each } z \in Z. \quad (1.b)$$

2.3. Consumers

For a native agent born in period $t-1$ when the state is z_{t-1}, the lifetime optimization problem is

$$\max_{d(z_{t-1}),A^y_{t-1}(z),A^m_t(z)} E_{t-1}\left\{u(c^m_t(z)) + \delta E_t\left[u(c^o_{t+1}(z))\right]\right\}$$

subject to,

$$\begin{aligned} d(z_{t-1}) + \sum_{z\in Z} q(z,z_{t-1})A^y_{t-1}(z) &\leq 0 && (2.a)\\ c^m(z_t) + \sum_{z\in Z} q(z,z_t)A^m_t(z) &= \omega(z_t)h^m_t + A^y_{t-1}(z_t) \quad \forall z_t \in Z && (2.b)\\ c^o(z_{t+1}) &= A^m_t(z_{t+1}) \qquad \forall z_{t+1}\in Z && (2.c)\\ h^m_t &= h[d(z_{t-1}), h^y_{t-1}] && (2.d) \end{aligned}$$

The first order conditions for the choice of $\mathcal{A}^y(z_{t-1}) = \{A^y_{t-1}(z), \text{ for all } z \in Z\}$ and $d(z_{t-1})$ boil down to

$$\begin{aligned} q(z,z_{t-1}) &= \frac{\pi(z|z_{t-1})u'(c^m_t(z))}{\sum_{z\in Z}\pi(z|z_{t-1})u'(c^m_t(z))\omega_t(z)h_d[d(z_{t-1}),h^y_{t-1}]} \quad \forall z\in Z && (3.a)\\ 1 &= \sum_{z\in Z} q(z,z_{t-1})\omega_t(z)h_d[d(z_{t-1}),h^y_{t-1}]. && (3.b) \end{aligned}$$

For each of the $A^m_t(z)$, the first order condition reads

$$q(z,z_t) = \frac{\pi(z|z_t)\delta u'(c^o_{t+1}(z))}{u'(c^m(z_t))} \quad \forall z\in Z. \qquad (3.c)$$

For a middle age immigrant, arriving in the state of the world z_t with human capital $\bar{h}^m_t = \gamma h^m_t$ and $A^y_{t-1}(z_t) = 0$, the maximization problem is:

$$\max_{A^m_t(z)} u(\bar{c}^m(z_t)) + E_t\left[\delta u(\bar{c}^o(z_{t+1}))\right]$$

subject to,

$$\begin{aligned} \bar{c}^m(z_t) + \sum_{z\in Z} q(z,z_t)\bar{A}^m_t(z) &= \omega(z_t)\bar{h}^m_t \quad \forall z_t\in Z && (2.b')\\ \bar{c}^o(z_{t+1}) &= \bar{A}^m_t(z_{t+1}) \quad \forall z_{t+1}\in Z. && (2.c') \end{aligned}$$

The first order conditions determining $\bar{A}^m(z_t)$ are analogous to those in (3.c):

$$q(z,z_t) = \frac{\pi(z|z_t)\delta u'(\bar{c}^o_{t+1}(z))}{u'(\bar{c}^m(z_t))} \quad \forall z\in Z. \qquad (3.c')$$

2.4. Financial markets

It should be clear from the budget constraints that the net financial position of the young is non-positive (i.e. $\sum_{z\in Z} q(z,z_{t-1})A^y_{t-1}(z) \leq 0$) and that of middle age non-negative (i.e. $\sum_{z\in Z} q(z,z_t)A^m_t(z) \geq 0$ and $\sum_{z\in Z} q(z,z_t)\bar{A}^m_t(z) \geq 0$) . When the latter is positive it corresponds to aggregate national saving, which is invested in the physical capital of firms and in the education of the young agents. The first order conditions for profit maximization of the firm imply

$$q(z,z_t) = \frac{\pi(z|z_t)p_{t+1}(z)}{\sum_{z\in Z}\pi(z|z_t)\varphi_{t+1}(z)} \text{ for each } z \in Z. \qquad ((4.a))$$

Multiplying $(4.a)$ by $F_K(K_{t+1}, H_{t+1}(z))$ and aggregating in $z \in Z$ we get

$$\sum_{z\in Z} q(z,z_t)\, F_K(K_{t+1}, H_{t+1}(z)) = 1. \qquad (4.b)$$

2.5. Competitive equilibrium

A *competitive equilibrium* is a mapping from the current state of the world into a distribution of quantities and prices at all t. For an initial condition $\left(K_0, H_0, z_0, N^o_0, A^y_{-1}(z_0), A^m_{-1}(\right.$ and a sequence $\{h^y_t(z)\}^\infty_{t=0}$, a competitive equilibrium is a collection of choices: (1) for native, $\{d(z_t), c^m_t(z), c^o_t(z), A^y_t(z), A^m_t(z)\}^\infty_{t=0}$, and immigrant, $\{\bar{c}^m_t(z), \bar{c}^o_t(z), \bar{A}^m_t(z)\}^\infty_{t=0}$, households; (2) for the representative firm, $\left\{K_t(z), H_t(z), A^f_t(z)\right\}^\infty_{t=0}$; as well as (3) prices, $\{p_t(z), q(z,z_t), \omega_t(z), \varphi_t(z)\}^\infty_{t=0}$; such that for all t and $z \in Z$, the consumers and the firm maximize their payoffs and the markets clear.

In each period t and state z there are three sets of markets to clear:

i) Output market:

$$C^m_t(z) + C^o_t(z) + d_t(z)\, N^y_t(z) + K_{t+1}(z) = F(K_t, H_t(z)). \qquad (5.a)$$

where $C^m_t(z) = [c^m_t(z) + \bar{c}^m_t(z)\, z]N^y_{t-1}$ and $C^o_t(z) = [c^o_t(z) + \bar{c}^o_t(z)\, z_{t-1}]N^y_{t-2}$.

ii) Labor market:

$$(1 + z\gamma)\, h^m_t N^y_{t-1} = H_t(z). \qquad (5.b)$$

iii) Capital market:

$$\sum_{z\in Z} q(z,z_t)A_t^f(z) = K_{t+1}, \tag{5.c}$$
$$A_t^f(z) = N_{t-1}^y A_t^m(z) + z_t N_{t-1}^y \bar{A}_t^m(z) + N_t^y A_t^y(z),$$
$$-\sum_{z\in Z} q(z,z_t)A_t^y(z) = d(z_t).$$

For each state $z \in Z$, the payoff from security $A_t^f(z)$ is:

$$b[A_t^f(z), z_{t+1} = z]A_t^f(z) = F(K_{t+1}, H_{t+1}(z)) - \omega_{t+1}(z)H_{t+1}(z) = F_K(K_{t+1}, H_{t+1}(z))K_{t+1}. \tag{5.d}$$

2.6. Example 1

Consider an economy with logarithmic utility and Cobb Douglas production functions: $u(c) = \log c$, $F(K,H) = AK^\alpha H^{1-\alpha}$ and $h(d,h^y) = Bd^\beta (h^y)^{1-\beta}$.

Write, $\widetilde{\omega}(z_t) = \omega(z_t)h(d_{t-1}, h_{t-1}^y)$. >From (3.$c$) we have, for a native middle age,

$$q(z,z_t) = \frac{\pi(z|z_t)\delta}{A_t^m(z)}\left[\widetilde{\omega}(z_t) + A_{t-1}^y(z_t) - \widetilde{A}^m(z_t)\right] \ \forall z \in Z,$$

where $\widetilde{A}^m(z_t) = \sum_{z\in Z} q(z,z_t)A_t^m(z)$. Multiplying by $A_t^m(z)$ and aggregating in $z \in Z$ we arrive at the total demand for contingent securities of a native middle age individual

$$\widetilde{A}^m(z_t) = \frac{\delta}{1+\delta}\left[\widetilde{\omega}(z_t) + A_{t-1}^y(z_t)\right].$$

The demand for consumption in middle-age, and the demand for each component $A_t^m(z)$ of $\mathcal{A}^m(z_t)$, are

$$c^m(z_t) = \frac{1}{1+\delta}\left[\widetilde{\omega}(z_t) + A_{t-1}^y(z_t)\right],$$
$$A_t^m(z) = \frac{\delta}{1+\delta}\left[\widetilde{\omega}(z_t) + A_{t-1}^y(z_t)\right]\frac{\pi(z|z_t)}{q(z,z_t)}.$$

For an immigrant in t we get

$$\begin{aligned}\bar{c}^m(z_t) &= \frac{1}{1+\delta}\gamma\widetilde{\omega}(z_t),\\ \widetilde{\bar{A}}^m(z_t) &= \frac{\delta}{1+\delta}\gamma\widetilde{\omega}(z_t),\\ \bar{A}_t^m(z) &= \frac{\delta}{1+\delta}\gamma\widetilde{\omega}(z_t)\frac{\pi(z|z_t)}{q(z,z_t)}.\end{aligned}$$

Now, using (5.c)-(5.d) and taking into account that $A_t^m(z) = \widetilde{A}^m(z_t)\pi(z|z_t)/q(z,z_t)$ and $\bar{A}_t^m(z) = \widetilde{\bar{A}}^m(z_t)\pi(z|z_t)/q(z,z_t)$, we have the demand for each component $A_{t-1}^y(z)$ of $\mathcal{A}^y(z)$:

$$A_{t-1}^y(z) = -d(z_{t-1})\frac{\pi(z|z_{t-1})}{q(z,z_{t-1})} + \frac{K_t}{N_{t-1}^y}\left[\frac{\varphi_t(z)}{p_t(z)} - \frac{\pi(z|z_{t-1})}{q(z,z_{t-1})}\right].$$

Furthermore, the expected return on human and physical capital must be equalized in equilibrium. Using this condition we have:

$$-\widetilde{A}^y(z_{t-1})N_{t-1}^y = d(z_{t-1})N_{t-1}^y = \eta K_t \Psi(z_{t-1}),$$

where $\Psi(z_{t-1}) = E_{t-1}\left\{p_t(z)(1+\gamma z)^{-\alpha}\right\}/E_{t-1}\left\{p_t(z)(1+\gamma z)^{1-\alpha}\right\}$ and $\eta = \beta(1-\alpha)/\alpha$. Therefore

$$A_{t-1}^y(z) = -d(z_{t-1})\frac{\pi(z|z_{t-1})}{q(z,z_{t-1})} + \frac{d(z_{t-1})}{\eta\Psi(z_{t-1})}\left[\frac{\varphi_t(z)}{p_t(z)} - \frac{\pi(z|z_{t-1})}{q(z,z_{t-1})}\right].$$

Finally, from (1) we obtain the equilibrium prices for each period t and state $z \in Z$:

$$\begin{aligned}\omega_t(z) &= (1-\alpha)AK_t^{\alpha}H_t(z)^{-\alpha}\\ \varphi_t(z) &= p(z_t)\alpha AK_t^{\alpha-1}H_t(z)^{1-\alpha}\\ q(z,z_t) &= \frac{\pi(z|z_t)p_{t+1}(z)}{E_t\left\{p_{t+1}(z)\alpha AK_{t+1}^{\alpha-1}H_{t+1}^{1-\alpha}(z)\right\}}.\end{aligned}$$

Note also that in equilibrium $p_t(\bar{z})c_t^m(\bar{z}) = p_t(0)c_t^m(0)$. Substituting the values of $c_t^m(\bar{z})$ and $c_t^m(0)$ we arrive to $p_t(\bar{z}) = p_t(0)\frac{(1+\gamma\bar{z})^{\alpha}}{1+\alpha\gamma\bar{z}}$, where $(1+\gamma\bar{z})^{\alpha} < 1+\alpha\gamma\bar{z}$ and we normalize $p_t(0) = 1$.

Set $h_t^y = H_t/N_t^y$ so that an autonomous system can be derived. Given initial conditions for $\left(K_0, H_0, N_0^o, A_{-1}^y(z_0), A_{-1}^m(z_0), A_{-1}^f(z_0)\right)$, the following system describes the dynamic of the economy for a given sequence of shocks $(z_0, z_1, ...)$,

$$\begin{aligned} K_{t+1} &= \Omega(z_t, z_{t-1}) A K_t^\alpha (H(z_t))^{1-\alpha}, \\ H(z_{t+1}) &= (1+\gamma z_{t+1}) B (\eta A \Omega(z_t, z_{t-1}) \Psi(z_t))^\beta K_t^{\alpha\beta} (H(z_t))^{1-\alpha\beta}. \end{aligned} \tag{6}$$

where

$$\Omega(z_t, z_{t-1}) = \frac{\delta}{1+\delta} \frac{1}{(1+\eta\Psi(z_t))} \left(1 - \frac{(1+\alpha\gamma z_t)}{(1+\gamma z_t)} \alpha \left[\pi(\bar{z}|z_t) \frac{(1+\gamma\bar{z})}{1+\alpha\gamma\bar{z}} + \pi(0|z_t)\right] (\eta\Psi(z_{t-1})+1)\right)$$

and

$$\Psi(z_t) = \frac{\pi(\bar{z}|z_t)\frac{1}{1+\alpha\gamma\bar{z}} + \pi(0|z_t)}{\pi(\bar{z}|z_t)\frac{(1+\gamma\bar{z})}{1+\alpha\gamma\bar{z}} + \pi(0|z_t)}.$$

Given a sequence of shocks $(z_0, z_1, ..)$ the evolution of the factor intensity ratio $X = K/H$ is given by

$$X_{t+1} = \frac{(\Omega(z_t, z_{t-1}) A)^{1-\beta}}{(1+\gamma z_{t+1}) B (\eta\Psi(z_t))^\beta} X_t^{\alpha(1-\beta)}.$$

Set $(z_t, z_{t+1}, z_{t+2}, ...) = (0, 0, 0, ...)$. The ray

$$X^* = \left[\frac{(\Omega(0,0) A)^{1-\beta}}{B(\eta\Psi(0))^\beta}\right]^{\frac{1}{1-\alpha(1-\beta)}}$$

defines a balanced growth path. For all initial conditions $(H_0, K_0) \in \Re_+^2$, iteration of (6) leads (H_t, K_t) to the ray X^*.

Along the balanced growth path, the two stocks of capital expand (or contract) at the factor

$$1 + g^* = \left[\frac{(\Omega(0,0) A)^\beta}{B(\eta\Psi(0))^\beta}\right]^{\frac{\alpha-1}{1-\alpha(1-\beta)}}.$$

2.6.1. Numerical example

Let us now consider the practical implications of an immigration shock using a numerical example. We assign reasonable values to preferences and technological parameters, but make no claim to be using empirically "certified" values. We compare two economies: an economy with no immigration $(z_0, z_1, z_2, ...) = (0, 0, 0, ...)$ and another with only one immigration shock $(z_0, z_1, z_2, ...) = (0, \bar{z}, 0, ...)$. We assume $\bar{z} = 0.3$, $\gamma = 0.7$, $\pi(\bar{z}|z_t) = \pi(0|z_t) = 0.5$. We normalize $N_0^m = 1$ and assume an annual growth rate of the population equal to 0. Recall that a period in this model is about 30 years. With respect to the production technology, α is fixed tat 0.3 and the scale parameter A is fixed at 1. In the human capital technology we set $\beta = 0.13$, which corresponds to an elasticity of output with respect of education of 0.09. The discount factor δ is set to match the ratio of investment over output $I/Y = 22\%$. This yields a value of $\delta = 0.904$, which corresponds to an annual discount rate of $0,996641$. We set the scale parameter B equal to 4 to get an annual rate of aggregate output growth equal to 3% along the balance growth path. With these parameter values we have an annual interest rate of $4,07\%$ (and a capital-output ratio equal to 1.69 in annual terms, which is certainly small when compared to the often used values). The fraction of production dedicated to education is equal to 6%, again a number on the low side for the US but not for many European countries such as Spain.

Assume the same initial conditions for both economies and assume they are already in their balance growth path from the start. Denote with "hat symbols" $(\widehat{x})$ the variables in the economy with an immigration shock in $t = 1$. In Table 1 we show the change (relative to the case of no shock) in utility and consumption, of middle age and old, caused by an immigration shock in period $t = 1$. Notice, first, that the generations alive when the shock hits consume more than in the economy with $(z_0, z_1, z_2, ...) = (0, 0, 0, ...)$, because output is much higher during that period and the insurance mechanism redistributes this extra income to both middle age people and old. The future generations, nevertheless, are worse off in the economy with $(z_0, z_1, z_2, ...) = (0, \bar{z}, 0, ...)$ because, as pointed out before, they could not purchase insurance against the immigration shock before being born. The immigration shock affects future generations negatively through two channels. First, because the increase in labor supply reduces per-capita wages in a way that is not compensated by the increase in the marginal productivity of capital that the abundance of labor in future periods entails. Second, because the new immigrant workers need to be endowed with productive capital, and this requires additional investment, which reduces consumption for at least one period.

More precisely, the mechanism at work is the following.

Our is a model of endogenous growth driven by constant returns to scale in the two reproducible factors, physical and human capital. The long run growth rate does not depend on the size of the economy, which increases after the immigration shock, but only on the technology and preferences parameters. Once the labor supply increases, equilibrium dictates that an extra amount of output must be allocated to endow these workers, and their offsprings, with physical and human capital in all periods following the one of the shock. Hence, saving rates must temporarily increase until the factor proportions reach again the balanced growth levels, after which growth resumes at the same rate as in the economy without shock. Because the additional investment, made necessary by the shock, reduces consumption for a few periods, after which consumption grows at the same rate as in the original equilibrium path, the new path will always stay below the original one. Hence, consumption levels, while growing at the same rate, will be lower forever. This explains the lower steady state utility for the representative agent in the economy with the immigration shock. Put it differently: because the immigrants come without physical capital (and less human capital) this must be provided, at least partially, by the natives, who give up some consumption during the adjustment periods. Once the adjustment is completed (which takes three periods, counting the one with the shock) in our calibration, growth resumes at the same rate but at a consumption level that is lower forever.

In Table 2 we show the effect of the immigration shock on the annual interest rate, wage and annual growth rate of aggregate output. As just argued, the immigration shock has a positive effect on the investment rate in the period in which the immigrants arrive and a temporarily negative (positive) effect on labor (capital) productivity. After the adjustment is completed, the growth rate and the marginal productivities resume their original long-run levels.

Table 1: Change in life-cycle utility and consumption

t	$\hat{U}_{t-1} - U_{t-1}$	$\hat{c}_t^m(z)/c_t^m(0)$	$\hat{c}_t^o(z)/c_t^o(0)$
0	0,0035	1	1
1	-0,006	1,0039	1,0039
2	-0,121	0,9401	0,9883
3	-0,126	0,9363	0,9363
4	-0,127	0,9353	0,9353
5	-0,127	0,9350	0,9350
6	-0,127	0,9350	0,9350

Table 2 Changes caused by an immigration shock in $t = 1$.

t	$\hat{r}_t$ annual	$\hat{\omega}_t$	$\hat{g}\ Y$ annual
0	0,0407	0,2500	0,03004
1	0,0453	0,2361	0,03660
2	0,0402	0,2517	0,03015
3	0,0406	0,2504	0,03007
4	0,0407	0,2501	0,03004
5	0,0407	0,2500	0,03004
6	0,0407	0,2500	0,03004

3. Equilibrium when credit and insurance markets are missing

The result here is substantially similar to what we find in Boldrin and Montes (2005). The only variation is that now we have uncertainty, hence one wants to analyze separately what happens when insurance markets are not available from what happens when also the lending/borrowing markets are not available and the only possible investment is to purchase the capital stock. This gives rise to the following two cases.

1. Young people cannot trade in $A^y(z)$, still they can borrow to invest d in human capital. Because there are no state contingent assets, they must repay their debt at a common interest rate, no matter if next period an immigration shock is or is not realized. That is to say: they can borrow but they cannot insure. In this case, even if the middle age people attempted to trade in state contingent $A^m(z)$ assets, it would not work for lack of a counterpart. The only entity they can trade those assets with is the firm, which cannot insure them against anything as it has no compensating sources of income in bad states. The income of old age people is now equal to the fixed return on d plus the random return on capital investment. Because these are linearly independent returns, and there are only two states of the world, middle age people can still use a portfolio composed of "educational bonds" and "shares of the firm" to span the whole state space for the period in which they are old. In other words, they can still fully insure their old age consumption (notice, though, that for particular parameters configuration this may require taking a negative position in one of the two assets, an

impossibility in this environment). This result is not general, though, as it follows from the special assumption of only two levels of the immigrations shock: nothing or positive. Should there be more, there would still be only two assets (the educational bond and the share) but many more states of the world, making spanning impossible. In either case, young people bear all the risk because, when middle age, they must reimburse a fixed amount $d(1+r)$, no matter what the state of the world is. This implies that, when there is immigration, middle age natives have less income to consume and save than in the complete markets case. As in the world with complete markets, their wage bill is lower (marginal productivity of labor decreases due to arrival of immigrants), but their debt payment is now higher, which leaves less for $c^m(z_t) + \sum_{z \in Z} q(z, z_t) A_t^m(z)$. Their lifetime utility is lower and, furthermore, total investment decreases.

2. Young people cannot borrow at all, hence middle age people can only invest in the physical capital. Obviously this implies that there is a much lower level of human capital in the economy, and there is no growth. In this case "workers" bear all the downside risk (i.e. they either do "normal" or do "worse") whereas the owners of capital bear all the upside (i.e. they either do "normal" or do "better").

4. The Welfare State of a Closed Economy

4.1. Missing credit and insurance markets

Let us begin with the case 2. of the previous section, in which all credit and insurance markets other than the market for physical capital have been shut down. In this case both, $F_H(K_t, H_t)$ and $F_K(K_t, H_t)$, are affected by the shock and neither one of the two factor owners can insure against it. We want to derive policies that are able to implement the sequentially complete market allocation (SCMA) of Section 2. They turn out not to be very different from those derived in Boldrin and Montes (2005), apart from the fact that contributions and benefits are now state contingent. Under uncertainty, we need to use the welfare state to also allocate risk efficiently between generations and not just, as in the deterministic case, to allow for intergenerational trade. Think of what happens when there is an unexpected flow of immigrants ($z = \overline{z}$): the marginal productivity of labor decreases and the marginal productivity of capital increases. If we simply levy a social security contribution in the amount $t_t^p = d_{t-1}^* R_t^*$ (starred symbols for now on refer to the SCMA) and nothing else, the per capita income of the middle age individuals decreases compared to the SCMA. In this class of models, immigration causes a redistribution from working to retired people; in general it redistributes from labor to capital. Furthermore, the decrease in net labor income will tend to reduce workers' saving, implying an under-investment in physical capital compared to the SCMA.

There can be gains from allowing the middle-age and old generations to share the immigration risk. We should stress here a relatively delicate point: an immigration shock causes aggregate uncertainty (it increases aggregate output) but, because in a competitive world it affects the two factors of productions differently, lower the wage rate and increases the rate of return on capital (it lowers the wage rate and increases the rate of return on capital) part of the aggregate uncertainty is insurable. In particular, the native workers face the risk of a reduced per capita income, while the native capital owners face the risk of an increased per-capita income. On the other hand, if there is no immigration, the native workers earn a higher per capita salary, while the capital owners earn a smaller share of output. Insurance, then, must work the following way: when there is immigration the old people (owning capital) pay something to the native middle age people, viceversa in those periods in which there is no immigration. We, therefore need to add a policy emulating the way in which intergenerational insurance markets would work.

Assume a period-by-period balanced budget and introduce two tax and transfer schemes similar to those studied in our earlier work; we call the first a "pension scheme" and the second an "education scheme". Write

$$t_t^p(z)\,N_{t-1}^y + \bar{t}_t^p(z)\,zN_{t-1}^y = b_t(z)\,N_{t-2}^y + \bar{b}_t(z)\,z_{t-1}N_{t-2}^y,$$

for the pension scheme, and

$$t_t^e(z)\,N_{t-1}^y + \bar{t}_t^e(z)\,zN_{t-1}^y = e_t(z)\,N_t^y,$$

for the education scheme. Let us start from the last equation. Here $e_t(z)$ denotes the educational transfer received from each member of the currently young generation. On the other side of the budget constraint, we find the contributions provided, respectively, by the middle age natives ($t_t^e(z)$) and by the middle age immigrants ($\bar{t}_t^e(z)$). In the optimal policy, we treat working immigrants differently from working natives, as they receive different wages in light of their different human capital. On the other hand, the optimal policy dictates treating all young people alike, those born to immigrants and those born to natives.

The budget constraint for the pension scheme can be interpreted similarly, but here we need treating natives and immigrants differently on either side. They pay different contributions ($t_t^p(z)$ and $\bar{t}_t^p(z)$, respectively) and receive different benefits when retired, $b_t(z)$ and $\bar{b}_t(z)$. Again, this mimic what would have happened in an economy like that of section 2, where markets were dynamically complete. The important point here is that, in both schemes, the contribution and benefit rates are state contingent, i.e. change depending on the immigration flow. The latter is an aggregate variable, hence the state contingent policy does not depend on any private information but on a state variable that should, at least in principle, be observable by the policy maker.

Under these policies, the budget constraints for the representative member of the generation born in period $t-1$ become

$$\begin{aligned}
&d(z_{t-1}) \leq e(z_{t-1}) \\
&c^m(z_t) + s(z_t) = \omega(z_t)h(d(z_{t-1}), h_{t-1}^y) - t^e(z_t) - t^p(z_t) \quad \forall z_t \in Z \\
&c^o(z_{t+1}) = s(z_t)R(z_{t+1}) + b(z_{t+1}) \quad \forall z_{t+1} \in Z
\end{aligned}$$

For an immigrant arriving in period t, the budget constraints read

$$\begin{aligned}
&\bar{c}^m(z_t) + \bar{s}(z_t) = \omega(z_t)\gamma h(d(z_{t-1}), h_{t-1}^y) - \bar{t}^e(z_t) - \bar{t}^p(z_t) \quad \forall z_t \in Z \\
&\bar{c}^o(z_{t+1}) = \bar{s}(z_t)R(z_{t+1}) + \bar{b}(z_{t+1}) \quad \forall z_{t+1} \in Z,
\end{aligned}$$

The symbol $s(z_t)$ is the investment in physical capital an individual makes in period t and state z, and $R(z_{t+1}) = \varphi(z_{t+1})/p(z_{t+1})$. If we set $e(z_{t-1}) = d^*(z_{t-1})$ (starred symbols refer to the SCMA), *i.e.* we transfer educational resources to the young generation up to the point at which the expected return on education is equal to the expected return on physical capital,

$$\sum_{z \in Z} \pi(z|z_{t-1})p_t(z)\, R_t(z) = \sum_{z \in Z} \pi(z|z_{t-1})p_t(z)\, \omega_t(z)h_d(d(z_{t-1}), h_{t-1}^y),$$

we reach the efficient level of human capital in period t. In Boldrin and Montes (2005) we show that in a deterministic world this policy, together with $t_t^p(z) =$

$d^*(z_{t-1})R_t^*(z)$, $\bar{t}_t^p(z) = 0$, $b_t(z) = t_{t-1}^e R_t^*(z)$ and $\bar{b}_t(z) = \bar{t}_{t-1}^e R_t^*(z)$, implements the efficient CMA overall. Pension benefits received (social security contributions) must correspond to the capitalized value of the lifetime contributions to aggregate human capital accumulation (educational services received). But this policy is not enough to achieve the appropriate amount of intergenerational risk sharing when we have shocks affecting the size of working population. We need to add a second mechanism allocating risk between generations.

Comparison of the last budget restrictions with the budget restrictions of the SCMA, $(2.a) - (2.d)$, shows that, if the lump-sum tax-transfer amounts satisfy

$$t^p(z_t) = A_{t-1}^{y*}(z_t), \qquad \bar{t}^p(z_t) = 0,$$

$$\begin{aligned} b(z_{t+1}) &= A_t^{m*}(z_{t+1}) - \frac{\lambda^*(z_t)K_{t+1}^*}{N_{t-1}^y} R^*(z_{t+1}), \\ \bar{b}(z_{t+1}) &= \bar{A}_t^{m*}(z_{t+1}) - \frac{\left(1 - \bar{\lambda}^*(z_t)\right) K_{t+1}^*}{z_t N_{t-1}^y} R^*(z_{t+1}), \end{aligned}$$

and

$$t^e(z_t) = \tilde{A}^{m*}(z_t) - \frac{\lambda^*(z_t)K_{t+1}^*}{N_{t-1}^y}, \quad \bar{t}^e(z_t) = \tilde{\bar{A}}^{m*}(z_t) - \frac{\left(1 - \bar{\lambda}^*(z_t)\right) K_{t+1}^*}{z_t N_{t-1}^y},$$

where $\lambda^*(z_t)$ $(\bar{\lambda}^*(z_t))$ is the portion of aggregate investment in physical capital that it is made by a native (immigrant) worker in period t in the SCMA, the competitive equilibrium under the new policy achieves the SCMA. Note how this is done: our pension system implements the investment in physical and human capital of the SCMA by "crowding-out" private saving by means of social security contributions.

We can interpret the efficient pension system as one with two components:

$$\begin{aligned} b(z_t) &= \underbrace{t^e(z_{t-1}) R^*(z_t)}_{\hat{b}(z_t)} + \underbrace{\left(A_{t-1}^{m*}(z_t) - \tilde{A}^{m*}(z_{t-1}) R^*(z_t)\right)}_{\tau^o(z_t)}, \\ t^p(z_t) &= \underbrace{d^*(z_{t-1}) R^*(z_t)}_{\hat{t^p}(z_t)} + \underbrace{\frac{\tau^o(z_t) N_{t-2}^y + \bar{\tau}^o(z_t) z_{t-1} N_{t-2}^y}{N_{t-1}^y}}_{\tau^m(z_t)}. \end{aligned}$$

For an immigrant we have $\widehat{\bar{b}}(z_t) = \bar{t}^e(z_{t-1}) R^*(z_t)$, $\widehat{\bar{t}}^p(z_t) = \widehat{\bar{\tau}}^m(z_t) = 0$ and $\bar{\tau}^o(z_t) = \left(\bar{A}^{m*}_{t-1}(z_t) - \widetilde{\bar{A}}^{m*}(z_{t-1}) R^*(z_t)\right)$.

The first component $\left(\widehat{t^p}(z_t), \hat{b}(z_t)\right)$ is used to repay the capitalized value of the educational debt to the lender. The second component $(\tau^m(z_t), \tau^o(z_t))$ is an insurance contract through which the middle-age and old generations share the immigration risk. The signs of $\tau^m(z_t)$ and $\tau^o(z_t)$ depend on the realization of the shock: when immigration is positive, $\tau^m(\bar{z}) < 0$ and $\tau^o(\bar{z}) < 0$, reflecting a transfer from retirees to workers; the opposite in the other case.

We have assumed lump sum taxes and transfers. The distortive effects of payroll taxes on the labor supply decision remain unmodeled. Interestingly, it turns out that in the special case we are studying, taxing the purchases of physical capital to finance education, while subsidizing the return from physical capital a period later, may implement the SCMA. The taxes collected from the middle age to finance pensions can be broken down into two parts. The first portion should be proportional to previous borrowing for education. The second portion serves an intergenerational insurance/redistribution purpose. Under this policy, the budget constraints for a native agent born in period $t-1$, when the state is z_{t-1}, read

$$
\begin{aligned}
& d(z_{t-1}) \leq e(z_{t-1}) \\
& c^m(z_t) + s(z_t)\left(1 + \hat{\tau}(z_t)\right) = \omega(z_t) h(d(z_{t-1}), h^y_{t-1}) - t^p(z_t) \quad \forall z_t \in Z \\
& c^o(z_{t+1}) = s(z_t)\left(1 + \hat{\tau}(z_t)\right) R(z_{t+1}) + \tau^o(z_{t+1}) \quad \forall z_{t+1} \in Z,
\end{aligned}
$$

whereas, for an immigrant arriving in period t, the budget constraints are

$$
\begin{aligned}
& \bar{c}^m(z_t) + \bar{s}(z_t)\left(1 + \hat{\tau}(z_t)\right) = \omega(z_t)\gamma h(d(z_{t-1}), h^y_{t-1}) - \bar{t}^p(z_t) \quad \forall z_t \in Z \\
& \bar{c}^o(z_{t+1}) = \bar{s}(z_t)\left(1 + \hat{\tau}(z_t)\right) R(z_{t+1}) + \bar{\tau}^o(z_{t+1}) \quad \forall z_{t+1} \in Z.
\end{aligned}
$$

Period-by-period balanced budget in the two systems (pensions and education) imply

$$
\begin{aligned}
e(z_t) N^y_t &= \hat{\tau}(z_t)\left[s(z_t) N^y_{t-1} + \bar{s}(z_t) z_t N^y_{t-1}\right], \\
t^p(z_t) N^y_{t-1} + \bar{t}^p(z_t) z_t N^y_{t-1} &= e(z_{t-1}) N^y_{t-1} R(z_t) + \tau^o(z_t) N^y_{t-2} + \bar{\tau}^o(z_t) z_{t-1} N^y_{t-2}.
\end{aligned}
$$

Set

$$
\begin{aligned}
\hat{\tau}(z_t) &= d^*(z_t) N_t^y / K_{t+1}^*, \qquad \bar{t}^p(z_t) = 0, \\
\tau^o(z_{t+1}) &= A_t^{m*}(z_{t+1}) - s(z_t)(1+\hat{\tau}(z_t)) R^*(z_{t+1}), \\
\bar{\tau}^o(z_{t+1}) &= \bar{A}_t^{m*}(z_{t+1}) - \bar{s}(z_t)(1+\hat{\tau}(z_t)) R^*(z_{t+1}) \quad \text{and} \\
t^p(z_{t+1}) &= e(z_t) R^*(z_{t+1}) + \frac{\tau^o(z_{t+1}) N_{t-1}^y + \bar{\tau}^o(z_{t+1}) z_t N_{t-1}^y}{N_t^y},
\end{aligned}
$$

and the SCMA is implemented[1].

Consider now the case in which, instead of financing education via taxation, the government issues one-period, ear-marked debt in the amount $d^*(z_t) N_t^y$ in each period. In the following period, the government pays back $d^*(z_t) R(z_{t+1}) N_t^y + \tau^o(z_{t+1}) N_{t-1}^y + \bar{\tau}^o(z_{t+1}) z_t N_{t-1}^y$ to the debt holders (where $\tau^o(z_{t+1}) = A_t^{m*}(z_{t+1}) - \tilde{A}^{m*}(z_t) R^*(z_{t+1})$ and $\bar{\tau}^o(z_{t+1}) = \bar{A}_t^{m*}(z_{t+1}) - \tilde{\bar{A}}^{m*}(z_t) R^*(z_{t+1})$). Also in this case, the repayment is financed by a tax on the middle-age individuals that should be parted into two components. The first component is proportional to the previous use of public education financing. The second component, again, is for intergenerational insurance. Notice that the net present value of this tax is effectively lump sum for the middle age worker, as it depends only on actions taken in the previous period and on the realization of an exogenous state of the world. In particular, it is not affected by individual's labor supply decisions. In this scheme the government effectively acts as a (somewhat special) financial institution, issuing the missing securities and using its taxing power to enforce repayment.

4.2. Missing insurance markets

The previous analysis shows that in case 1 in section 3, i.e. when agents have access to credit markets to finance education but insurance is not being offered, a PAYGO pension system that always transfers resources from workers to retirees is not efficient. In the absence of private insurance markets, we need a system of intergenerational taxes-transfers contingent on the realization of the immigration shock. Call it $\tau_t^m(z), \tau_t^o(z), \bar{\tau}_t^o(z)$. The balance budget of this system reads

[1] For the economy considered in the example 1 the choice of $\hat{\tau}(z_t) = \eta\Psi(z_t)$, $\tau^o(z_{t+1}) = \tilde{A}^{m*}(z_t)\left[E_t\left\{p_{t+1}^*(z) R_{t+1}^*(z)\right\}/p_{t+1}^*(z) - R_{t+1}^*(z)\right]$, $\bar{\tau}^o(z_{t+1}) = \tilde{\bar{A}}^{m*}(z_t)\left[E_t\left\{p_{t+1}^*(z) R_{t+1}^*(z)\right\}/p_{t+1}^*(z) - R_{t+1}^*(z)\right]$ and $t^p(z_t) = d^*(z_{t-1}) R^*(z_t) + \left[\tau^o(z_t) N_{t-2}^y + \bar{\tau}^o(z_t) z_{t-1} N_{t-2}^y\right]/N_{t-1}^y$ is sufficient to implement the SCMA.

$$\tau_t^m(z)\,N_{t-1}^y = \tau_t^o(z)\,N_{t-2}^y + \bar{\tau}_t^o(z)\,z_{t-1}N_{t-2}^y.$$

The budget constraints for the representative member of the generation born in period $t-1$ become

$$\begin{aligned}
&d(z_{t-1}) \le e(z_{t-1})\\
&c^m(z_t) + s(z_t) = \omega(z_t)h(d(z_{t-1}), h_{t-1}^y) - d(z_{t-1})R(z_t) - \tau^m(z_t) \quad \forall z_t \in Z\\
&c^o(z_{t+1}) = s(z_t)R(z_{t+1}) + \tau^o(z_{t+1}) \quad \forall z_{t+1} \in Z
\end{aligned}$$

For an immigrant arriving in period t, the budget constraints read

$$\begin{aligned}
&\bar{c}^m(z_t) + \bar{s}(z_t) = \omega(z_t)\gamma h(d(z_{t-1}), h_{t-1}^y) \quad \forall z_t \in Z\\
&\bar{c}^o(z_{t+1}) = \bar{s}(z_t)R(z_{t+1}) + \bar{\tau}^o(z_{t+1}) \quad \forall z_{t+1} \in Z.
\end{aligned}$$

Market clearing is

$$s(z_t)N_{t-1}^y + \bar{s}(z_t)z_tN_{t-1}^y = K_{t+1} + d(z_t)N_t^y.$$

To implement the SCMA we must set

$$\begin{aligned}
\tau^o(z_t) &= \left(A_{t-1}^{m*}(z_t) - \tilde{A}^{m*}(z_{t-1})\,R^*(z_t)\right), \quad \text{and}\\
\bar{\tau}^o(z_t) &= \left(\bar{A}_{t-1}^{m*}(z_t) - \widetilde{\bar{A}}^{m*}(z_{t-1})\,R^*(z_t)\right).
\end{aligned}$$

For the economy studied in Example 1, to implement the SCMA, is sufficient to pick $\tau^o(z_{t+1}) = \tilde{A}^{m*}(z_t)\left[E_t\left\{p_{t+1}^*(z)\,R_{t+1}^*(z)\right\}/p_{t+1}^*(z) - R_{t+1}^*(z)\right]$ and $\bar{\tau}^o(z_{t+1}) = \widetilde{\bar{A}}^{m*}(z_t)\left[E_t\left\{p_{t+1}^*(z)\,R_{t+1}^*(z)\right\}/p_{t+1}^*(z) - R_{t+1}^*(z)\right]$.

5. The Welfare State of an Open Economy.

How should the previous analysis be altered in the case of an open economy? It is easy to realize that it may change completely if capital mobility is both instantaneous and perfect, as capital will flow into the country at the same speed at which immigrants do, so as to restore equality between the internal rate of return on capital and the one established on the international capital markets. When this is the case, the immigration shock has no economic relevance whatsoever as neither the wage of the native workers nor the return on capital of the native capital owners will be affected by the arrival of new workers. Efficient and perfectly frictionless capital markets may act, in this context, as insurance devices rendering the state contingent assets essentially redundant. This is an interesting result as it suggests that, in the light of the simulations presented earlier, the Spanish trade deficit was beneficial, in terms of consumption and overall utility, to both the native households and the immigrant ones.

This observation helps explaining, at least in part, what we observed in Spain during the last twelve years or so: as the flow of immigration into the country continued and even increased, Spain external trade deficit balooned while productivity did not move. Along the lines of our model, these facts have the following interpretation: capital flew into Spain if not at the same rate at which labor was entering, certainly at a very high rate, thereby preventing the K/H ratio from falling and the real wage rate with it. Analysis of the actual data is difficult, not to say impossible, by means of a model as simplified as this, in which there is no distinction between durable and non durable goods and one period lasts roughly thirty years of which, since the immigration shock first hit, we have observed at most half. To put it differently: if one takes our model literally, there is no sense in which it can be used to study the Spanish case because we have not yet observed even a single "model period" in the actual data.

Nevertheless, a simple back of the envelope calculation based on our model should tell us how much the immigration shock contributed to the Spanish trade deficit of the last 13 years. Assume, therefore, that capital flew into Spain at roughly the rate needed, year after year, to keep the internal K/H ratio constant. We do know that immigrants probably have a slightly lower human capital level than native, but we do not have good estimates of the ratio (γ in our notation) so, let us assume for simplicity that $\gamma = 1$.

The bottom line of the calculation is the following. In Spain the annual K/Y ratio without housing is 2.8, and higher than 4.0 with housing. Employment in 1996 was still about 12.8 M while today it is 20.3 M, of which 2.7 M are immigrants. The K/Y ratio has not changed much since then (recent data show it has slightly increased). Hence, with respect to the original work force the immigrants added almost 21%, while they are about 13% of the current work force. Take a number in between (i.e. 17%) to account for the fact that this took place over, roughly, a decade (in fact less). Reader, please be careful and use intuition: in the model a period is 25-30 years, we have experienced at best 12, so you have to take "flow averages" to compute what a possible mean over a 25 years period would be.

This implies that, if (i) the immigrants came without any K; (ii) the saving rate of the natives remained constant (it roughly did) meaning that, if Spain was in steady state before 1996, national saving just supplied the K needed by the natives; (iii) the final K/H ratio for immigrants is similar to those for natives; then Spain would have had to borrow from abroad the resources needed to increase the original stock of K of about 17%. In fact, the number is larger, because we have, on top of the 2.7 M immigrant workers, about other 4.0 M native workers that become employed and were not such, at least officially, before. The analytical problem with these native workers is more complicated as part of them were probably underground (hence, their K already existed), part had accumulated savings they invested in their own K, and so on and so forth. In any case, because the employment growth due to natives only adds to our estimates, the latter will be a very reasonable lower bound of the amout of capital that Spain needed to import during the last twelve years or so.

Bottom line, at least about 17% of the new stock of capital had to be imported. Notice that "imported" here means "net import", as there is no export in our model: import in the model is equal to the trade deficit in the national income accounts. Now, keep in mind what this means: over a period of about 12 years Spain had to import a little more than 1/6 of its current stock of capital. Another way of saying it, is that it imported about 1.4% of its capital stock per year, during each one of the 12 years that go from 1996 to 2007. Given the current K/Y ratios (use 2.8 and 4.0 as the upper and lower boundary), this implies that each year something in the neighborhood of 4.0-5.7% of GNP had to be imported. That is a cumulated total import of between 48% and 68% of current Spanish GNP, everything else equal. Between 1995 and 2007, the actual trade deficit, in percentage of GNP, adds up to 50.7% with an annual average of 4.2%. Hence, our back of the envelope calculation is pretty much nailing it. Chance? Maybe, who knows!

6. Conclusions

Moral number 1: the trade deficit and borrowing from abroad were a substitute for the missing internal insurance markets. Spain received a very large numbe of immigrants and this would have had a dramatic impact on productivity and income distribution if the trade deficit had not allowed the country to accumulate capital stock much faster than the national saving rate allows.

Moral number 2: immigration shocks have large impacts not only on aggregate output but also on its composition and on income distribution. Absent complete financial markets, such impacts should be properly managed by well planned government policies, of the form we have described. It is at least dubious that such policies were implemented in Spain during the last fourteen years or so. The absence of such policies has clear detrimental effects not only on welfare but also on human capital accumulation and overall economic growth.

Moral number 3: the current debate on the impact of immigration on Spanish society and economy seems to be missing some key aspects. In the model presented here we have outlined some of them, with particular focus on education and pensions. In particular, we have shown that an optimal policy response to a large immigration flow requires a reduction of pension payments and an increase of the investment in education. As far as we can tell, neither of these two policies have been implemented in Spain.

Moral number 4: even simple stylized models can help thinking about difficult issues in economic policy and are capable of shed new light on issues that are often forgotten or considered too complicated to be addressed formally.

Moral number 5: more and better work needs to be done, more detailed and disaggregated models need to be built and better data should be collected.

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