# Why does the Pirate launch the Copy before the Incumbent launches the Original?*

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**Abstract**

We analyze pirate's decision in a vertical product differentiation model with price competition. We first show that the monopoly never maximizes the social welfare. Second, we find that the government's role is very important in the market where the chance of pirating exists. Third, the incumbent has a key role to avoid the pirate's entry through prices. Fourth, the incumbent's incentives to install an antipiracy system decrease as increases the government's expense in monitoring the piracy. Fifth, the government not permits the incumbent to install it, since the monopoly gives the worst social welfare. Finally, the pirate's decision depends on the government's technology in monitoring the piracy, the incumbent's ability to anticipate the pirate can be leader in prices, and profit like follower is bigger than leader in prices.

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# 1 Introduction

Over the past few years, it is usual to find that most digital products have been copied and distributed without the authorization of legal owners. To such an extent that it is possible to find a new product pirated before it is launched on the market. We point out two news extracted from Spanish newspaper EL PAIS: "The new García Márquez pirated in Colombia before yours presentation,"(2004b) and, "A pirated version of start game of Xbox to Christmas appear in internet"(2004a).

The most of studios about piracy reached the conclusion that there are high and different levels of piracy across countries and high losses on industry. This studios assume that the purchase of a copy imply the purchase of an original product when the copy is no available.

In this paper, we look for the causes and incentives of a pirate to launch the copy of a original product before the incumbent launches it on the market. We analyze them in a vertical product differentiation model with two firms that compete in prices: the incumbent, who creates a the original product, and the pirate, who illegally copies the original product and decides when to launch the copy in the market.

Piracy has previously been analyzed to understand its effects on the incumbent's behaviour and social welfare. Some papers have supposed that the copy is exclusively made by final consumers (Bae and Choi (2003), Johnson (1985), Peitz and Waelbroeck (2003), Shy and Thisse (1999)). However, we consider that the copy is made by a only firm that sells it in the market, like Banerjee (2003) and Poddar(2003).

We also examine the strategies to prevent the piracy by the incumbent. Firstly, we consider the incumbent's possibility of dissuading the pirate from enter in the market through prices. And, next we consider the possibility of installing an antipiracy system with a fixed cost, like Banerjee (2003). We show, like Banerjee (2003), that the incumbent installs an antipiracy system if its cost is lower, and, an increase in the copy's quality increases the incumbent's incentive to install it. In a different approach to this issues, Poddar (2003) have analyzed a model where the original developer (incumbent) undertakes some costly R&D in order to raise the marginal cost of producing a copy by the pirate. He shows that the pirate survives in the market, when the cost of piracy is not too high and the copy is moderately reliable and moderately differentiated from the original product available in the market.

Since the government is who establish copyright, the question arise what the government's role is when copyright are violated. So we include the government in our analysis and supposes, like Banerjee (2003), that it monitors and penalizes the illegal activity carried out by the pirate. Banerjee (2003) show that if the monitoring cost is very low then the monopoly may be the socially optimal outcome, and if it is profitable for a monopolist (incumbent) to prevent piracy by installing a protective device then the government not monitors it in equilibrium.

Our analysis shows first that the monopoly never maximizes the social welfare, i.e. the government prefers that the pirate enters in the market to the incumbent gets the monopoly without additional costs. Even though, it can prefer that the incumbent only sells the product when he sets a price enough lower to dissuade the entry of the pirate, i.e. the government can prefer that the incumbent gets a monopoly when he takes a cost to avoid the piracy. Second, the government's final decision depends on the monitoring technology the piracy, which is represented by the monitoring cost. Third, the incumbent's behaviour depends on the government's expense in monitoring the piracy. Fourth, the incumbent's incentives to install an antipiracy system decrease as increases the government's expense in monitoring the piracy. Fifth, the government not permits the incumbent to install it, since the monopoly gives the worst social welfare. Finally, the pirate decides to launch the copy before the incumbent launches the original product on the market according to the government's decision and the incumbent's ability to anticipate the pirate can be leader in prices.

Contrary to the majority studios of piracy assume and some courts has ruled,[1] in this paper we find that the purchase of a copy at a lower price does not imply the purchase of an original at a higher price when the copy is not available, like Bae and Choi (2003), Banerjee (2003) and Shy and Thisse (1999).

The rest of the paper is organized as follows. Section 2 describes the model formally. Section 3 looks for the market equilibrium. Section 4 looks for the optimal policy. Section 5 analyzes the optimal strategies. Section 6 considers the possibility of installing an antipiracy system by the incumbent. Section 7 extends the model to the case where the incumbent has the opportunity to decide to when enter. Finally, Section 8 concludes.

## 2 The model

We consider four types of agents: consumers, the developer of a original product (incumbent), a pirate who illegally reproduces and sells it, and the government, which is responsible for monitoring and penalizing the pirate.

There is a continuum of consumers indexed by $\theta$, $\theta \in \left[0, \overline{\theta}\right]$.[2] $\theta$ is assumed to follow a uniform distribution, and represents the difference in the consumers' tastes for the quality of the product. Each consumer is assumed to buy only one unit of the good or not buy.

The utility of a type $\theta$ consumer is,

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{if the consumer buys the original product} \\ \theta q_p - p_p & \text{if the consumer buys the pirated product} \\ 0 & \text{if the consumer does not buy} \end{cases} \quad (1)$$

where $p_i$, $q_i$, $p_p$ and $q_p$ are the price and quality of the original and the pirated product, respectively. We assume $q_i > q_p > 0$.

Let's $x_i = p_i/q_i$ and $x_p = p_p/q_p$ the incumbent's and pirate's hedonic prices, respectively. Since qualities are common knowledge, decisions on prices are equivalent to decisions on hedonic prices. Let $r = q_i/q_p > 1$ be the ratio of qualities. Without loss of generality, we also assume that $x_i, x_p \in \left[0, \overline{\theta}\right]$.

Firms' demand functions are obtained as follows. Let's $\theta_o$ the consumer who is indifferent between buying the original and the pirated product. From (1), $\theta_o = (rx_i - x_p) / (r-1)$. Let's $\theta_i$ the consumer

[1] One of the reasons why the court ruled that Napster harmed the music industry is the loss of sales of CDs (see Peitz and Waelbroeck (2005)).

[2] We consider that the market is covered, ie. it means that always exists at least a consumer does not buy at all.

indifferent between buying from the incumbent and not buying at all, that is, $\theta_i = x_i$. Let's $\theta_p$ the consumer indifferent between buying from the pirate and not buying at all, that is, $\theta_p = x_p$.

The demands faced by the incumbent and the pirate are

$$D_i(x_i, x_p) = \begin{cases} \overline{\theta} - x_i & \text{if } x_i \leq x_p \\ \overline{\theta} - \min\left\{\theta_o, \overline{\theta}\right\} & \text{if } x_i \geq x_p \end{cases} \tag{2}$$

$$D_p(.) = \begin{cases} 0 & \text{if } x_i \leq x_p \\ \min\left\{\theta_o, \overline{\theta}\right\} - \theta_p & \text{if } x_i \geq x_p \end{cases} \tag{3}$$

We assume that the consumers do not face the risk of prosecution from the use of copies because they not copied and distributed the original product.[3] The government is responsible for monitoring and penalizing the pirate. Let $\alpha$ and $G$ be the monitoring rate and the penalty. So, $\alpha$ is the probability to detect the pirate. We assume $0 \leq G \leq \overline{G}$, where $\overline{G}$ is the maximum penalty. Let $C(\alpha)$ be the cost of monitoring. We assume $C(0) = 0, C'(\alpha) > 0, C''(\alpha) > 0$. The government chooses $\alpha$ and $G$ to maximize the social welfare.

We assume that a firm remains in the market if and only if it is making positive profit. If the pirate's illegal operations are detected, which occurs with probability $\alpha$, he must pay the penalty $G$ and loses his income. The expected profits of the original firm and the pirate, taken into account that the detection is made after the sales, are,

$$\pi_i(.) = q_i x_i D_i(x_i, x_p), \quad \pi_p(.) = (1 - \alpha)\, q_p x_p D_p(x_i, x_p) - \alpha G \tag{4}$$

Note that the firms are not subject to a quality-improvement cost like Wauthy (1996) and Banerjee (2003), but different from Ronnen (1991), Motta (1993) and Crampes and Hollander (1995). The cost incurred by the incumbent to develop the original product is a sunk cost, and the production cost after it has been developed is assumed to be zero. We also assume the pirate can copy without costs the original product before it is launched by the incumbent.

Let $R$ be the net expected revenue of the government $R = \alpha G + \alpha\delta I_p(x_i, x_p) - C(\alpha)$, where $I_p(x_i, x_p) = q_p x_p D_p(x_i, x_p)$ represents the pirate's revenue and $\delta \in [0, 1]$ represents the government's capacity to reuse the revenue seized from the pirate. In the absence of monitoring, the penalty is irrelevant. So we assume $G = 0$ if $\alpha = 0$.

The social welfare is the sum of the profits of the incumbent and the pirate, the consumer surplus and the net expected revenue of the government.

The two stage complete information game is the following. The government announces $\alpha$ and $G$ to maximize social welfare, and both firms observe the policy variables. Then, the pirate decides whether to price first or not. If he decides to act first, the incumbent becomes a follower and prices the original product taking into account the pirate's price which becomes a leader (l-subgame). But if the pirate decides to wait, the incumbent prices first the original product, and the pirate becomes a follower that

[3] It is according to penal code most of countries (for instance, see from article 270 to 272 of Spanish penal code).

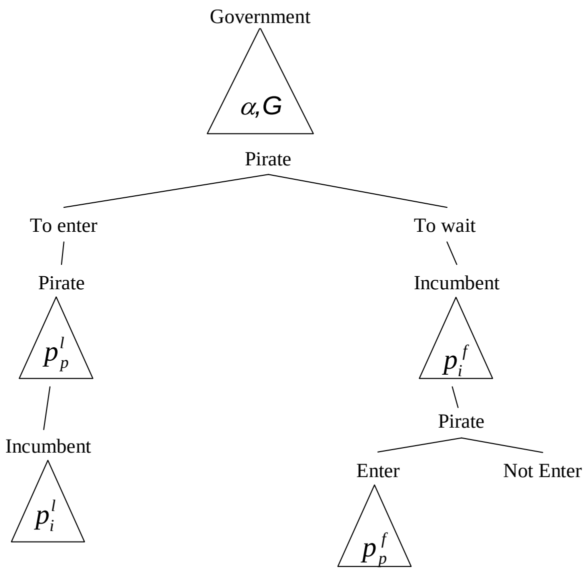


Figure 1: The timing of the game

decides whether to price the pirated product (i.e., to enter) or not (f-subgame).[4] Next, the consumers observe the firms' price and decide to buy the original product, pirated product or not buy at all.

In the next section, we look for the subgame perfect equilibrium (SPE) of the game by backward induction. First, we solve f-subgame. Second, we solve l-subgame. Next, the pirate decides whether to price first or not. Finally, the government chooses the optimal policy $(\alpha, G)$ anticipating the equilibrium of the continuation game.

# 3 Market Equilibrium

## 3.1 F-subgame

The f-subgame is reached when the pirate decides to wait, so the incumbent becomes a leader in prices.

The pirate's optimal hedonic price, given the incumbent's choice, is obtained by maximizing the pirate's profit. It is similar to the one computed by Ronnen (1991):

$$x_p^{BR}(x_i) = \begin{cases} x_i/2 & \text{if } 0 \leq x_i \leq \frac{2\overline{\theta}(r-1)}{2r-1} \\ rx_i - (r-1)\,\overline{\theta} & \text{if } \frac{2\overline{\theta}(r-1)}{2r-1} \leq x_i \leq \frac{\overline{\theta}(2r-1)}{2r} \\ \overline{\theta}/2 & \text{if } \frac{\overline{\theta}(2r-1)}{2r} \leq x_i \end{cases} \quad (5)$$

By replacing (5) into the pirate's profit, we obtain the maximal pirate's profit $\pi_p^c(x_i) = (1-\alpha)\, q_i \gamma(x_i) - \alpha G$, where

[4] We do not consider price competition with simultaneous decisions because this strategy is ruled out by the possibility of being leader (or follower).

$$\gamma\left(x_i\right)=\begin{cases}\frac{x_i^2}{4(r-1)} & \text{if } 0\leq x_i\leq\frac{2\overline{\theta}(r-1)}{2r-1}\\ \left(rx_i-(r-1)\overline{\theta}\right)\left(\overline{\theta}-x_i\right) & \text{if } \frac{2\overline{\theta}(r-1)}{2r-1}\leq x_i\leq\frac{\overline{\theta}(2r-1)}{2r}\\ \frac{\overline{\theta}^2}{4r} & \text{if } \frac{\overline{\theta}(2r-1)}{2r}\leq x_i\end{cases}\tag{6}$$

The pirate decides to enter in the market when $\pi_p^c\left(p_i\right)>0$, i.e. when $x_i>x_i^{ne}$. Where $x_i^{ne}$ is the no-entry hedonic price and is equal to

$$x_i^{ne}=\begin{cases}\sqrt{4\left(r-1\right)g\left(\alpha,G\right)} & \text{if } 0\leq g\left(\alpha,G\right)\leq\frac{\overline{\theta}(r-1)}{(2r-1)^2}\\ \frac{\overline{\theta}(2r-1)-\sqrt{\overline{\theta}^2-4rg}}{2r} & \text{if } \frac{\overline{\theta}(r-1)}{(2r-1)^2}\leq g\left(\alpha,G\right)\leq\frac{\overline{\theta}}{4r}\\ +\infty & \text{if } \frac{\overline{\theta}}{4r}<g\left(\alpha,G\right)\end{cases}\tag{7}$$

Where $g\left(\alpha,G\right)=\alpha G/q_i\left(1-\alpha\right)$ indicates the government's effort to avoid the piracy. In consequence, the pirate's optimal decision is: to enter and price $x_p^{BR}(x_i)$ if $x_i>x_i^{ne}$; and not to enter if $x_i\leq x_i^{ne}$.

According to the previous pirate's optimal decision the incumbent anticipates profits

$$\pi_i^c\left(x_i\right)=\begin{cases}q_ix_i\left(\overline{\theta}-x_i\right) & \text{if } 0\leq x_i\leq x_i^{ne}\\ q_ix_iD_i\left(x_i,x_p^{BR}(x_i)\right) & \text{if } x_i^{ne}<x_i\leq\overline{\theta}\end{cases}\tag{8}$$

When the government's effort against the piracy is very little ($g\left(.\right)$ very low), the pirate will enter always the market and price $x_p^f$, and the incumbent will price $x_i^f$ by taking into account the pirate's optimal reaction. However, when the government's effort is very strong ($g\left(.\right)$ very high), the pirate will never enter, and the incumbent will act as a monopolist and set the monopoly price, $x_i^m$. Moreover, for an intermediate range of government's effort levels, the incumbent will find optimal to avoid piracy by choosing the intermediate price $x_i^{ne}$. The optimal strategies are summarized in the following proposition.

**Proposition 1** *In any SPE, the optimal strategies of incumbent and pirate are:*

*(a) The pirate will enter the market only if $x_i>x_i^{ne}$, where $x_i^{ne}$ is defined in (7), and he will price according to (5).*

*(b) The incumbent will price $x_i^*=x_i^f>x_i^{ne}$ if $g\left(.\right)<g_L$, where*

$$g_L=\frac{\overline{\theta}^2\left(r-\sqrt{2r-1}\right)}{8\left(r-1\right)\left(2r-1\right)}.\tag{9}$$

*(c) The incumbent will price $x_i^*=x_i^{ne}$ if $g_L\leq g\left(.\right)<g_m$, where*

$$g_m=\begin{cases}\frac{\overline{\theta}^2(2-r)}{4} & \text{if } 1<r\leq 3/2,\\ \frac{\overline{\theta}^2}{16(r-1)} & \text{if } 3/2\leq r.\end{cases}\tag{10}$$

*(d) The incumbent will price $x_i^*=x_i^m$ if $g_m\leq g\left(.\right)$.*

Proof: see Appendix A.

Note that there is no piracy when $g^{ne} \leq g(.)$. When $g^m \leq g(.)$, the piracy is only eliminated because of a high government's expense in preventing it, so the incumbent can sets monopoly price. Nevertheless, when $g^{ne} \leq g(.) < g^m$, as well as the government's intervention, it is also necessary the incumbent sets a low enough price; i.e., the incumbent shares with the government the cost to eliminate the piracy.

When the differentiation is low ($1 < r \leq 3/2$) and the incumbent sets $x_i^m$, the pirate's best response is to expel the incumbent from the market through prices,.although this price is bigger than $x_i/2$. Therefore, the government has to do a lower effort to lead to monopoly.

We obtain the following values:

$$\begin{array}{l} x_i^f = \frac{\overline{\theta}(r-1)}{2r-1}, \quad \pi_i^f = \frac{\overline{\theta}^2 q_i(r-1)}{2(2r-1)}, \quad x_p^f = \frac{\overline{\theta}(r-1)}{2(2r-1)}, \quad I_p^f = \frac{\overline{\theta}^2 q_i(r-1)}{4(2r-1)^2}, \quad \theta_o^f = \frac{\overline{\theta}}{2}, \\ x_i^m = \theta_i^m = \frac{\overline{\theta}}{2}, \quad \pi_i^m = \frac{\overline{\theta}^2 q_i}{4} \end{array} \tag{11}$$

## 3.2 L-subgame

In this subgame the pirate became the leader. So the incumbent prices the original product taking into account the pirate's choice.

The incumbent's optimal hedonic price, given the pirate's choice, is obtained by maximizing the incumbent's profit. It is similar to one computed by Ronnen (1991):

$$x_i^{BR}(x_p) = \begin{cases} \left(\overline{\theta}(r-1) + x_p\right)/2r & \text{if } 0 \leq x_p \leq \frac{\overline{\theta}(r-1)}{2r-1} \\ x_p & \text{if } \frac{\overline{\theta}(r-1)}{2r-1} \leq x_p \leq \overline{\theta}/2 \\ \overline{\theta}/2 & \text{if } \overline{\theta}/2 \leq x_p \end{cases} \tag{12}$$

The pirate incorporates the incumbent's reaction function into its profit function and chooses the price that maximizes his profit. From the first order condition the hedonic prices, the indifferent consumers and profits are as follows:

$$x_i^l = \frac{\overline{\theta}(r-1)(4r-1)}{4r(2r-1)}, \quad \pi_i^l = \frac{\overline{\theta}^2 (q_i - q_p)(4r-1)^2}{16(2r-1)^2}, \quad x_p^l = \frac{\overline{\theta}(r-1)}{2(2r-1)}, \quad I_p^l = \frac{\overline{\theta}^2 q_i(r-1)}{8r(2r-1)}, \quad \theta_o^l = \frac{\overline{\theta}(4r-3)}{4(2r-1)} \tag{13}$$

Since the incumbent's profit is not negative he always enters in the market. Note that $\pi_p^l > 0$ if and only if $g(.) < I_p^l = g_0$.

## 3.3 Pirate: To be leader or follower

In this subsection we look for the optimal pirate's decision, taking into account the profits obtained for each subgame. He decides between pricing first or waiting until to become a follower.

If the pirate waits, he anticipates the profit $\pi_p^F = (1-\alpha)\frac{\overline{\theta}^2 q_i(r-1)}{4(2r-1)^2} - \alpha G > 0$, when $g(.) < g_L$, and $\pi_p^F = 0$ when $g_L \leq g(.)$. If the pirate prices first, he anticipates the profit $\pi_p^L = (1-\alpha)\frac{\overline{\theta}^2 q_p(r-1)}{8(2r-1)} - \alpha G$, which is positive if and only if $g(.) < g_0$. Since $c < g_0$, to obtain the pirate's optimal decision, we have to compare $\pi_p^F$ with $\pi_p^L$ on the three regions given by $g(.) < g_L$, $g_L \leq g(.) < g_0$, and $g_0 \leq g(.)$.

For $g(.) < g_L$, we have $\pi_p^F = (1-\alpha)\frac{\bar{\theta}^2 q_p r(r-1)}{4(2r-1)^2} - \alpha G$, and $\pi_p^L = (1-\alpha)\frac{\bar{\theta}^2 q_p(r-1)}{8(2r-1)} - \alpha G$. Since $\pi_p^F > \pi_p^L$, the pirate decides to wait to copy the new product.

For $g_L \leq g(.) < g_0$, we have $\pi_p^F = 0$ and $\pi_p^L = (1-\alpha)\frac{\bar{\theta}^2 q_p(r-1)}{8(2r-1)} - \alpha G$. Since $\pi_p^L > \pi_p^F$, the pirate prices the original product before the incumbent.

For $g_0 \leq g(.)$, we have $\pi_p^F = 0$ and $\pi_p^L < 0$. So the pirate decides to wait becoming a follower that will not enter in the market.

The following proposition summarizes the optimal strategies:[5]

**Proposition 2** *In any SPE, the optimal strategies of pirate and incumbent are the following:*

*(a) If $g(.) < g_L$, the pirate will wait and price the copy like a follower $x_p^* = x_p^f$.*

*(b) If $g_L \leq g(.) < g_0$, the pirate will become leader and price $x_p^* = x_p^l$.*

*(c) If $g_0 \leq g(.)$, the pirate become a follower, which later on will not enter. And, the incumbent become a monopolist.*

*(d) The incumbent will price $x_i^*$, where*

$$x_i^* = \begin{cases} x_i^f & \text{if } g(.) < g_L \\ x_i^l & \text{if } g_L \leq g(.) < g_0 \\ x_i^{ne} & \text{if } g_0 \leq g(.) < g_m \\ x_i^m & \text{if } g_m \leq g(.) \end{cases}$$

The pirate's decision, like incumbent's decision, depends on level of government's expense to avoid the piracy. When $g(.) < g_L$, the pirate waits until the incumbent prices the original product since like follower the profit is bigger. Nevertheless, when $g_L \leq g(.) < g_0$, he anticipates that his profit like a follower is zero, since the incumbent dissuades him from entering in the market through prices, and like a leader is positive, since when he prices first he restricts himself to force the incumbent to not dissuade him, so that he prices first $x_i^l$.

The government's optimal policy is analyzed in the next section.

# 4 Optimal Policy: analysis of social welfare

The government chooses the optimal policy that maximizes social welfare, anticipating the equilibrium of the continuation game. The social welfare is the sum of the profits of the incumbent and the pirate, the consumer surplus and the net expected revenue of the government. The consumer surplus is

[5] In $g(.) = g_0$, the pirate is indiferent between become a leader, and wait and became a follower, because his expected profits are zero. Nevertheless, to exist the maximum of social welfare for every values of $g$, is necessary that the pirate become a follower which later on will not enter. when $g(.) = g_0$.

$$CS = \int_{\theta_p}^{\theta_o} (\theta q_p - p_p)\, d\theta + \int_{\theta_o}^{\overline{\theta}} (\theta q_i - p_i)\, d\theta \tag{14}$$

The consumer surplus obtained from different optimal strategies both incumbent and pirate is

$$CS^f = \frac{\overline{\theta}^2 q_i \left(4r^2+r-1\right)}{8(2r-1)^2} \quad CS^{ne} = \frac{q_i \overline{\theta}^2 \left(16r^3+12r^2-15r+3\right)}{32r(2r-1)^2} \quad CS^{ne} = \frac{q_i \left(\overline{\theta}-x_i^{ne}\right)^2}{2} \quad CS^m = \frac{\overline{\theta}^2 q_i}{8} \tag{15}$$

The social welfare of continuation in SPE, taking the optimal pirate's decision, is:

$$SW = \begin{cases} CS^f + \pi_i^f + (1-\alpha+\alpha\delta)\, I_p^f - C(\alpha) & \text{if } g(.) < g_L, \\ CS^l + \pi_i^l + (1-\alpha+\alpha\delta)\, I_p^l - C(\alpha) & \text{if } g_L \le g(.) < g_0, \\ CS^{ne} + \pi_i^{ne} - C(\alpha) & \text{if } g_0 \le g(.) < g_m \\ CS^m + \pi_i^m - C(\alpha) & \text{if } g_m \le g(.) \end{cases} \tag{16}$$

where $CS^k$ and $\pi_i^k$ are the value of consumer surplus, incumbent's profit and pirate's revenue, respectively, in the four state $k \in \{f, l, ne, m\}$ stipulate in the previous sections, and $I_p^k, k \in \{f, l\}$ is the pirate's revenue.

Note that social welfare is decreasing in $\alpha$ and $G$ on the intervals $[0, g_L), [g_L, g_0), [g_0, g_m)$ and $[g_m, +\infty)$, since (i) $g(\alpha, G) = \alpha G / q_i (1-\alpha)$ is increasing in $\alpha$ and $G$; (ii) the values $CS^k$,$\pi_i^k, I_p^k, k \in \{f, l, m\}$ are independent of $\alpha$ and $G$; and (iii) the sum $CS^{ne} + \pi_i^{ne} = q_i \left(\overline{\theta}^2 - (x_i^{ne})^2\right)/2$ is decreasing in $g(.)$ because$x_i^{ne}$ is increasing in $g(.)$. So in order to maximize the social welfare the government will choose the minimum monitoring rate and penalty that leads to different situations. As we assume $0 \le G \le \overline{G}$, the maximum must reach in $G = \overline{G}$, since a bigger penalty not entails a bigger cost, and in $\alpha \in \{\alpha_f, \alpha_L, \alpha_0, \alpha_m\}$, where $\alpha_f = 0, \alpha_L = \frac{q_i g_L}{q_i g_L + \overline{G}}, \alpha_0 = \frac{q_i g_0}{q_i g_0 + \overline{G}}, \alpha_m = \frac{q_i g_m}{q_i g_m + \overline{G}}$. As a result, the value of $g$ in the maximum is reached in $\{0, g_L, g_0, g_m\}$. Since $0 < g_L < g_0 < g_m$, it is true $\alpha_f < \alpha_L < \alpha_0 < \alpha_m$.

The maximum social welfare is obtained from comparing the following values:

$$\begin{aligned} SW^f &= CS^f + \pi_i^f + I_p^f, \\ SW^l &= CS^l + \pi_i^l + (1-\alpha_L+\alpha_L\delta)\, I_p^l - C(\alpha_L), \\ SW_0^{ne} &= CS_0^{ne} + \pi_{i0}^{ne} - C(\alpha_0), \\ SW^m &= CS^m + \pi_i^m - C(\alpha_m) \end{aligned} \tag{17}$$

where $CS_0^{ne} + \pi_{i0}^{ne} = \frac{\overline{\theta}^2 q_i \left(3r^2-1\right)}{4r(2r-1)}$ is the gross social welfare obtained from $x_i^{ne}$ and $g_0$.

We can easily check the following relationships:

$$CS^m + \pi_i^m < CS^f + \pi_i^f + I_p^f < CS^l + \pi_i^l + I_p^l < CS_0^{ne} + \pi_{i0}^{ne} \tag{18}$$

From (18), we can observe the pure monopoly (without restriction in prices) provides the worst social welfare due to excessive power of the incumbent in the market. Therefore, the government never chooses $\alpha_m$, and the pure monopoly is not part of equilibrium.

The social welfare when the pirate is the follower can be bigger than when he is leader in prices. It depends on the relationship between the profit and loss. In particular, when $CS^l + \pi_i^l + (1 - \alpha_L + \alpha_L \delta) I_p^l < CS^f + \pi_i^f + I_p^f$ the social welfare where the pirate is the follower is bigger. It is because the government not incurs in a monitoring cost and can reuse little revenue seized from the pirate. Therefore, when the gains are bigger than the increase of monitoring cost, dissuading the entry to the pirate by the incumbent provides the biggest social welfare; otherwise, let enter the pirate like a follower is the biggest social welfare.

Otherwise, the situation where the pirate is the leader can also maximize the social welfare. Just like the previous case, the biggest social welfare depends on the difference between the gains come from the increase of competence and the increase of monitoring cost. Therefore, no-entry situation maximizes the social welfare when $C(\alpha_0)$ is small enough compared to the gain of social welfare that it generates; letting enter the pirate, like a leader, maximizes the social welfare when $C(\alpha_L)$ is small enough compared to the gain of social welfare that it generates; and, finally, letting enter the pirate, like a follower, maximizes the social welfare when $C(\alpha_0)$ and $C(\alpha_L)$ are big enough compared to the gains of social welfare associated to no-entry and l-situations.

Although the equilibrium depends on $C(\alpha)$ and the parameters of the model, taking into account $\alpha_L = \alpha_0$ for $\overline{G} = 0$ and when $\overline{G} \to +\infty$, we have the following implications:

1. If $\overline{G}$ is big enough we have $SW_0^{ne} > SW^l > SW^f$. The equilibrium is $\alpha = \alpha_0$.

2. Accepting $C(1) = +\infty$, if $\overline{G}$ is small enough we have $SW^f > SW_0^{ne} > SW^l$. The equilibrium is $\alpha = \alpha_f = 0$.

3. Only for intermediates values of $\overline{G}$, the equilibrium can be $\alpha = \alpha_L$. For this purpose, it is necessary:

$$C(\alpha_L) < CS_0^{ne} + \pi_{i0}^{ne} - CS^f - \pi_i^f - I_p^f < C(\alpha_0) \tag{19}$$

Note that the social welfare can be maximum when the incumbent dissuades the entry of the pirate through prices, although the government incurs in a budgetary deficit to reach it. This deficit is due to the government incurs in a monitoring cost, and it does not obtain revenue because the pirate does not enter in the market.

In the next section we analyze the optimal strategies.

# 5 Analysis of optimal strategies

## 5.1 Optimal strategies

The value of $x_i^{ne}$ and $\pi_i^{ne}$ obtained from $g(\alpha, G) = g_0$ is:

$$x_{i0}^{ne} = \sqrt{\frac{\overline{\theta}^2 (r-1)^2}{2r(2r-1)}}, \quad \pi_{i0}^{ne} = \sqrt{\frac{\overline{\theta}^4 q_i^2 (r-1)^2}{2r(2r-1)}} - \frac{\overline{\theta}^2 q_i (r-1)^2}{2r(2r-1)} \tag{20}$$

We obtain that the sales of the original product do not decrease for the threat of piracy[6] like Bae and Choi (2003), Banerjee (2003) and Shy and Thisse (1999). But unlike Shy and Thisse (1999), we do not assume network effects between the original and the copy. This is due to the effect of the threat of piracy on the incumbent's pricing behaviour, and that the copy's quality is lower than the original's quality. Therefore, the purchase of a copy at a lower price does not imply the purchase of a original at a higher price when the copy is not available. This result is compatible with the results in Johnson (1985), where copying reduces the original's sales in spite of the incumbent's pricing behaviour. It is because Johnson (1985) assumes the copy is the same quality than the original.

Note that the leader's demand not changes when the qualities change, independently he is the incumbent or the pirate. The incumbent's demand like a leader $\left(D_i^f = \overline{\theta}/2\right)$ consists of the same consumers, unlike the pirate's demand $\left(D_p^l = \overline{\theta}/4\right)$, where some consumers pass from buying incumbent to buying pirate, and others pass from buying pirate to not buying at all, or vice versa. In particular, in l-subgame, when the differentiation increases, on the one hand, the pirate take the incumbent away some consumers, and on the other, he loses consumers who decide to not buy at all. So sell the best product and be the leader ensures a consumer group against changes in the qualities. Nevertheless, to be a pirate and leader ensures the same demand independently of qualities.

Note that both the incumbent and the pirate prefer to be follower than leader in prices.[7] And like Shaked and Sutton (1982), the top quality firm (incumbent) enjoys greater revenue than its rival (pirate). We also can observe that the pirate establishes the same price independently he is a leader or follower.

A measure of price competition is obtained by taking the ratio of prices. From (21) we deduce that an increase in the quality ratio relaxes price competition and leads to price rises.

$$\frac{p_i^f}{p_p^f} = 2r; \quad \frac{p_i^l}{p_p^l} = \frac{4r-1}{2}; \quad \frac{x_i^f}{x_p^f} = 2; \quad \frac{x_i^l}{x_p^l} = \frac{4r-1}{2r} \tag{21}$$

It is clear from (11), (13), (20) and (21) that hedonic prices increase in $r$, while their ratio is constant in f-subgame and increasing in l-subgame. It is because the pirate, like a follower, always prices equal to the half incumbent's price, and like a leader he can't make it because the last decision is of the incumbent.

Incumbent's hedonic prices are lower in f-subgame than monopoly, lower in l-subgame than f-subgame, and lower in no-entry situation than l-subgame.[8] Accordingly, the consumer surplus in no-entry situation is the highest. It is because the threat of entry a pirate forces the incumbent to set lower prices to avoid it, so when the pirate enters the incumbent can set bigger prices because he can't take him out from the market. It explains the reason why the no-entry situation is the best option for the government when

[6] If we consider covered market, we obtain $D_i^m > D_i^f$.

[7] If we consider covered market, we obtain incumbent's leader profit can be bigger than follower profit.

[8] This result is compatible with Mussa and Rosen (1978)'s results, where the monopoly price is larger than the competitive price.

the monitoring cost is lower. That is to say, when the monitoring cost is lower, the government prefers the incumbent only produces and distributes the product a smallest price than the product is sold to different qualities and prices.

### 5.2 Comparative static

Considering that the optimal government's policy not changes, we analyze the effects of a increase in the products' quality. Next proposition summarizes the results.

**Proposition 3** *In any SPE, we have:*

*(a) An increase in $q_i$ leads to higher prices and revenue both the incumbent and the pirate, in every optimal strategies.*

*(b) Moreover, an increase in $q_i$ leads to lower demand both the incumbent and the pirate except when they are leader in prices.*

*(c) The effect of $q_p$ is opposed to $q_i$, although the effect on the price and revenue of pirate depend on the initial level of differentiation. In particular, for a initial high differentiation it is positive, otherwise it is negative.*

*(d) When the government decides $g(\alpha, G) = 0$, the social welfare increase in $q_i$ and $q_p$.*

Proof: see Appendix B.

A bigger differentiation leads to a greater market power both incumbent and pirate. So that they can set a higher price and obtain a bigger revenue.[9] Nevertheless, an increase in the pirate's quality forces the incumbent to reduce the price of the original product because of the threat of entry of the pirate. In this case incumbent's profit fall despite increasing his demand. Moreover, since a rise in $q_p$ implies an increase in the value of consumers of the pirated product and a decrease in the market power both firms, for a high initial differentiation the pirate rises the price because he still keeps a high market's power, otherwise he decreases it.

## 6 Antipiracy System

Since the incumbent want to keep the monopoly's profit, we now consider the case where he can install a protection device to avoid the piracy of his product (antipiracy system).

Following Banerjee (2003), we assume that the original's quality is not damaged by the antipiracy system and the installation cost is fixed. So the incumbent installs it when the increase in profit exceeds the fixed cost $F$.

[9] These results are the same as the vertical product differentiation literature, where it states that the firms prefers to differentiate its products from those of competitors to restore some monopolistic power (Champsaur and Rochet (1989), Shaked and Sutton (1982)).

The incumbent's incentives to install an antipiracy system when the pirate enters (f-subgame or l-subgame) or he is dissuaded from entering in the market (no-entry situation) are $\pi_i^m - \pi_i^f$, $\pi_i^m - \pi_i^l$ and $\pi_i^m - \pi_i^{ne}$, respectively. Unlike Banerjee (2003), the incumbent's incentives to install it decreases as the government's expense in monitoring the piracy increases, since

$$\pi_i^m - \pi_i^f > \pi_i^m - \pi_i^l > \pi_i^m - \pi_i^{ne}$$

The social welfare when the incumbent installs the antipiracy system is:[10]

$$SW_{ApS}^m = \frac{3\overline{\theta}^2 q_i}{8} - F$$

Unlike Banerjee (2003), the government always prefers that the pirate enters like follower than the incumbent gets the monopoly through the antipiracy system $\left(SW_0^f > SW_{ApS}^m\right)$. So the government forbids the incumbent to install it. Therefore, the government's optimal policy is the same that previous section.

Moreover, an increase in copy's quality rises the incumbent's incentives to install the antipiracy system, just like Banerjee (2003).

# 7 Shall the incumbent give to the pirate the opportunity to be a leader?

So far we have assume the pirate is the only one that can decide to when enter in the market. In this section, we extend the model to the case where the incumbent has also the opportunity to decide to when enter. Therefore, the question arise is whether the incumbent continue to give to the pirate the opportunity to be a leader.

The extended game is the following. The government announces $\alpha$ and $G$ to maximize social welfare, and both firms observe the policy variables. Then, the incumbent decides to immediately enter or wait. If he immediately enters, the pirate observes the price and become a follower, and the game follows like a subgame similar to f-subgame. But, if he waits, the game follows the game plan analyzed previously, i.e. if the incumbent waits the pirate can decides to when enter in the market. When the pirate acts first, the game follows the analyzed l-subgame, where the pirate become a leader. But when he waits, the games follows the analyzed f-subgame, where the pirate is follower.

In same way subsection 3.3, we get the following proposition.

**Proposition 4** *In any SPE of the extended game, the hedonic prices given $g(.)$ are as follows: $x_i = x_i^f$ and $x_p = x_p^f$ if $g(.) \in [0, g_L)$; $x_i = x_i^l$ and $x_p = x_p^l$ if $g(.) \in [g_L, g_I)$; $x_i = x_i^{ne}$ if $g(.) \in [g_I, g_m)$; $x_i = x_i^m$ if $g(.) \in [g_m, +\infty)$. Where*

[10] We assume that the government not monitors and penalizes the piracy in this case.

$$g_I = \frac{\overline{\theta}^2 q_i}{64\,(r-1)} \left(2 - \sqrt{\frac{8r^2 - 5r + 1}{r\,(2r-1)^2}}\right)^2 \in (g_L, g_0)\,. \tag{22}$$

*The incumbent is indifferent between to wait and to act as a leader in prices for any* $g\,(.)$ *except on the interval* $[g_L, g_i)$*, where he prefers to wait in order to allow the pirate to act first as a leader.*

Just like section 4, the social welfare of continuation in SPE is:

$$SW = \begin{cases} CS^f + \pi_i^f + (1-\alpha+\alpha\delta)\, I_p^f - C\,(\alpha) & \text{if } g\,(.) < g_L, \\ CS^l + \pi_i^l + (1-\alpha+\alpha\delta)\, I_p^l - C\,(\alpha) & \text{if } g_L \leq g\,(.) < g_I, \\ CS^{ne} + \pi_i^{ne} - C\,(\alpha) & \text{if } g_I \leq g\,(.) < g_m \\ CS^m + \pi_i^m - C\,(\alpha) & \text{if } g_m \leq g\,(.) \end{cases} \tag{23}$$

Since the social welfare is decreasing in $g\,(.)$ on the intervals $[0, g_L)\,, [g_L, g_I)\,, [g_I, g_m)$ and $[g_m, +\infty)$, in SPE we have $g\,(.) \in \{0, g_L, g_I, g_m\}$. Therefore, the maximum social welfare will be $SW^f, SW^l, SW_I^{ne}$ and $SW^m$. Where the value of $SW^f, SW^l$ and $SW^m$ is the same than in (17), and $SW_I^{ne}$ is

$$SW_I^{ne} = CS_I^{ne} + \pi_{iI}^{ne} - C\,(\alpha_I)$$

Given $g_I \in (g_L, g_0)$ and $SW^{ne}$ is decreasing in $g\,(.)$, we have $SW_I^{ne} > SW_0^{ne}$. In such a way that the relationship (18) not changes.

Therefore, in the same way section 4, we reached the same conclusions, i.e.:

1. The pure monopoly is never a equilibrium.

2. The equilibrium depends on $C\,(\alpha)$.

3. If $\overline{G}$ is big enough we have $SW_I^{ne} > SW^l > SW^f$. The equilibrium is $\alpha = \alpha_0$.

4. Accepting $C\,(1) = +\infty$, if $\overline{G}$ is small enough we have $SW^f > SW_I^{ne} > SW^l$. The equilibrium is $\alpha = \alpha_f = 0$.

5. Only for intermediates values of $\overline{G}$, the equilibrium can be $\alpha = \alpha_L$. For this purpose, it is necessary:

$$C\,(\alpha_L) < CS_I^{ne} + \pi_{iI}^{ne} - CS^f - \pi_i^f - I_p^f < C\,(\alpha_I) \tag{24}$$

# 8 Conclusions

We have analyzed the causes and incentives of a pirate to launch the copy of a original product before the incumbent launches it on the market, and, the government's role. The framework of analysis has been a duopoly model of vertical product differentiation with price competition, where both the incumbent and the pirate are committed in their prices.

The pirate's final decision depends on the government's expense in monitoring the piracy, which depends on its monitoring technology it. In particular, when the government spends a small sum of resources in monitoring, the pirate decides to set first price. Moreover, the pirate's decision also depends on the incumbent's ability to anticipate the pirate can be leader in prices, and profit like follower is bigger than leader in prices.

Our analysis shows that the monopoly never maximizes the social welfare; when the incumbent sets a low enough price so the pirate does not enter in the market the social welfare is maximum if the monitoring cost is smaller than the gain of social welfare that it generates; letting enter the pirate like a leader maximizes the social welfare when the monitoring cost is small enough compared to the gain of social welfare that it generates; and, finally, letting enter the pirate like a follower maximizes the social welfare when the monitoring cost is big enough compared to the gains of social welfare associated to no-entry and l-situations.

We also show that the incumbent installs an antipiracy system if its cost is lower, and, that the incumbent's incentives to install it decreases as increases the government's expense in monitoring the piracy. However, the government not permits the incumbent to install it since the monopoly provides the worst social welfare.

Our results shows that the great variation in the piracy rates across countries is a consequence of different technologies for monitoring the piracy by the governments.[11]

A interesting result, contrary to majority of studios about piracy and some sentence, is obtained in this paper. Which is that the sales of a original product do not decrease for the threat of piracy.[12] This is because the effect of the threat of piracy on the incumbent's pricing behaviour and that the copy's quality is lower than the original's quality. Therefore, the purchase of a copy at a lower price does not imply the purchase of an original at a higher price when the copy is not available.

The results, in this paper, suggest that when the monitoring cost is small and the piracy exists, the government must try hard, but not too much to avoid the monopoly, or, the incumbent must reduce the original's price to prevent the pirate's entry, so the threat of piracy is latent. In this case, as well as government's intervention is needed the incumbent's participation.

Like every previous piracy model, we assume price competition. Nevertheless, when we assume quantity competition we obtain that leader's profit is bigger than follower's profit. In such a way that the pirate enters before the incumbent if he get information about the original before it is launched on the market.

When we do not include the pirate's profit in the social welfare, the monopoly is still provided the worst social welfare; the government's final decision depends on monitoring technology; and, the equilibrium candidates are no-entry situation, when the pirate enters like leader and when he enters like follower in prices.

We assume that both the incumbent and the pirate are in the same economy. If we consider the incumbent as a foreign firm, we obtain that the no-entry situation, as well as monopoly, never maximizes

[11] See Global Software Piracy Study 2005.

[12] One of the reasons why the court ruled that Napster harmed the music industry is the loss of sales of CDs (see Peitz and Waelbroeck (2005)). And, a judge of Alicante considers that the piracy (top-manta) not harm the record company's sales.

the social welfare. So that the government's optimal decision is let the pirate to enter like leader or follower, according to the government's capacity to reuse the revenue seized from the pirate ($\delta$) and the monitoring technology ($C(\alpha)$). The conclusion when the pirate is a foreign firm is the same that when we do not include the pirate's profit in the social welfare.

In this paper we implicitly raise the controversy between full and partial protection of the incumbent, since he want to get monopoly and the consumers not.

One interesting application of the present model is to the matter of tax evasion and smuggling in international trade. Let $t$ be the tariff on imports and $\bar{t} > \alpha$ the tariff which makes indifferent the foreign firm between enter illegally or no. We obtain the foreign firm never enters illegally in the market when $t < \bar{t}$, in particular when $t < \alpha$. Therefore, there is a maximum threshold on $t$ to avoid smuggling. Let's $\bar{t}^h$ and $\bar{t}^l$ the maximum threshold of the high quality and low quality firm, respectively. Since $\bar{t}^h < \bar{t}^l$, the high quality firm's incentive to enters illegally is bigger than the low quality firm, because of high quality firm's revenue is greater than low quality firm's revenue.

Of course, the scope of our results is limited by our assumptions. It would be interesting, for instance, to extend our analysis to the case of multi-product firms, network externalities and switching cost.

# Appendix A

**Proof of Proposition 1.** To maximize (8), we must obtain first the possible local maxima restricted to the intervals $I_1 = [0,\, x_i^{ne}]$ and $I_2 = [x_i^{ne},\, \bar{\theta}]$. Let $x_{ki}$ and $\pi_{ki}$ be the incumbent's hedonic price and profit in the maximum of $\pi_i^c(\cdot)$ on the interval $I_k$ for $k = 1, 2$, respectively.

First, consider the maximization on $I_1$. Since the monopoly hedonic price is $x_i^m = \bar{\theta}/2$, we will have $x_{1i} = x_i^m$ if $x_i^m \leq x_i^{ne}$ and $x_{1i} = x_i^{ne}$ otherwise. In consequence, we have:

$$x_{1i} = \begin{cases} x_i^m & \text{if } g^m \leq g, \\ x_i^{ne} & \text{otherwise,} \end{cases} \tag{25}$$

where $g^m = \gamma(\bar{\theta}/2)$, and the maximal value is $\pi_{1i} = \phi_1(x_{1i})$, where $\phi_1(x_i) = q_i x_i(\bar{\theta} - x_i)$ is the monopoly profit function.

Second, consider the maximization on $I_2$. This case is feasible only when $g \leq \bar{\theta}^2/4r$ holds. From (5), it is easy to see that $x_i \geq x_p^{BR}(x_i)$ holds for any $x_i \in [0,\, \bar{\theta}]$. Therefore, from (2) and (5), the maximization on $I_2$ is equivalent to maximize

$$\hat{\pi}_i^c(x_i) = \begin{cases} \phi_2(x_i) & \text{if } x_i \leq \frac{2(r-1)}{2r-1}\bar{\theta},\ x_i \in I_2 \\ 0 & \text{if } \frac{2(r-1)}{2r-1}\bar{\theta} \leq x_i \leq \bar{\theta}, \end{cases} \tag{26}$$

where $\phi_2(x_i) = q_i x_i(\bar{\theta} - \frac{2r-1}{2(r-1)} x_i)$ is the incumbent's profit function when the pirate enters the market and reacts optimally.

Let $x_i^f = (r-1)\bar{\theta}/(2r-1)$ be the the maximum of $\phi_2(\cdot)$ over $[0,\,\bar{\theta}]$. Therefore $x_i^f$ is the incumbent's optimal hedonic price when the pirate enters the market and chooses optimally his hedonic price. Suppose $g \le \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}$. In this case, $x_i^{ne} \le 2(r-1)\bar{\theta}/(2r-1)$ holds, and the maximum of $\pi_i^c(\cdot)$ on $I_2$ is reached at $x_{2i} = \max(x_i^{ne}, x_i^f)$, with a maximal value equal to $\pi_{2i} = \phi_2(x_{2i})$.Suppose $\frac{(r-1)\bar{\theta}^2}{(2r-1)^2} \le g \le \frac{\bar{\theta}^2}{4r}$ or, equivalently, $2(r-1)\bar{\theta}/(2r-1) \le x_i^{ne}$. Since (26) becomes $\hat{\pi}_i^c(x_i) = 0$, $\forall x_i \in [x_i^{ne}, \bar{\theta}]$, the maximum of $\pi_i^c(\cdot)$ on $I_2$ is reached at any point in $I_2$ with a maximal value equal to $\pi_{2i} = 0$.

To summarize the arguments, the maximization of $\pi_i^c(\cdot)$ in (8) leads us to compare

$$\pi_{1i} = \begin{cases} \phi_1(x_i^{ne}) & \text{if } 0 \le g \le g^m, \\ \phi_1(x^m) & \text{if } g^m \le g, \end{cases} \tag{27}$$

with

$$\pi_{2i} = \begin{cases} \phi_2(x_i^f) & \text{if } 0 \le g \le \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}, \\ \phi_2(x_i^{ne}) & \text{if } \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2} \le g \le \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}, \\ 0 & \text{if } \frac{(r-1)\bar{\theta}^2}{(2r-1)^2} \le g \le \frac{\bar{\theta}^2}{4r}, \\ +\infty & \text{if } \frac{\bar{\theta}^2}{4r} \le g. \end{cases} \tag{28}$$

Note that the parabola $\phi_1$ is always above the parabola $\phi_2$ on $[0,\,\bar{\theta}]$. The maximum of $\phi_1$ is reached at $x_i = x_i^m = \bar{\theta}/2$, whereas the maximum of $\phi_2$ is reached at $x_i = x_i^f$.

The situation of $\gamma(\cdot)$, relative to the points $\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}$, $\frac{(r-1)\bar{\theta}^2}{(2r-1)^2}$ and $\frac{\bar{\theta}^2}{4r}$, can be easily obtained. From the definition of $\gamma(\cdot)$ in (6), we have $g^m = \frac{\bar{\theta}^2}{16(r-1)}$ if $r \ge 3/2$, and $g^m = \frac{\bar{\theta}^2(2-r)}{4}$ if $1 < r \le 3/2$. This implies the inequalities $\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2} \le g^m \le \frac{\bar{\theta}^2}{4r}$, and we have that $\frac{(r-1)\bar{\theta}^2}{(2r-1)^2} <$ (resp. $>$) $g^m$ if and only if $1 < r < 3/2$ (resp. $3/2 < r$).

In the rest of the proof, we compare $\pi_{1i}$ with $\pi_{2i}$, by separating the arguments into several cases, corresponding to different intervals for the values of $g$.

(Case 1) Consider first any value $g \in [0,\, \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2})$. Expressions (27-28) imply $\pi_{1i} = \phi_1(x_i^{ne})$ and $\pi_{2i} = \phi_2(x_i^f) = \pi_i^f = \frac{q_i(r-1)\bar{\theta}^2}{2(2r-1)}$ and, from (7), we have $x_i^{ne} < x_i^f$. By solving the corresponding second-degree polynomial equation, we can find a unique point $x_i^0 = \bar{\theta}(1-(2r-1)^{-1/2})/2 < x_i^f$ such that $\phi_1(x_i^0) = \pi_i^f$. Since $\pi_i^f$ is the maximal value of $\phi_2(\cdot)$, we can see that $\phi_1(x_i^{ne})$ is lower (resp. higher) than $\phi_2(x_i^f) = \pi_i^f$ if and only if $x_i^{ne}$ is lower (resp. higher) than $x_i^0$. Additionally, from (7) we can show that $x_i^{ne}$ is lower (resp. higher) than $x_i^0$ if and only if $g$ is lower (resp. higher) than $g^{ne} \in \left(0,\, \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}\right)$, which is defined in (9).

For the present case, we conclude that the incumbent's equilibrium hedonic price is $x_i^* = x_i^f$ (and the pirate will enter) if $g < g^{ne}$, because then $\pi_i^f > \phi_1(x_i^{ne})$. When $g = g^{ne}$, it can be $x_i^* = x_i^{ne}$ (and the pirate will not enter) or, indifferently, $x_i^* = x_i^f$ (and the pirate will enter), because $\phi_1(x_i^{ne}) = \pi_i^f$. If $g^{ne} < g \le \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}$, we have $x_i^* = x_i^{ne}$ (and the pirate will not enter), because $\phi_1(x_i^{ne}) > \pi_i^f$.

Note that, although the pirate becomes indifferent between to enter or not to enter when the incumbent prices $x_i^{ne}$, it is necessary for the existence of SPE that the pirate choose not to enter for that hedonic price.

Whenever $g = \frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}$, the incumbent's optimal hedonic price must be $x_i^* = x_i^{ne}$ and the pirate must not enter. Here, we have $x_i^{ne} = x_i^f$, but $\phi_1(x_i^{ne}) > \pi_i^f$ holds.

(Case 2) Consider $g \in \left(\frac{(r-1)\bar{\theta}^2}{4(2r-1)^2}, \min(g^m, \frac{(r-1)\bar{\theta}^2}{(2r-1)^2})\right)$. Expressions (27-28) imply $\pi_{1i} = \phi_1(x_i^{ne}) > \phi_2(x_i^{ne}) = \pi_{i2}$. In this case, the incumbent will price $x_i^* = x_i^{ne}$ and the pirate will not enter in the SPE.

(Case 3, feasible for $3/2 \leq r$) Consider $g \in \left[g^m, \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}\right]$. Here expressions (27-28) leads us to $\pi_{1i} = \phi_1(x^m)$ and $\pi_{2i} = \phi_2(x_i^{ne})$. Evidently, the incumbent will choose $x_i^* = x_i^m$ and the pirate will not enter because $x_i^m < x_i^{ne}$.

(Case 4, feasible for $1 < r \leq 3/2$) Consider $g \in \left[\frac{(r-1)\bar{\theta}^2}{(2r-1)^2}, g^m\right]$. From (27-28), we obtain here $\pi_{1i} = \phi_1(x_i^{ne}) > 0 = \pi_{2i}$. Therefore, the incumbent will price $x_i^* = x_i^{ne}$, and the pirate will not enter.

(Case 5) Consider $g \in \left(\max(g^m, \frac{(r-1)\bar{\theta}^2}{(2r-1)^2}), \frac{\bar{\theta}^2}{4r}\right]$. From (27-28), we obtain here $\pi_{1i} = \phi_1(x_i^m) > 0 = \pi_{2i}$. Therefore, the incumbent will price $x_i^* = x_i^m$, and the pirate will not enter.

(Case 6) Consider $\frac{\bar{\theta}^2}{4r} < g$. In this case, not to enter will be optimal for the pirate given any incumbent's hedonic price. The incumbent will price $x_i^* = x_i^m$.

Finally, note that the critical values of $g$ that determine the incumbent's optimal hedonic price in $\{x_i^f, x_i^{ne}, x_i^m\}$ are $g^{ne}$ and $g^m$. ■

# Appendix B

**Proof of Proposition 4.** From (20-13-11) we take partial derivatives with respect to $q_i$ and $q_p$

$$\begin{aligned}
\frac{\partial p_{i0}^{ne}}{\partial q_i} &= \sqrt{\frac{\overline{\theta}^2(4r^2-3r+1)^2}{8r(2r-1)^3}} > 0 & \frac{\partial p_{i0}^{ne}}{\partial q_p} &= -\sqrt{\frac{\overline{\theta}^2 r(3r-1)^2}{8(2r-1)^3}} < 0 \\
\frac{\partial \theta_{o0}^{ne}}{\partial q_i} &= \sqrt{\frac{\overline{\theta}^2(3r-1)^2}{8q_i^2 r(2r-1)^3}} > 0 & \frac{\partial \theta_{oo}^{ne}}{\partial q_p} &= -\sqrt{\frac{\overline{\theta}^2 r(3r-1)^2}{8q_i^2(2r-1)^3}} < 0 \\
\frac{\partial \pi_{i0}^{ne}}{\partial q_i} &= \sqrt{\frac{\overline{\theta}^4(4r^2-3r+1)^2}{8r(2r-1)^3}} - \frac{\overline{\theta}^2 r(r-1)}{(2r-1)^2} > 0 & \frac{\partial \pi_{i0}^{ne}}{\partial q_p} &= -\sqrt{\frac{\overline{\theta}^4 q_i(3q_i-q_p)^2}{8(2q_i-q_p)^3}} + \frac{\overline{\theta}^2(3r-1)(r-1)}{2(2r-1)^2} < 0
\end{aligned} \tag{29}$$

$$\begin{aligned}
\frac{\partial p_i^l}{\partial q_i} &= \frac{\overline{\theta}\left(8r^2-8r+3\right)}{4(2r-1)^2} > 0 & \frac{\partial p_p^l}{\partial q_i} &= \frac{\overline{\theta}}{2(2r-1)^2} > 0 \\
\frac{\partial p_i^l}{\partial q_p} &= -\frac{\overline{\theta}(2r(3r-2)+1)}{4(2r-1)^2} < 0 & \frac{\partial p_p^l}{\partial q_p} &= \frac{\overline{\theta}\left(2r^2-4r+1\right)}{2(2r-1)^2} \lesseqgtr 0 \\
\frac{\partial \theta_o^l}{\partial q_i} &= \frac{\overline{\theta}}{2q_p(2r-1)^2} > 0 & \frac{\partial \theta_p^l}{\partial q_i} &= \frac{\overline{\theta}}{2q_p(2r-1)^2} > 0 \\
\frac{\partial \theta_o^l}{\partial q_p} &= -\frac{\overline{\theta} r}{2q_p(2r-1)^2} < 0 & \frac{\partial \theta_p^l}{\partial q_p} &= -\frac{\overline{\theta} r}{2q_p(2r-1)^2} < 0 \\
\frac{\partial \pi_i^l}{\partial q_i} &= \frac{\overline{\theta}^2(4r-1)((r-1)(8r-2)+3)}{16(2r-1)^3} > 0 & \frac{\partial \pi_i^l}{\partial q_p} &= -\frac{\overline{\theta}^2(4r-1)(2r(2r-1)+1)}{16(2r-1)^3} < 0 \\
\frac{\partial I_p^l}{\partial q_i} &= \frac{\overline{\theta}^2}{8(2r-1)^2} > 0 & \frac{\partial I_p^l}{\partial q_p} &= \frac{\overline{\theta}^2\left(2r^2-4r+1\right)}{8(2r-1)^2} \lesseqgtr 0
\end{aligned} \tag{30}$$

$$
\begin{array}{ll}
\frac{\partial p_i^f}{\partial q_i} = \frac{\overline{\theta}(2r(r-1)+1)}{(2r-1)^2} > 0 & \frac{\partial p_p^f}{\partial q_i} = \frac{\overline{\theta}}{2(2r-1)^2} > 0 \\
\frac{\partial p_i^f}{\partial q_p} = -\frac{\overline{\theta}r^2}{(2r-1)^2} < 0 & \frac{\partial p_p^f}{\partial q_p} = \frac{\overline{\theta}\left(2r^2-4r+1\right)}{2(2r-1)^2} \lesseqgtr 0 \\
\frac{\partial \theta_p^f}{\partial q_i} = \frac{\overline{\theta}}{2q_p(2r-1)^2} > 0 & \frac{\partial \pi_i^f}{\partial q_i} = \frac{\overline{\theta}^2(2r(r-1)+1)}{2(2r-1)^2} > 0 \\
\frac{\partial \theta_p^f}{\partial q_p} = -\frac{\overline{\theta}r}{2q_p(2r-1)^2} < 0 & \frac{\partial \pi_i^f}{\partial q_p} = -\frac{\overline{\theta}^2 r^2}{2(2r-1)^2} < 0 \\
\frac{\partial CS^f}{\partial q_i} = \frac{\partial CS^f}{\partial r} = \frac{\overline{\theta}^2\left(8r^3-12r^2+1\right)}{8(2r-1)^3} \gtreqless 0 & \frac{\partial I_p^f}{\partial q_i} = \frac{\overline{\theta}^2}{4(2r-1)^3} > 0 \\
\frac{\partial CS^f}{\partial q_p} = \frac{\overline{\theta}^2 r^2(10r-3)}{8(2r-1)^3} > 0 & \frac{\partial I_p^f}{\partial q_p} = \frac{\overline{\theta}^2 r^2(2r-3)}{4(2r-1)^3} \gtreqless 0 \\
\frac{\partial SW^f}{\partial q_i} = \frac{\overline{\theta}^2(12r(r-1)(2r-1)+4r-1)}{8(2r-1)^3} > 0 & \frac{\partial SW^f}{\partial q_p} = \frac{\overline{\theta}^2 r^2(6r-5)}{8(2r-1)^3} > 0
\end{array}
\qquad (31)
$$

■

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