Political Campaign and Preferences over Political Issues*

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Abstract

We explore how the political campaign affects voters' preferences and has impact on electoral outcomes. We introduce a spatial model of political competition with two office-seeking political parties and a continuum of voters with well-defined preferences on policies. The parties' campaign strategies consist on emphasizing some of the political issues over others. In doing so, the political parties not only influence voters' preferences but also their final vote-share. We analyze the political consequences of having an advantage on some of the political issues and we compare the electoral results with and without a political campaign. The main predictions of the model are consistent with the empirical evidence.

Key-words: Election campaign, political issues, preferences manipulation, positional voting.
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1 Introduction

There are several theories that aim at explaining how political campaign expenditure persuades voters:
(1) Campaign expenditure influences a fixed fraction of voters who are uninformed about the parties' political positions (see, e.g., Baron, 1994, and Grossman and Helpman, 1996).
(2) Campaign expenditure clarifies the political positions of the candidates and alleviates risk averse voters' uncertainty (see, e.g., Austen-Smith, 1987).
(3) Campaign expenditure is a signal of the high valence of an incumbent candidate (see, e.g., Prat, 2002).
(4) Campaign expenditure affects an "electoral production function" and increases the probability of winning the elections (see, e.g., Friedman, 1958, Brams and Davis, 1973, Snyder, 1989).[1]

Explanations (1) and (2) relate campaign activities to information acquisition, Explanation (3) interprets campaign expenditure as a signal, and Explanation (4) provides a more general setting where campaign activities work as an input to produce votes. None of these explanations, however, describes how political campaigns may affect or distort voters' preferences. Thus, in (1) and (3) voters' preferences are not described, and in (2) and (4) campaign expenditure reduces voters' uncertainty.[2]

In this paper we aim at modelizing an important aspect which has not been considered so far: how political campaigns may affect voters' preferences. Our proposal does not go through those campaign activities that aim at providing information or that aim at providing a signal. It however focuses on campaign activities that aim at persuading voters by means of distorting voters' taste on political issues. Thus, our proposal resembles, to a certain extent, some of the effects that the advertising activities have on market economies, where advertising is assumed to change the taste of the consumers (see, for instance, Dixit and Norman, 1978).

From the empirical evidence (see Laver and Hunt, 1992; Budge, 1993; Riker, 1993; Petrocik, 1996) and from recent political campaigns in modern democracies, we deduce that competition of political parties is increasingly based on political issues. Indeed, we often observe that the political parties

[1]As pointed out by Snyder (1989), a particular instance of an "electoral production function" is provided by the rent-seeking literature. See, for instance, Tullock (1981).

[2]In Explanation (1), the informed voters have well-defined preferences on the political space while the non-informed voters cannot be described by an ideal policy.

use political campaigns to promote those issues by which they can capture a greater amount of votes.[3] This is the case, for instance, of some recent electoral campaigns in countries such as the United States, the United Kingdom, and Spain, where the economic issue as opposed to the position in Iraq's war (or the fight against terrorism) have become the main two issues in the electoral race. We therefore hypothesize that campaign expenditure may affect the relative intensity that voters assign to one issue over another.

Empirical Evidence

Based on the empirical evidence, Riker (1993) and Petrocik (1996) provide some ideas on how political parties compete in political issues. From the analysis of the national campaign of the U.S. for the ratification of the Constitution, Riker (1993) argues that, (i) when one party has a clear-cut advantage on an issue, it regularly emphasizes that issue while the other party abandons it (dominance principle), and (ii) when neither side has a clear advantage on an issue, both abandon it (dispersion principle). In a similar vein, from the analysis of the U.S. presidential elections between 1960 and 1992, Petrocik (1996) provides the idea of "issue ownership", which is based on the perception that voters have as to how a party handles certain political issues (or political problems). A party that is viewed as better qualified to handle an issue is said to have ownership of that issue. Thus, it is expected that candidates emphasize issues in which they are advantaged.

None of the above mentioned authors, however, develops a formal theoretical model explaining the observed evidence. Thus, with this paper we complement their analysis by means of providing a theoretical model which tries to capture all the basic features of parties' competition in political issues.

An outline of the model

Our model is based on the one proposed by Riker and Ordeshook (1973). We extend this model to allow for the effect of campaign expenditure affecting voters' preferences.

We consider a two-dimensional spatial model of political competition between two parties. The parties' political positions are common knowledge. Parties aim at maximizing votes. Voters are represented by their own ideal policies, and they vote sincerely for the party that matches their own ideal

[3] In the same vein, Roemer (1998) argues that political parties try to increase the salience of some issues as a mean of pulling voters away from the other competing party.

policy more accurately.[4] The strategies of the parties consist of allocating campaign funds between the political issues. In this way, the parties affect the relative importance that voters assign to one of the political issues over the other. We study the Nash equilibria of this allocation of campaign funds game.

Main results

We show that the electorate can be partitioned into two groups: partisan voters (those who will vote for one of the parties irrespectively of the parties' campaign strategies), and issue voters (those whose vote is influenced by the campaign strategies, and whom the parties are aiming to influence via campaign expenditures).

Firstly we show equilibrium existence of the proposed campaign game. We then study the votes obtained by each party as a function of the campaign strategies. We analyze the shape of these functions and we characterize the equilibrium strategies.

We find that when a party has an advantage in both political issues it wins the elections for sure. Even though, its rival spend all its campaign resources on one of the political issues. When each party has an advantage in a different political issue, we find that they both spend all their campaign resources on their advantageous issue. In this case, the electoral victory can be achieved by either of the political parties.

We compare the electoral results with and without the political campaign. When there is no political campaign, the party whose policy position is closer to the median voter always obtains a majority of votes. However, even when this party has more campaign funds than its opponent, it may not win the elections when there is a political campaign.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 provides an equilibrium analysis. Section 4 compares the electoral results with and without political campaign. Section 5 provides the conclusions.

[4]We follow the proximity model where preferences on political parties follow directly from "closeness". An alternative is the directional model proposed by Rabinowitz and Macdonald (1989) where preferences are defined by one direction on the policy space.

2 The model

There is a society with a continuum of voters which shall select a representative to serve in the legislature by popular election. There are two political parties, A and B, with fixed political positions on a two-dimensional policy space, that aim at maximizing votes by spending campaign resources. There are two political issues, 1 and 2 that describe two salient problems of the society.[5]

Political parties

Each party $j \in \{A, B\}$ has a fixed and known **political position** $x_j = (x_{j1}, x_{j2}) \in [0,1]^2$, where $x_{jr} \in [0,1]$ is the political position of party j on issue $r \in \{1,2\}$. We assume, without loss of generality, that $x_{A1} < x_{B1}$ and $x_{A2} < x_{B2}$.

Each party j is endowed with some fixed **campaign funds** $\bar{c}_j > 0$.[6] Campaign funds are devoted to the advertising campaign and each party emphasizes those issues which can persuade a greater amount of voters. We define a **campaign strategy** of party j as a vector $c_j \in C_j = \{(c_{j1}, c_{j2}) \in \mathbb{R}^2_+ : c_{j1} + c_{j2} \leqslant \bar{c}_j\}$, which indicates how the party allocates its funds between the two different issues.[7] Let $c = (c_A, c_B) \in C_A \times C_B = C$ denote a profile of campaign strategies. For each $c \in C$ and each $r \in \{1,2\}$, let $c_r = c_{Ar} + c_{Br}$ be the total funds spent on issue r.

Voters

Each voter i has a fixed and known **ideal political position** $\pi_i = (\pi_{i1}, \pi_{i2}) \in [0,1]^2$ where $\pi_{ir} \in [0,1]$ is the ideal political position of voter i on issue r. Voters' ideal political positions are uniformly distributed on $[0,1]^2$. The median voter is denoted by m, and his ideal policy is given by $\pi_m = (\frac{1}{2}, \frac{1}{2})$.

Each voter prefers the party that matches his own ideal policy more accurately. Besides that, campaign strategies also have an influence on voters' preferences. Thus, one of the crucial assumptions of this model is that the intensity of voters' preferences over each issue r depend on the campaign

[5] As pointed out by authors as Poole and Rosenthal (1991), adding a third dimension may explain little more

[6] We can assume that each party is aligned with a set of interest groups which provide campaign funds.

[7] The campaign strategy c_j can be also interpreted as the time that party j devotes to advertising each political issue.

expenditure on that issue, c_r. In particular, the preferences of each voter i over the political parties are represented by the following utility function:

$$u_i(j,c) = -\alpha_1(c_1)[x_{j1} - \pi_{i1}]^2 - \alpha_2(c_2)[x_{j2} - \pi_{i2}]^2 \qquad (1)$$

where, for each issue r, $\alpha_r(.)$ is a continuously differentiable function of the campaign expenditure on issue r that indicates the weight that voters assign to that issue. We will refer to $\alpha_r(.)$ as the **influence function** on issue r, where $\alpha_r(0) > 0$ and $\frac{\partial \alpha_r(c_r)}{\partial c_r} > 0$. When there is no political campaign, we assume that both political issues are equally weighted, i.e., $\alpha_1(0) = \alpha_2(0)$.

Note that we have made the simplifying assumption that all voters are equally influenced by the campaign expenditure, i.e., for any issue r, the influence function $\alpha_r(.)$ does not vary among voters. This assumption can be justified on the basis that all voters have equal access to advertising activities. Riker and Ordeshook (1973), pointed out that there is one relaxation that is generally permitted which consists of considering that there exists some average level of concern for each of the political issues.[8]

Figure 1 illustrates an example of voters' indifference curves. The solid curves represent the indifference curves when there is no campaign expenditure. Expending campaign funds can vary the relative importance that voters assign to each issue. Thus, the narrow doted curves represent the indifference curves when campaign expenditure makes issue 1 more relevant, while the wide doted curves represent the indifference curves when campaign expenditure makes issue 2 more relevant.

Given any profile of campaign strategies $c \in C$, voter i casts his ballot for party j when $u_i(j,c) > u_i(k,c)$ (where $k \neq j$).

The utility function of each voter i can be rewritten as:

$$u_i(j,c) = -T(c)\left[x_{j1} - \pi_{i1}\right]^2 - \left[x_{j2} - \pi_{i2}\right]^2 \qquad (2)$$

where $T(c) = \frac{\alpha_1(c_1)}{\alpha_2(c_2)}$ can be interpreted as the relative intensity of voters' preferences over issue 1 when the profile of campaign strategies is c. Thus, the greater $T(c)$ is, the more relevant issue 1 is compared to issue 2 in voters' preferences.

[8]Note also that, while the campaign expenditure determines the intensity of voters' preferences over issues, it has no influence on their ideal political positions. This restriction is probably the smallest step one could take to analyze the effect of campaign expenditure when there are several issues (and it is still reasonable in many settings).

[FIGURE 1]

From (2), voter i is indifferent between the two parties when his ideal political position satisfies the following condition:

$$\pi_{i2} = \frac{T(c)\left[x_{A1}^2 - x_{B1}^2\right] + \left[x_{A2}^2 - x_{B2}^2\right]}{2\left[x_{A2} - x_{B2}\right]} - \frac{T(c)\left[x_{A1} - x_{B1}\right]}{\left[x_{A2} - x_{B2}\right]}\pi_{i1}. \tag{3}$$

Equation (3) allows us to distinguish between those voters that vote for party A and those voters that vote for party B. Figure 2 shows an example of that. In a slight abuse of the notation, we use $T(c)$ to denote the line defined by Equation (3). Any voter whose ideal political position is located on this line is indifferent between the two parties. If the ideal political position of a voter is located to the left of that line, he votes for party A. Similarly, if the ideal political position of a voter is located to the right of that line, he votes for party B.

[FIGURE 2]

Note that the available campaign funds define some minimum and maximum values for $T(c)$: since each party j can expend at most $\bar{c}_j$, we have $\frac{\alpha_1(0)}{\alpha_2(\bar{c}_A+\bar{c}_B)} \leqslant T(c) \leqslant \frac{\alpha_1(\bar{c}_A+\bar{c}_B)}{\alpha_2(0)}$ for all $c \in C$. We denote $T_{\min} = \frac{\alpha_1(0)}{\alpha_2(\bar{c}_A+\bar{c}_B)}$ and $T_{\max} = \frac{\alpha_1(\bar{c}_A+\bar{c}_B)}{\alpha_2(0)}$ the minimum and maximum values of $T(c)$. These values are the key to knowing the subgroup of voters that may change their vote according to the specific profile of campaign strategies. A voter located in the midpoint of the distance between the political position of party A and party B, i.e., $\pi_i = \left(\frac{x_{A1}+x_{B1}}{2}, \frac{x_{A2}+x_{B2}}{2}\right)$, is always indifferent between both parties, whatever the campaign strategies are.

Consider the example depicted in Figure 3. Again, we abuse of notation and use $T_{\min}$ (respectively $T_{\max}$) to denote the line defined by Expression (3) when $T(c) = T_{\min}$ (respectively $T(c) = T_{\max}$). In this example, any voter i whose ideal political position is located below lines $T_{\min}$ and $T_{\max}$ is such that $u_i(A,c) > u_i(B,c)$ for all $c \in C$, and then he always votes for party A, no matter what the profile of campaign strategies is. Similarly, any voter whose ideal political position is located above lines $T_{\min}$ and $T_{\max}$ is such that $u_i(B,c) > u_i(A,c)$ for all $c \in C$, and then he always votes for party B. We call these voters **partisan voters**.

Any voter located between lines $T_{\min}$ and $T_{\max}$ is such that $u_i(A, c) > u_i(B, c)$ for some $c \in C$ and $u_i(B, c') > u_i(A, c')$ for some $c' \in C$, and then his vote will depend on the particular profile of campaign strategies. We call these voters **issue voters**. The campaign expenditure on a particular issue can move the vote of an issue voter towards the party that best fits his preferences on that issue. Note that $\frac{\partial T_{\min}}{\partial(\bar{c}_A+\bar{c}_B)} < 0$ and $\frac{\partial T_{\max}}{\partial(\bar{c}_A+\bar{c}_B)} > 0$, and then, the greater the campaign funds are, the greater the set of issue voters is.

[FIGURE 3]

Campaign game

Each party objective is maximizing votes. Given any profile of campaign strategies $c \in C$, let $V_j(c)$ be the percentage of votes that party j obtains in the elections. Since voters are uniformly distributed on $[0,1]^2$, $V_A(c)$ can be measured by the area located below the line defined by $T(c)$, and $V_B(c)$ by the area located above this line. Since every line that goes through $\left(\frac{1}{2}, \frac{1}{2}\right)$ divides the policy space into two equal areas, the party that obtains the vote of the median voter, $\pi_m = \left(\frac{1}{2}, \frac{1}{2}\right)$, always achieves a strict majority of votes.

Our equilibrium concept in this paper is Nash equilibrium. A profile of campaign strategies $c^* \in C$ is a (Nash) **equilibrium** if, for all party j and all $c'_j \in C_j$, $V_j(c^*_j, c^*_k) \geqslant V_j(c'_j, c^*_k)$ (where $k \neq j$).

3 Equilibrium Analysis

We firstly show equilibrium existence of the proposed campaign game. For that, we need to study the votes that each party obtains as a function of their campaign strategies. To simplify our notation let $x_1 = x_{A1} - x_{B1}$, $x_2 = x_{A2} - x_{B2}$, $z_1 = x_{A1}^2 - x_{B1}^2$, and $z_2 = x_{A2}^2 - x_{B2}^2$.

Lemma 1 *The percentage of votes for party A as a function of the campaign strategies, $V_A(c)$, is given by the following expressions:*

(a) If $x_{A1} + x_{B1} + x_{A2} + x_{B2} \leq 2$, then:

$$V_A(c) = \begin{cases} \frac{z_2}{2x_2} + \frac{T(c)[z_1 - x_1]}{2x_2} & ;\ \textit{if}\ T(c) \leq \frac{z_2}{2x_1 - z_1} \\ \frac{z_1 z_2}{4x_1 x_2} + \frac{T(c) z_1^2}{8x_1 x_2} + \frac{z_2^2}{T(c) 8x_1 x_2} & ;\ \textit{if}\ \frac{z_2}{2x_1 - z_1} \leq T(c) \leq \frac{2x_2 - z_2}{z_1} \\ \frac{z_1}{2x_1} + \frac{z_2 - x_2}{T(c) 2x_1} & ;\ \textit{if}\ T(c) \geq \frac{2x_2 - z_2}{z_1} \end{cases}$$

(b) If $x_{A1}+x_{B1}+x_{A2}+x_{B2}\geq 2$, then:

$$V_A(c)=\begin{cases}\frac{z_2}{2x_2}+\frac{T(c)[z_1-x_1]}{2x_2} & ;\ if\ T(c)\leq\frac{2x_2-z_2}{z_1}\\ \frac{4x_1x_2-[2x_1-z_1][2x_2-z_2]}{4x_1x_2}-\\ -\frac{T(c)[2x_1-z_1]^2}{8x_1x_2}-\frac{[2x_2-z_2]^2}{T(c)8x_1x_2} & ;\ if\ \frac{2x_2-z_2}{z_1}\leq T(c)\leq\frac{z_2}{2x_1-z_1}\\ \frac{z_1}{2x_1}+\frac{z_2-x_2}{T(c)2x_1} & ;\ if\ T(c)\geq\frac{z_2}{2x_1-z_1}\end{cases}$$

The proof of this result appears in the Appendix. As it follows from Figure 2, the function described in Lemma 1 measures the area below the line of the indifferent voters.

The vote functions $V_A(.)$ and $V_B(.)=1-V_A(.)$ are continuous and, since the space of strategies C_A and C_B are compact, we can affirm that the campaign game always possesses a Nash equilibrium (see, for example, Glicksberg, 1952).

Proposition 1 *The campaign game always has an equilibrium.*

As it follows from the functions $V_A(.)$ and $V_B(.)=1-V_A(.)$ characterized in Lemma 1, the percentage of votes obtained by each party not only depends on the campaign strategies, but also on the political position of the parties. We say that party has an advantage on an issue when its political position on that issue is closer to the median voter on that issue (and therefore it would obtain a simple majority on the hypothetical one-issue election).

If the midpoint of the parties' political positions on issue r, $\frac{x_{Ar}+x_{Br}}{2}$, is greater (respectively smaller) than the ideal political position of the median voter on that issue, $\frac{1}{2}$, then party A (respectively party B) has an advantage on that issue.[9] We distinguish four different cases depending on the location of the midpoint $(\frac{x_{A1}+x_{B1}}{2},\frac{x_{A2}+x_{B2}}{2})$ (see Figure 4).

[FIGURE 4]

[9] If $\frac{x_{Ar}+x_{Br}}{2}>\frac{1}{2}$ then, since $x_{Ar}<x_{Br}$, $\left|x_{Ar}-\frac{1}{2}\right|<\left|x_{Br}-\frac{1}{2}\right|$. Similarly, if $\frac{x_{Ar}+x_{Br}}{2}<\frac{1}{2}$ then $\left|x_{Ar}-\frac{1}{2}\right|>\left|x_{Br}-\frac{1}{2}\right|$.

If the midpoint of the parties' political positions is in Area 1 of Figure 4 then party A has an advantage on both political issues. If the midpoint is in Area 2 then party A has an advantage on issue 1 while party B has an advantage on issue 2. If the midpoint is in Area 3 then party B has an advantage on both issues. Finally, if the midpoint is in Area 4 then party A has an advantage on issue 2 while party B has an advantage on issue 1.

In order to characterize the equilibrium strategies, we start analyzing the shape of function $V_A(.)$ when the parties' political positions are described by each of the proposed cases.

Lemma 2 *If a party has an advantage on both political issues, then the percentage of voters that cast their ballots for that party is a single-peaked function of the relative intensity of voters' preferences over issue* 1 *(i.e., the percentage of votes for that party is a strictly increasing function of* $T(c)$ *up to some point and strictly decreasing beyond that point).*

Lemma 3 *If each party has an advantage on a different issue, then the percentage of voters that cast their ballots for the party that has an advantage on issue* 1 *is a strictly increasing function of the relative intensity of voters' preferences over issue* 1*,* $T(c)$*.*

The Appendix contains the proofs of these lemmas. By Lemma 2, when a party has an advantage on both political issues, the greatest vote-share that such party can achieve is obtained by means of emphasizing not only one but both political issues. By Lemma 3, however, when each party has an advantage on a different political issue, the greatest vote-share for each party is achieved when its campaign resources are entirely devoted to the political issue where it has an advantage.

The following propositions describe some of the parties' equilibrium strategies and electoral results. The proofs are in the Appendix.

Proposition 2 *If a party has an advantage on both political issues, then it obtains a strict majority of votes in every equilibrium of the campaign game. Furthermore, the equilibrium strategy of the party that loses the elections is such that it spends all its campaign funds on only one of the political issues.*

Proposition 3 *If each party has an advantage on a different issue, then the unique equilibrium of the campaign game is such that each party spends all* *its funds on the issue in which it has an advantage.*[10]

A party that has an advantage on both political issues always achieves a majority of the votes. However, when each party has an advantage on a different political issue, the electoral results cannot be deduced since in equilibrium either party can obtain a majority of the votes. In this last case, the electoral victory will depend not only on each party amount of campaign resources but also on the influence functions.

Regarding equilibrium strategies, when a party has an advantage on either one or none of the political issues, it has unique equilibrium strategy. When a party has an advantage on both political issues, however, the equilibrium may not be unique. In this case, its equilibrium strategy aims at achieving the peak of its vote-share function (i.e., $V_j(.)$) and it can consist of investing some or all of its campaign resources on one or both political issues.[11]

4 The Electoral Results

The Electoral Results without a Political Campaign

When there is no political campaign, we have assumed that all the voters assign equal weight to both political issues, i.e., $\alpha_1(0) = \alpha_2(0)$, and thus, $T(0) = 1$. Our assumption implies that without a political campaign, each voter casts his ballot for the party that is closer to his ideal political position.

Note that, when there is no political campaign, the indifferent voters are those whose ideal policy is equidistant to the parties' political positions. Let T_0 be the line containing the indifferent voters when there is no political campaign. As we show in Figure 5, T_0 is perpendicular to the line that joins the parties' political positions. When T_0 is above (respectively below) the ideal political position of the median voter, $\pi_m = \left(\frac{1}{2}, \frac{1}{2}\right)$, party A (respectively party B) obtains a strict majority of votes. Therefore, the party with

[10] When either $\frac{x_{A1}+x_{B1}}{2} = \frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2} < \frac{1}{2}$ or $\frac{x_{A1}+x_{B1}}{2} > \frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2} = \frac{1}{2}$, the function $V_A(.)$ is weakly increasing and $(c^*_{A1}, c^*_{A2}) = (\bar{c}_A, 0)$ $(c^*_{B1}, c^*_{B2}) = (0, \bar{c}_B)$ is an equilibrium but it may not be the unique equilibrium. In the same way, when either $\frac{x_{A1}+x_{B1}}{2} = \frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2} > \frac{1}{2}$ or $\frac{x_{A1}+x_{B1}}{2} < \frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2} = \frac{1}{2}$, the function $V_A(.)$ is weakly decreasing and $(c^*_{A1}, c^*_{A2}) = (0, \bar{c}_A)$ $(c^*_{B1}, c^*_{B2}) = (\bar{c}_B, 0)$ is an equilibrium but it may not be unique.

[11] If we assume that parties derive desutility from spending campaign resources, we can recover, in this case, equilibrium uniqueness.

a political position closer to the median voter obtains a strict majority of votes.

Proposition 4 *When there is no political campaign, the party with a political position closer to the median voter always obtains a majority of the votes.*

[FIGURE 5]

In Figure 5, party A is located closer to the median voter than is party B. In this example, party A obtains a majority of the votes when there is no political campaign

The Electoral Results with a Political Campaign

When there is a political campaign, the political issues may be weighted differently according to voters' preferences. In this case, the party located closer to the median voter may not obtain a majority of the votes. Note that when a party has an advantage on both political issues, this party is always closer to the median voter than its opponent. However, the party that is closer to the median voter can have a political position on one of the political issues that is more distant from the median voter than that of its opponent (for instance, in Figure 5, party A is closer to the median voter on issue 1 whereas party B is closer to the median voter on issue 2). In this case each party has an advantage on a different political issue, and the political campaign can provide a majority of the votes to the party that would lose the elections without political campaign (for instance, in the example depicted in Figure 5, the equilibrium campaign strategies could induce line T_1 and, in this case, it would be Party B which obtains a strict majority of the votes). As a matter of fact, the party located closer to the median voter can end up losing the elections even when it has more campaign funds than its opponent.

Proposition 5 *When there is a political campaign, the party with a political position closer to the median voter can lose the elections even when it has more campaign funds than its opponent.*

To prove Proposition 5, consider the example illustrated in Figure 5 and suppose that the parties have the following political positions and campaign funds:

Party A		Party B	
$x_{A1} = \frac{1}{2}$	$x_{A2} = \frac{1}{3}$	$x_{B1} = \frac{5}{6}$	$x_{B2} = \frac{1}{2}$
$\bar{c}_A = 100$		$\bar{c}_B = 95$	

Suppose also that the influence functions are the following:

Issue 1	Issue 2
$\alpha_1(c_1) = 1 + \sqrt{c_1}$	$\alpha_2(c_2) = 1 + 5\sqrt{c_2}$

Note that party A is closer to the median voter than party B, and that the midpoint of the parties' political positions, $\left(\frac{x_{A1}+x_{B1}}{2}, \frac{x_{A2}+x_{B2}}{2}\right) = \left(\frac{2}{3}, \frac{5}{12}\right)$, is located in Area 2 of Figure 4, and so, party A has an advantage on issue 1 and party B has an advantage on issue 2. Then, by Proposition 3, the unique equilibrium of the campaign game is given by $(c^*_{A1}, c^*_{A2}) = (100, 0)$ and $(c^*_{B1}, c^*_{B2}) = (0, 95)$. We obtain that $T(c^*) = \frac{\alpha_1(100)}{\alpha_2(95)} = 0.221$, and since $\frac{2x_2 - z_2}{z_1} = 0.4375$, by Lemma 1 the percentage of votes obtained by party A is given by $V_A(c^*) = \frac{z_2}{2x_2} + \frac{T(c^*)[z_1 - x_1]}{2x_2} \simeq 0.49 < V_B(c^*)$. Therefore, party A loses the elections.

In this example the influence functions are such that issue 2 is more sensitive than issue 1 to campaign expenditure. Thus, although party A has more campaign resources than party B, its advantage on issue 1 is not enough to obtain a majority of votes. The campaign expenditure on issue 2 is more efficient and so, party B can win the elections even when it has less campaign funds.

5 Conclusion

This paper proposes a model through which political campaign expenditure affects the electoral results. In doing so, we have focused on the role that the advertising campaign plays on the weight that voters assign to each of the political issues.

Voters who lack partisan identification are called issue voters. In contrast to other works such as Baron (1994) or Grossman and Helpman (1996), in our model the fraction of issue voters depends on the amount of campaign resources. The weight that the issue voters assign to each political issue is crucial to determine their vote.

A party has an advantage on an issue when such party would obtain a simple majority on the hypothetical one-issue election. Our results concerning the equilibrium analysis can be summarized in the following table.

	Electoral outcome	**Equilibrium strategies**
A party has an advantage on both issues	This party wins the elections	Its rival spends all its funds on one issue
Each party has an advantage on a different issue	Either of the parties can win the elections	Each party spends all its funds on the issue on which it has an advantage

Table I. Equilibrium analysis.

The equilibrium obtained when each party has an advantage on a different political issue is coherent with the empirical evidence. On the one hand, it satisfies the 'dominance principle' suggested by Ricker (1993), where a party emphasizes the issue where it has an advantage while the other party abandons it. On the other hand, if the advantage on a political issue is interpreted as the ability of a party to handle a political issue, the obtained equilibrium is also coherent with the issue-ownerhip theory proposed by Petrocik (1996).

Our model suggests that political issues where both parties share an equal position become irrelevant from the point of view of designing the advertising campaign. This observation is also coherent with the empirical evidence since by Ricker's 'dispersion principle', both parties abandon a political issue when they do not have a clear advantage.

When a party has an advantage on both political issues it always obtains a strict majority of the votes. In this case the electoral campaign can just modify the parties' vote-share. When each party has an advantage on a different political issue, the political campaign not only can modify the parties vote-share, but it can also give the electoral victory to either of the political parties. In particular, a closer position to the median voter does not guarantee a strict majority of the votes.

In many modern democracies, opinion polls contain information about parties' support on every single issue. Our model suggests that such information together with a measure of the voters' influence functions are the relevant tools to properly design the parties' campaign strategy.

APPENDIX

PROOF OF LEMMA 1:

From Expression (3), a voter i will vote for party A if and only if his ideal political position satisfies the following condition:

$$\frac{T(c)[x_{A1}-x_{B1}]}{[x_{A2}-x_{B2}]}\pi_{i1}+\pi_{i2}-\frac{T(c)\left[x_{A1}^2-x_{B1}^2\right]+\left[x_{A2}^2-x_{B2}^2\right]}{2[x_{A2}-x_{B2}]}<0 \tag{4}$$

Let $Y_i=\alpha(c)\pi_{i1}+\pi_{i2}-\beta(c)$, where $\alpha(c)=\frac{T(c)[x_{A1}-x_{B1}]}{[x_{A2}-x_{B2}]}$ and $\beta(c)=\frac{T(c)\left[x_{A1}^2-x_{B1}^2\right]+\left[x_{A2}^2-x_{B2}^2\right]}{2[x_{A2}-x_{B2}]}$. It can be shown that the distribution function of Y_i is given by the following expression:

$$F(y)=\begin{cases} 0 & ;\text{ if } y\leq-\beta(c) \\ \frac{[y+\beta(c)]^2}{2\alpha(c)} & ;\text{ if } -\beta(c)\leq y\leq\min\{1,\alpha(c)\}-\beta(c) \\ \frac{2y+2\beta(c)-\min\{1,\alpha(c)\}}{2\max\{1,\alpha(c)\}} & ;\text{ if } \min\{1,\alpha(c)\}-\beta(c)\leq y\leq \max\{1,\alpha(c)\}-\beta(c) \\ 1-\frac{[\alpha(c)-\beta(c)-y+1]^2}{2\alpha(c)} & ;\text{ if } \max\{1,\alpha(c)\}-\beta(c)\leq y\leq 1+\alpha(c)-\beta(c) \\ 1 & ;\text{ if } y\geq 1+\alpha(c)-\beta(c) \end{cases} \tag{5}$$

Evaluating the previous distribution function in zero, we obtain the percentage of voters that cast their ballots for party A as a function of the campaign strategies.

$$V_A(c)=\begin{cases} 0 & ;\text{ if } \beta(c)\leq 0 \\ \frac{\beta(c)^2}{2\alpha(c)} & ;\text{ if } 0\leq\beta(c)\leq\min\{1,\alpha(c)\} \\ \frac{2\beta(c)-\min\{1,\alpha(c)\}}{2\max\{1,\alpha(c)\}} & ;\text{ if } \min\{1,\alpha(c)\}\leq\beta(c)\leq\max\{1,\alpha(c)\} \\ 1-\frac{[\alpha(c)-\beta(c)+1]^2}{2\alpha(c)} & ;\text{ if } \max\{1,\alpha(c)\}\leq\beta(c)\leq 1+\alpha(c) \\ 1 & ;\text{ if } \beta(c)\geq 1+\alpha(c) \end{cases} \tag{6}$$

Let $x_1=x_{A1}-x_{B1}$, $x_2=x_{A2}-x_{B2}$, $z_1=x_{A1}^2-x_{B1}^2$, and $z_2=x_{A2}^2-x_{B2}^2$. Note that the following relations are satisfied:

(i) $0\leq\beta(c)\leq 1+\alpha(c)$ for all c,[12]

[12] Note that $\beta(c)<0$ if and only if $T(c)<-\frac{z_2}{z_1}\leq 0$, which never occurs, since $T(c)>0$ for all c. Similarly, $\beta(c)>1+\alpha(c)$ if and only if $T(c)<\frac{z_2-2x_2}{2x_1-z_1}\leq 0$.

(ii) $\alpha(c) \geq 1$ if and only if $T(c) \geq \frac{x_2}{x_1}$,
(iii) $0 \leq \beta(c) \leq 1$ if and only if $T(c) \leq \frac{2x_2-z_2}{z_1}$,
(iv) $1 \leq \beta(c) \leq \alpha(c)$ if and only if $T(c) \geq \max\left\{\frac{2x_2-z_2}{z_1}, \frac{z_2}{2x_1-z_1}\right\}$,
(v) $\alpha(c) \leq \beta(c) \leq 1+\alpha(c)$ if and only if $T(c) \leq \frac{z_2}{2x_1-z_1}$,
(vi) $0 \leq \beta(c) \leq \alpha(c)$ if and only if $T(c) \geq \frac{z_2}{2x_1-z_1}$,
(vii) $\alpha(c) \leq \beta(c) \leq 1$ if and only if $T(c) \leq \min\left\{\frac{2x_2-z_2}{z_1}, \frac{z_2}{2x_1-z_1}\right\}$, and
(viii) $1 \leq \beta(c) \leq 1+\alpha(c)$ if and only if $T(c) \geq \frac{2x_2-z_2}{z_1}$.

Then, Expression (6) can be rewritten as:

$$V_A(c) = \begin{cases} \frac{z_1 z_2}{4x_1x_2} + \frac{T(c)z_1^2}{8x_1x_2} + \frac{z_2^2}{T(c)8x_1x_2} & ;\text{if } \frac{z_2}{2x_1-z_1} \leq T(c) \leq \frac{x_2}{x_1} \\ & \text{or } \frac{x_2}{x_1} \leq T(c) \leq \frac{2x_2-z_2}{z_1} \\ \frac{z_1}{2x_1} + \frac{z_2-x_2}{T(c)2x_1} & ;\text{if } T(c) \geq \max\left\{\frac{z_2}{2x_1-z_1}, \frac{2x_2-z_2}{z_1}, \frac{x_2}{x_1}\right\} \\ \frac{z_2}{2x_2} + \frac{T(c)[z_1-x_1]}{2x_2} & ;\text{if } T(c) \leq \min\left\{\frac{z_2}{2x_1-z_1}, \frac{2x_2-z_2}{z_1}, \frac{x_2}{x_1}\right\} \\ \frac{4x_1x_2-[2x_1-z_1][2x_2-z_2]}{4x_1x_2} - & ;\text{if } \frac{x_2}{x_1} \leq T(c) \leq \frac{z_2}{2x_1-z_1} \\ \frac{T(c)[2x_1-z_1]^2}{8x_1x_2} - \frac{[2x_2-z_2]^2}{T(c)8x_1x_2} & \text{or } \frac{2x_2-z_2}{z_1} \leq T(c) \leq \frac{x_2}{x_1} \end{cases} \tag{7}$$

Note that either $\frac{z_2}{2x_1-z_1} \leq \frac{x_2}{x_1} \leq \frac{2x_2-z_2}{z_1}$ or $\frac{2x_2-z_2}{z_1} \leq \frac{x_2}{x_1} \leq \frac{z_2}{2x_1-z_1}$. Moreover, $\frac{z_2}{2x_1-z_1} \leq \frac{2x_2-z_2}{z_1}$ if and only if $x_{A1}+x_{B1}+x_{A2}+x_{B2} \leq 2$. Therefore $V_A(c)$ is given by the expressions in the statement of this lemma.

PROOF OF LEMMA 2:

Suppose, without loss of generality, that party A has an advantage on both political issues. We will show that V_A is a strictly increasing function of $T(c)$ for all $T(c) < \frac{2x_2-z_2}{2x_1-z_1}$, and strictly decreasing in $T(c)$ for all $T(c) > \frac{2x_2-z_2}{2x_1-z_1}$.

If party A has advantage on both political issues then $x_{A1}+x_{B1}+x_{A2}+x_{B2} > 2$ (since $\frac{x_{A1}+x_{B1}}{2} > \frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2} > \frac{1}{2}$). Note that $\frac{x_{A1}+x_{B1}}{2} > \frac{1}{2}$ implies $\frac{2x_2-z_2}{z_1} \leq \frac{2x_2-z_2}{2x_1-z_1}$, and $\frac{x_{A2}+x_{B2}}{2} > \frac{1}{2}$ implies $\frac{2x_2-z_2}{2x_1-z_1} < \frac{z_2}{2x_1-z_1}$.

Let us first show that when $T(c) < \frac{2x_2-z_2}{2x_1-z_1}$, then $\frac{\partial V_A(c)}{\partial T(c)} > 0$. From Lemma 1, if $T(c) \leq \frac{2x_2-z_2}{z_1}$, then

$$\frac{\partial V_A(c)}{\partial T(c)} = \frac{z_1-x_1}{2x_2} = \frac{(x_{A1}-x_{B1})[(x_{A1}+x_{B1})-1]}{2(x_{A2}-x_{B2})} \tag{8}$$

Since $x_{Ar} < x_{Br}$ for all $r \in \{1,2\}$, and $(x_{A1}+x_{B1}) > 1$, it follows that $\frac{\partial V_A(c)}{\partial T(c)} > 0$.

If $\frac{2x_2 - z_2}{z_1} < T(c) < \frac{2x_2 - z_2}{2x_1 - z_1}$, then

$$\frac{\partial V_A(c)}{\partial T(c)} = \frac{1}{8x_1 x_2} \left[-(2x_1 - z_1)^2 + \frac{(2x_2 - z_2)^2}{T(c)^2} \right] \tag{9}$$

where $8x_1x_2 > 0$ and since $T(c) < \frac{2x_2 - z_2}{2x_1 - z_1}$, the term in brackets is also positive. Hence, $\frac{\partial V_A(c)}{\partial T(c)} > 0$.

Let us secondly show that when $T(c) > \frac{2x_2 - z_2}{2x_1 - z_1}$, then $\frac{\partial V_A(c)}{\partial T(c)} < 0$. If $\frac{2x_2 - z_2}{2x_1 - z_1} < T(c) < \frac{z_2}{2x_1 - z_1}$, then the derivative $\frac{\partial V_A(c)}{\partial T(c)}$ is given by Expression (9). Since $T(c) > \frac{2x_2 - z_2}{2x_1 - z_1}$, it follows directly that $\frac{\partial V_A(c)}{\partial T(c)} < 0$.

If $T(c) \geq \frac{z_2}{2x_1 - z_1}$, then

$$\frac{\partial V_A(c)}{\partial T(c)} = -\frac{z_2 - x_2}{T(c)^2 2x_1} = -\frac{(x_{A2} - x_{B2})[(x_{A2} + x_{B2}) - 1]}{T(c)^2 2(x_{A1} - x_{B1})} \tag{10}$$

Since $x_{Ar} < x_{Br}$ for all $r \in \{1, 2\}$ and $(x_{A2} + x_{B2}) > 1$, it follows that $\frac{\partial V_A(c)}{\partial T(c)} < 0$.

PROOF OF LEMMA 3:

Suppose, without loss of generality, that party A has an advantage on issue 1 and party B has an advantage on issue 2. We will show that the percentage of voters that cast their ballots for party A is a strictly increasing function of $T(c)$.

From Lemma 1 we have:

$$\frac{\partial V_A(c)}{\partial T(c)} = \begin{cases} \frac{z_1 - x_1}{2x_2} & ;\text{if } T(c) \leq \min\left\{\frac{z_2}{2x_1 - z_1}, \frac{2x_2 - z_2}{z_1}\right\} \\ \frac{1}{8x_1x_2}\left[-(2x_1 - z_1)^2 + \frac{(2x_2 - z_2)^2}{T(c)^2}\right] & ;\text{if } \frac{2x_2 - z_2}{z_1} < T(c) < \frac{z_2}{2x_1 - z_1} \\ \frac{1}{8x_1x_2}\left[z_1^2 - \frac{z_2^2}{T(c)^2}\right] & ;\text{if } \frac{z_2}{2x_1 - z_1} < T(c) < \frac{2x_2 - z_2}{z_1} \\ -\frac{z_2 - x_2}{T(c)^2 2x_1} & ;\text{if } T(c) \geq \max\left\{\frac{z_2}{2x_1 - z_1}, \frac{2x_2 - z_2}{z_1}\right\} \end{cases}$$

Since $\frac{x_{A1} + x_{B1}}{2} > \frac{1}{2}$ it follows that $\frac{z_1 - x_1}{2x_2} > 0$, and since $\frac{x_{A2} + x_{B2}}{2} < \frac{1}{2}$ it follows that $-\frac{z_2 - x_2}{T(c)^2 2x_1} > 0$. Furthermore, when $\frac{x_{A2} + x_{B2}}{2} < \frac{1}{2}$ then $\frac{z_2}{2x_1 - z_1} < \frac{2x_2 - z_2}{2x_1 - z_1}$. Therefore, if $\frac{2x_2 - z_2}{z_1} < T(c) < \frac{z_2}{2x_1 - z_1}$ we have $T(c) < \frac{2x_2 - z_2}{2x_1 - z_1}$, in which case $\frac{1}{8x_1x_2}\left[-(2x_1-z_1)^2+\frac{(2x_2-z_2)^2}{T(c)^2}\right]>0$. Finally, when $\frac{x_{A1}+x_{B1}}{2}>\frac{1}{2}$ then $\frac{z_2}{z_1}<\frac{z_2}{2x_1-z_1}$. Therefore, if $\frac{z_2}{2x_1-z_1}<T(c)<\frac{2x_2-z_2}{z_1}$ we have $T(c)>\frac{z_2}{z_1}$, which implies that $\frac{1}{8x_1x_2}\left[z_1^2-\frac{z_2^2}{T(c)^2}\right]>0$.

PROOF OF PROPOSITION 2:

Suppose, without loss of generality, that $\frac{x_{A1}+x_{B1}}{2}>\frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2}>\frac{1}{2}$, i.e., party A has an advantage on both political issues. By Lemma 2, function $V_A(.)$ is single-peaked. Moreover, from Lemma 1 we have $\underset{T(c)\to 0}{lim}\ V_A=\frac{(x_{A2}+x_{B2})}{2}>\frac{1}{2}$ and $\underset{T(c)\to\infty}{lim}\ V_A=\frac{(x_{A1}+x_{B1})}{2}>\frac{1}{2}$. Therefore, party A always obtains a strict majority of votes.

Next, we will show that the party losing the elections (i.e., party B) spends all its campaign funds on one issue. Let $c^*=(c_A^*,c_B^*)\in C$ be such that $c_{B1},c_{B2}>0$. In this case, by means of unilateral deviations, party B can always move up and down the value of $T(c)$ and then, it can increase its votes (since by Lemma 2, the function $V_B(.)$ is single-dipped). Hence, c^* is not an equilibrium.[13]

PROOF OF PROPOSITION 3:

Suppose, without loss of generality, that party A has an advantage on issue 1 and party B has an advantage on issue 2, i.e., $\frac{x_{A1}+x_{B1}}{2}>\frac{1}{2}$ and $\frac{x_{A2}+x_{B2}}{2}<\frac{1}{2}$. By Lemma 3, the payoff function of party A is strictly increasing in $T(c)$, and therefore $(c_{A1}^*,c_{A2}^*)=(\bar{c}_A,0)$ is a strictly dominant strategy for party A and $(c_{B1}^*,c_{B2}^*)=(0,\bar{c}_B)$ is a strictly dominant strategy for party B.

[13]Note that the result is valid even if we consider that the losing political party plays mixed strategies (in any equilibrium in mixed strategies, the losing political party will play a combination of the pure strategies $c_j=(\bar{c}_j,0)$ and $c_j=(0,\bar{c}_j)$).

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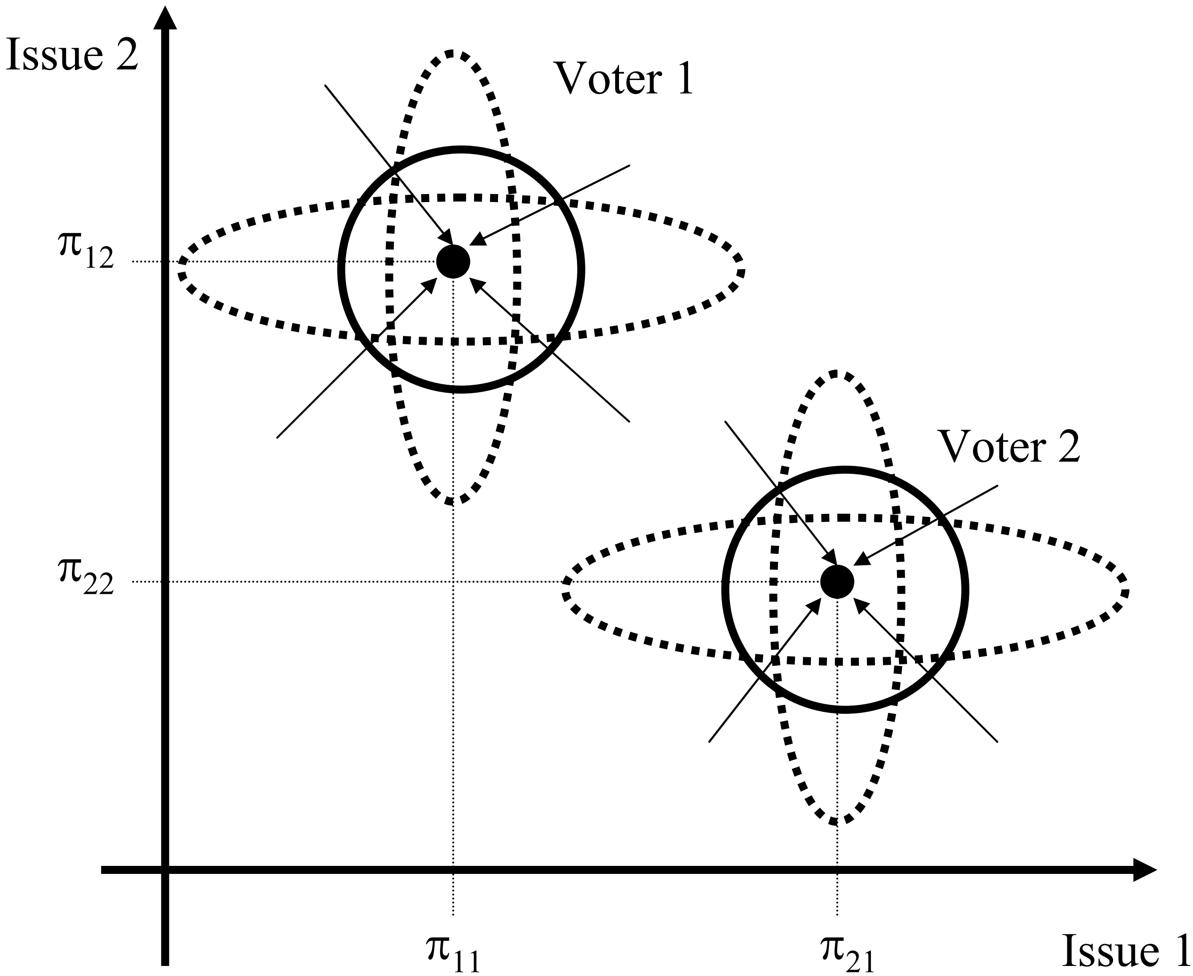

Figure 1. Example of voters' indifference curves.

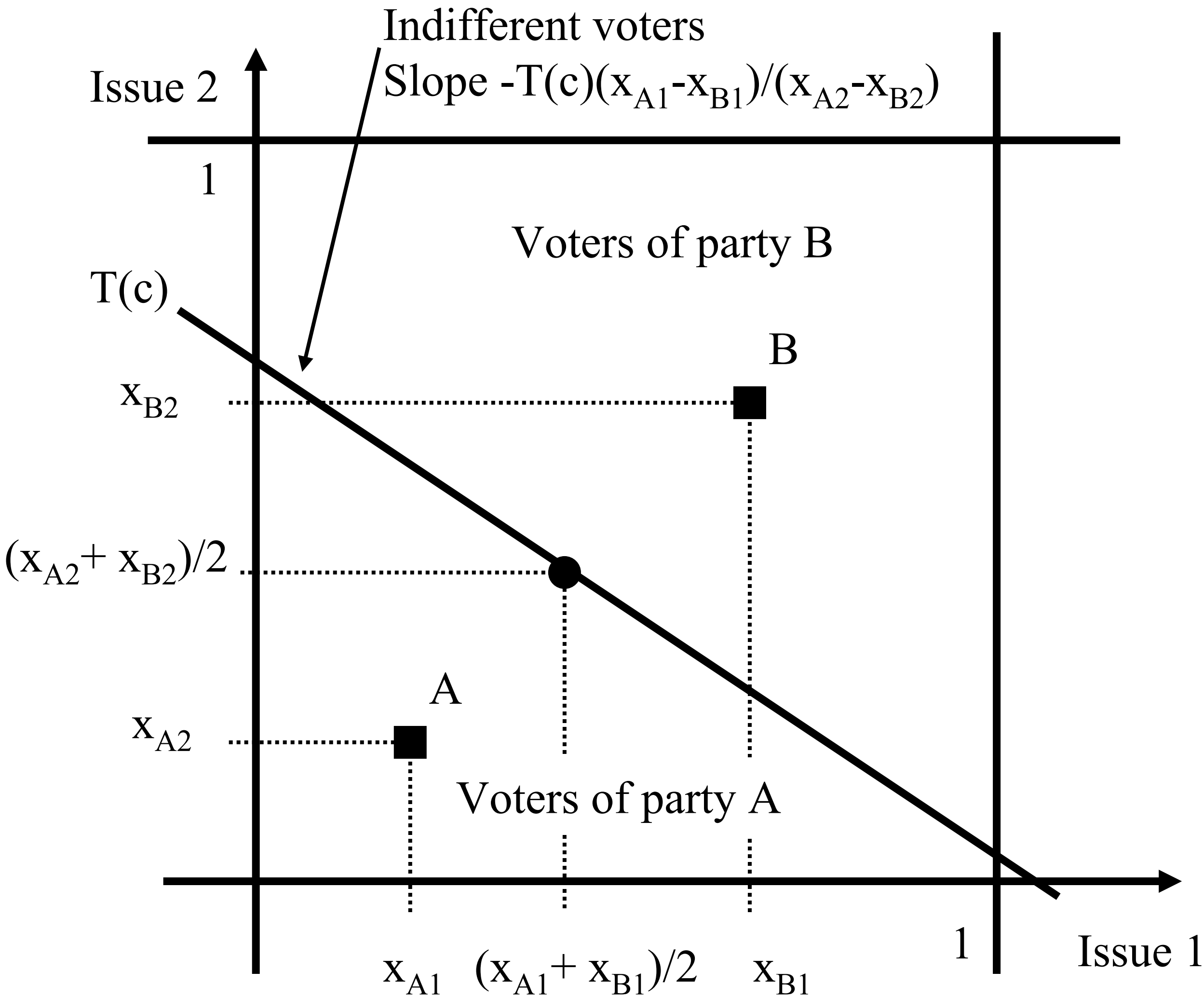

Figure 2. Example of voters of party A and party B.

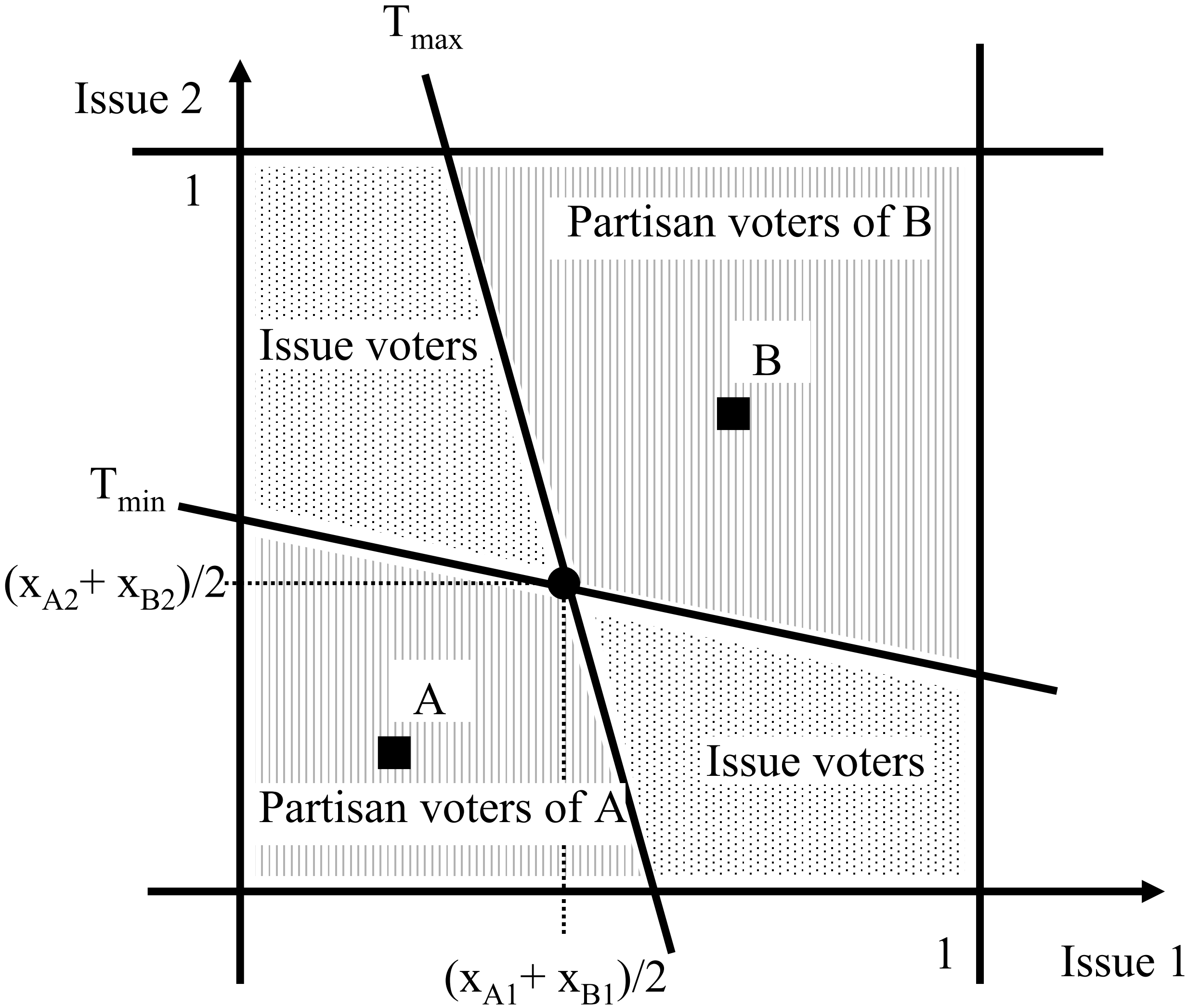

Figure 3. Example of partisan voters and issue voters.

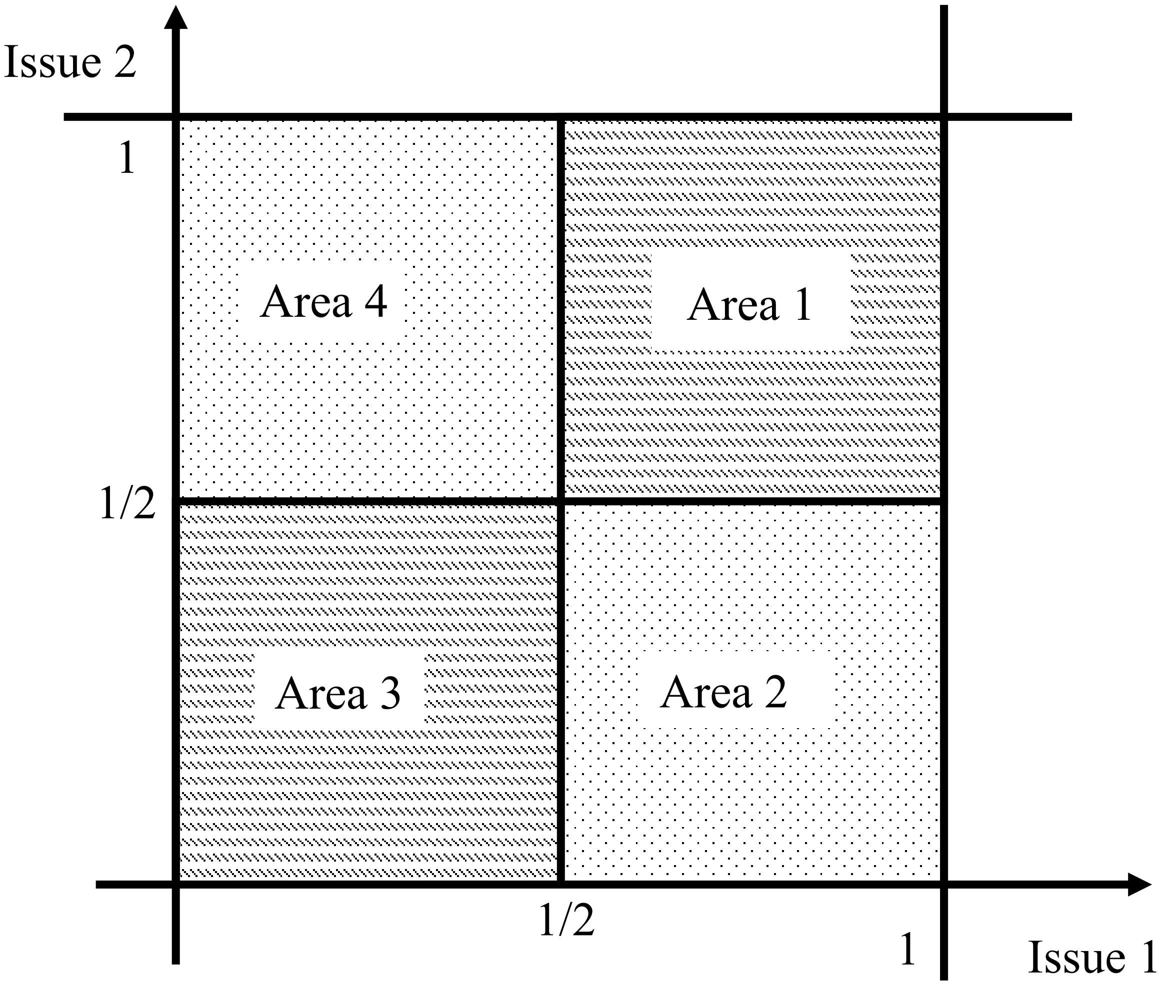

Figure 4. Political areas.

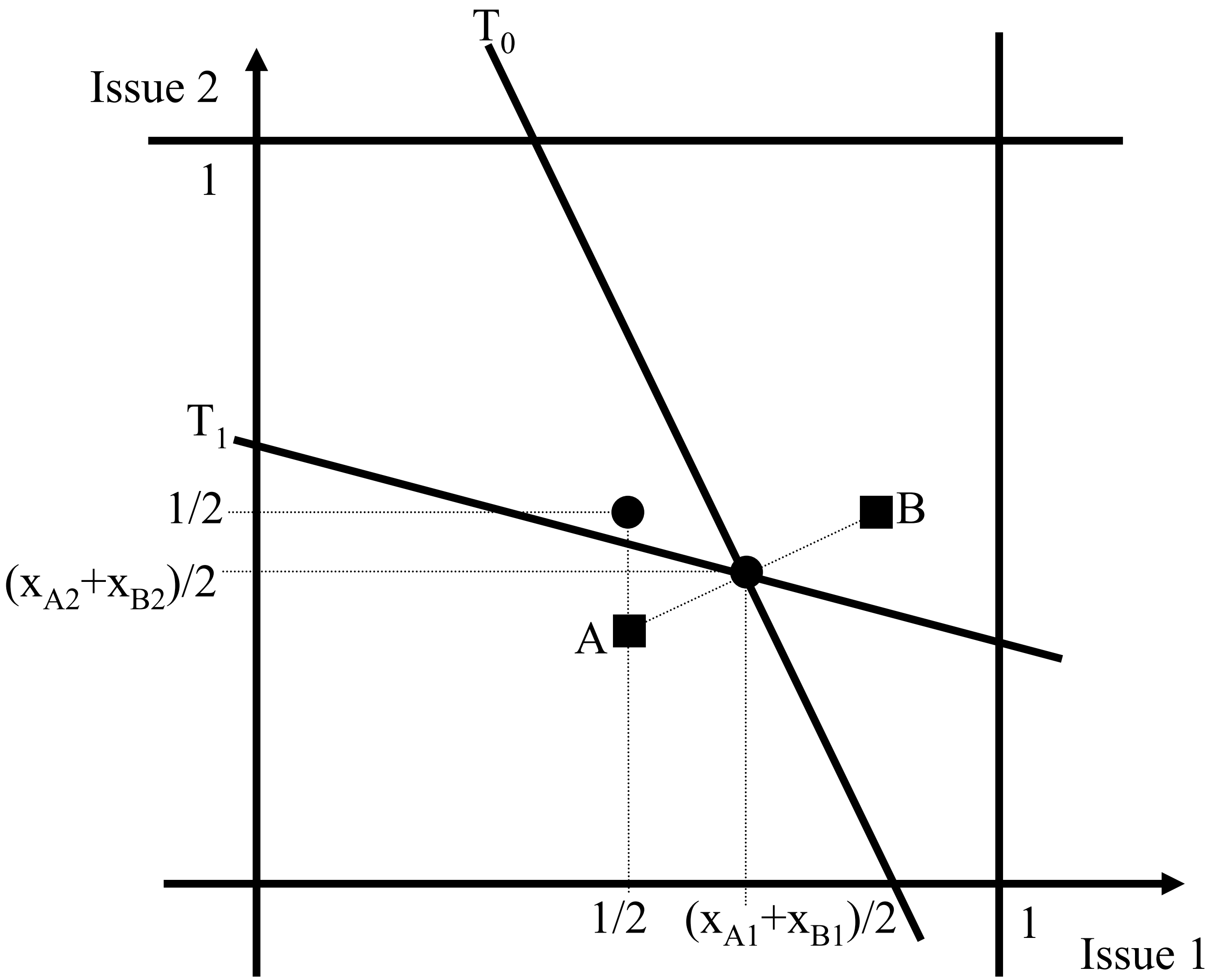

Figure 5. Electoral result with and without electoral campaign.