Competition Versus Information under the Word-of-Mouth Effect*

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PRELIMINARY

Abstract

One of the most conspicuous changes in the economic environment over the last decades has been the increase in the number of brands available to consumers. In this paper we argue that if consumers' uncertainty about the quality of goods is significant, a larger number of brands may bring about less average information about quality; this in turn may lower the incentives to produce high-quality goods. The goal of this paper is to investigate this possible trade-off between the number of firms and the average quality of goods when informational issues are important. We focus on a monopolistic competition market where word-of-mouth is the source of reliable information. We obtain that, under some reasonable conditions, a larger number of firms reduces the average quality of goods in equilibrium, as well as consumers' surplus. Moreover, in a free-entry setting, we obtain that an increase in the entry cost of bad-quality producers may lead to an equilibrium with a higher proportion of bad-quality producers.

Keywords: Monopolistic Competition, Word of Mouth, Quality Uncertainty, Asymmetric Information.

JEL Classification: L13, L15

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1 Introduction

One of the most conspicuous changes in the economic environment over the last decades has been the increase in the number of brands and models available to consumers in most goods. Some technological changes making increasing returns to scale in production less important, the larger size of domestic markets allowing for a larger number of brands, and more importantly, the remarkable expansion of international trade are some of the reasons that have prompted this plethora of goods. The abundance of firms and brands is one of the economists' most cherished market conditions. It increases competition and consumer diversity, which usually means higher social welfare.[1] Whenever uncertainty about the quality of the goods in the market is significant, a larger number of brands and models may bring about less average information about the quality of the goods, however. In turn, less average information may lead to lower incentives for firms to produce high-quality goods. Hence, the increasing abundance of brands and models may have the negative consequence of a reduction in the average quality of goods. The goal of this paper is to investigate this possible trade-off. We will focus on markets where word-of-mouth (WOM) is an important source of information about the goods characteristics.

Uncertainty about good quality may be quite significant for those goods that are only sparsely purchased (such as durables; but also for quite different goods such as attorney services, travel companies, etc.). The flow of new models, brands and suppliers is so intense in many markets that when one is to buy a new unit (of a car, an electrical appliance, or a flight to an infrequent destination), the market supply of brands and models may have changed drastically and the model that one bought last time may not even exist anymore. In these markets, WOM recommendations are very important. WOM consists in the passing of information in an informal, person-to-person manner. WOM information has a key advantage with respect to other common sources of information about goods characteristics, such as advertising: the person providing the information does not have, in general, incentives to distort the information. This makes WOM information specially reliable. Moreover, the personal relationship and knowledge

[1] Though, as noted by Salop (1979) it may also convey higher aggregate costs and prices.

about the person giving the information helps appraising the value and significance of that information.[2] On the other hand, the main problem of WOM information is its limited scope (or more precisely, its inelasticity). Since there is no market for this information (which is a pure externality stemming from social relationships), it is inelastic to the increasing need of information raised by the increasing number of brands and models of the different goods. In other words, the flow of information provided by WOM tends to be proportional to the circle of relatives, friends, fellow workers, etc. independently of the informational needs (or the value of that information). Our main result in this paper is that, in this context, a larger number of firms in a market may reduce consumers' surplus due to the negative effect on consumers' average information on goods quality which in turn brings about a negative effect on the composition of total output in terms of quality.

Classical models of reputation assume that all consumers share the information about goods that have been previously sold by a monopolist.[3] The monopolist can choose the quality he wants to produce and sell, and the consumers' expectations about future quality depend on past quality. Thus, the monopolist may have incentives to build himself a reputation in order to charge a higher price in the future. More recently, Rob and Sekiguchi (2001) study reputation in the context of a duopoly model where firms choose a level of investment that influences their realized quality in a repeated game. However, none of these models consider the possibility that the information about past quality may be imperfect.

"Word-of-mouth" phenomena have been considered in game-theoretical models of learning (see for example Ellison and Fudenberg (1995) or Banerjee and Fudenberg (2004)). From the point of view of the industrial organization, the word-of-mouth effect has been investigated in the context of a monopoly by Kennedy (1994), Vettas (1997), and Navarro (2005) among others. While Kennedy analyzes the word-of-mouth communication combined with the role of prices as a signal of quality, Vettas emphasizes the

[2] Note that we are not considering as WOM information some other sources of information such as ratings and reviews in internet sites. These sources fail to grant the reliability of familiar person-to-person communication that we emphasize as a distinctive characteristic of WOM.

[3] See for example Klein and Leffler (1981) and Shapiro (1983).

role of quantities in the transmission of the word-of-mouth effect. In particular, Vettas (1997) studies a model where consumers learn about the quality of a good sold in the previous period with a probability proportional to the number of sales of the product. Therefore, producers take into account this communication when they decide each period production. A high quality firm spreads its production through time producing larger quantities in the first periods, while a low-quality producer randomizes the periods during which it produces a positive quantity. Navarro (2005) studies the incentives of a monopolist to produce high quality goods in a model where consumers may learn about the quality of the goods through the social network where they belong to. She shows that under some conditions a more "dense" social network may lower the incentives to produce high quality.

To the best of our knowledge, the analysis of word-of-mouth effect in the context of oligopoly has been restricted to duopoly models. In particular, the combination of the word-of-mouth effect with the informative role of market share has been considered in a two-period duopoly model by Caminal and Vives (1996). These authors illustrate each duopolist's incentive to increase its market share to signal that it produces a high-quality good, which involves an equilibrium which is more competitive than in the case where market shares do not play an informative role. In a more dynamic version or this duopoly model, Caminal and Vives (1999) show that consumers learn slowly about quality differentials and that prices converge also slowly to full-information levels. In those duopoly models it is assumed that some "rational" consumers update their believes about the quality of each firm, based on the previous information on market shares and on word-of-mouth communication, while other consumers make their decisions at random.

In contrast with those previous duopoly approaches, in our model we consider an oligopoly where the trade-off between the degree of competition (represented by the number of firms) and the accuracy of information (captured by the word-of-mouth effect) is explicitly considered. The basic assumption in our model is that the amount of information each consumer can obtain about past quality is limited. As a consequence, the signal about the quality of each product becomes noisier as the number of competitors increases, which tends to reduce consumers' surplus. In our analysis we identify the conditions under which this negative word-of-mouth effect, associated to a higher number of firms, dominates or is dominated by the positive effect that competition usually has on consumer's welfare.

The rest of the paper is organized as follows. In the following section we present our simplest model, taking the number and distribution of firms in the market as given. In Section 3 we extend the model to endogenize the number and distribution of firms, assuming free entry. In the last section we gather our conclusions.

2 A Simple Model

We consider the circular city model by Salop (1979) with two periods. Firms are uniformly distributed around a circular city with perimeter equal to 1 and set their prices simultaneously. There are n firms with identical cost functions $C(x_i) = cx_i$, where x_i is the production by firm i. Firms' output is horizontally differentiated and may also differ in quality (which may be either good, denoted $q = 1$, or bad $q = 0$). Moreover, firms may be high type H or low type L. High-type firms' output is always of good quality. However, only a fraction $\beta > 0$ of each low-type firm's output is of good quality (and thus, a fraction $1 - \beta$ of each firm's output is of bad quality). At the beginning of period 1, nature decides the type of each firm, which is high with probability $\Pr(H) = \rho$ and of low with probability $\Pr(L) = 1 - \rho$. We assume that β and ρ are common knowledge.

There is a continuum of consumers of density $S = 1$ located uniformly around the circle. As in Salop's model we assume a linear transportation cost tx, where x is the distance from the consumer to the chosen firm, and $t = 1$. Consumers wish to buy one unit of the good as long as its cost (price plus transportation) does not exceed the value they assign to the good, which we assume to be its expected quality.

In the first period, consumers ignore the type of each firm, and each one buys one unit of the good. We assume that each consumer is in contact with some $A - 1$ other consumers with whom he shares the information about the quality of the good they have consumed. We assume the number A to be exogenous (it would be given by the average number of friends, relatives, fellow workers, etc.) and that consumers distribute uniformly their purchases across firms in the first period (since the information they have about each firm is the same). Hence at the beginning of the second period, each consumer has a sample of size $m = A/n$ of the quality of each firm's output.

Based on this information the consumer updates, according to Bayes's rule, the probability that each firm is high or low type. Thus, as the number of firms increases, the sample corresponding to each firm output is smaller and the accuracy of information is reduced.

In the first period each firm produces good quality with (unconditional) probability $\Pr(q_i = 1) = \rho + (1-\rho)\beta$ and bad quality with probability $\Pr(q_i = 0) = (1-\rho)(1-\beta)$.

Therefore, we have

$$E_1(q_i) = \rho + (1-\rho)\beta$$

from the previous calculations it is easy to obtain the Nash equilibrium and the consumers' surplus associated to this equilibrium. However, we now focus on the second period in order to investigate the effect of the number of firms on the consumer surplus.

At the beginning of the second period, each consumer examines her samples. For each brand, there are two possibilities: 1) If $q_i = 0$ for at least one observation, then the consumer knows with certainty that the firm is of type L; 2) We denote by M the event that $q_i = 1$ for all the m observations. If M happens, then the (conditional) probabilities that the firm is of high or low type are, respectively,

$$\begin{aligned}
\Pr(H/M) &= \frac{\Pr(M/H)\cdot\Pr(H)}{\Pr(M/H)\cdot\Pr(H)+\Pr(M/L)\cdot\Pr(L)} = \frac{\rho}{\rho+(1-\rho)\beta^m} \\
\Pr(L/M) &= 1-\Pr(H/M) = 1-\frac{\rho}{\rho+(1-\rho)\beta^m}
\end{aligned}$$

Therefore, the probability distribution of q_i, conditional to M, is given by

$$\begin{aligned}
\Pr((q_i &= 1)/M) = [\Pr((q_i=1)/H)]\cdot[\Pr(H/M)] + [\Pr((q_i=1)/L]\cdot[\Pr(L/M)] \\
&= \frac{\rho}{\rho+(1-\rho)\beta^m} + \beta(1-\frac{\rho}{\rho+(1-\rho)\beta^m}) = \frac{\rho+(1-\rho)\beta^{m+1}}{\rho+(1-\rho)\beta^m} \\
\Pr((q_i &= 0)/M) = 1-\Pr((q_i=1)/M) = 1-\frac{\rho+(1-\rho)\beta^{m+1}}{\rho+(1-\rho)\beta^m}
\end{aligned}$$

We can also obtain the expectation of q_i conditional on M, for the second period:

$$E_2(q_i/M) = \frac{\rho}{\rho+(1-\rho)\beta^m} + \beta\left(1-\frac{\rho}{\rho+(1-\rho)\beta^m}\right)$$

we denote this expected quality by $a(m)$, so simplifying

$$a(m) = \frac{\rho + (1-\rho)\beta^{m+1}}{\rho + (1-\rho)\beta^{m}} \tag{1}$$

Differentiating $a(m)$ with respect to m, it is easy to see that the higher m, the higher the expected quality conditional on M, that is, $a'(m) > 0$.

Let us compare our results with those obtained in the limit case where A tends to infinity, given n. This is equivalent to assume that each consumer has complete information about each firm's type. In this case, standard calculations show that the expected quality of firm i, at period 2 is given by

$$\begin{aligned} E_2(q_i/H) &= 1, \\ E_2(q_i/L) &= \beta. \end{aligned}$$

When A is finite, the consumers know the identity of a fraction of the low-type firms: they are those who have sold at least one unit of bad product in the previous period. If a firm i produces bad quality goods, the probability that the consumers have not received any bad signal from this firm is β^m. Therefore, the consumers receive a good sample from all high quality firms and from a fraction β^m of the set of bad quality firms.

It is possible that L-type firms would like to sell their products at a different price than that of the H-type firms (of course, this would imply to set a price $P_l < \beta$). We assume that it is not in the interest of L-type firms to do so. A sufficient condition for this is $c > \beta$, (marginal cost greater than β, so that "bad" firms are not willing to set a separating equilibrium price that would allow them to sell all their products). We make this assumption.

Assumption 1: $c > \beta$.

Assumption 1 is not a necessary condition, but it is sufficient to eliminate the possibility of a separating equilibrium.[4] Note that this assumption implies that a consumer buying from an L-type firm has a negative expected utility, since the price he pays for the good is larger than the good's expected quality.

In the standard model of Salop's, the Nash equilibrium in prices implies:

$$P^* = \frac{t}{n} + c,$$

[4] In fact, it may happen that even if the L-type firms could sell at a price below β they prefer to mix up their products with those of the H-type firms, in order to charge a higher price for them.

and the consumer surplus is

$$CS = S\left(E(q) - P^{*}\right) - T(n),$$

where $E(q)$ is the good's expected quality, $T(n)$ the total transportation cost at equilibrium, $T(n) = \frac{S*t}{4n}$, and S the size of the market.[5]

>From the above expression we can easily derive the consumer surplus in both the first and the second period of our model. There are only two differences between them. First, the expected quality, which in the first period is unconditional on the initial number of firms, while in the second period it depends on it (or rather, on the size of the sample, m) and is given by equation (1). The second difference is the number of firms operating in each period: in the second period, a low quality producer can only sell its products if the consumers have not received a bad signal from it in the first period. Therefore, only a fraction β^m of the L-type firms can sell their products, while the rest are expelled from the market. Hence, in the second period there will only be n' firms, where $n' = [\rho + (1-\rho)\beta^m]\, n$.

Substituting $E(q)$, P^{*}, n, S, and $T(n)$ by its values, taking into account that $n = \frac{A}{m}$, and rearranging, we can write the consumer surplus in periods 1 and 2 as:

$$CS_1 = \rho + (1-\rho)\beta - \frac{5}{4}\frac{m}{A} - c \tag{2}$$

$$CS_2 = \frac{\rho + (1-\rho)\beta^{m+1} - \frac{5}{4}\frac{m}{A}}{\rho + (1-\rho)\beta^{m}} - c \tag{3}$$

Note that, given A, the larger m implies the smaller n and vice versa: $A = m * n$. A higher n implies more competition, a lower price and a lower average transportation cost, but it also implies a lower m, and therefore worse information and a lower expected quality in the second period. Differentiating equation (3) with respect to m, it is easy to derive Proposition 1, which establishes that in the second period the consumer surplus is maximum for an intermediate value of m:

Proposition 1 *For A large enough there is a unique m^{*} such that the consumer surplus given by equation (3) is strictly increasing in m for $m < m^{*}$*

[5]Note that a consumer faces a transportation cost of at most $\frac{1}{2n}$ (which is the maximal distance to the nearest firm). The aggregate transportation cost is therefore $\int_0^{1/2n} x dx$ at each side of each firm, which multiplied by n gives $\frac{1}{4n}$.

and strictly decreasing for $m > m^*$. Moreover, m^* is strictly increasing in A, $\frac{\partial m^*}{\partial A} > 0$.

Proof: See Appendix.

In Proposition 2 we have focused on the second period, when a fraction of the L-type firms operating in the first period has exited the market. In the first period even if there are two types of firms, the consumers cannot distinguish them, so that all goods have the same expected quality, $\rho + (1-\rho)\beta$. Thus, in the first period the consumer surplus is increasing in the number of firms. To consider the total consumer surplus we can weigh the surplus in each period with a parameter δ, that can be understood as a discount factor. We give weight δ to the profits in the first period and $1-\delta$ to those of the second period. As we have seen, the consumer surplus is decreasing in m in the first period (i.e. increasing in n) and has a bell shape in the second period. It is clear that it will be maximal for a large n if δ is close to 1, while if δ is close to zero our result will hold: a large n (small m) hurts the consumer surplus. An interesting interpretation of the parameter δ is as follows: we can consider that there is a *learning period* where some consumers (the most eager ones) buy the good. This would be our first period. In the second period the good becomes popular an everybody buys it. Then, δ and $1-\delta$ may be interpreted as the size of the market in the two periods, and in general, $1-\delta$ will be larger.

Note that one of the main conclusions of the Salop's model is that, without barriers to entry other that an entry cost, the market provides too many firms from the social welfare point of view. However, in Salop's model the consumer surplus is strictly increasing in the number of firms. In our framework this is not true any more because many firms reduce the quality of the information, thus preventing the consumers from choosing the best firms to buy from. Although we focus in the consumers' surplus, in our model social welfare must also be decreasing in the number of firms at some point, even when the entry cost is zero. This is so because at equilibrium the firms' profits go to zero, so that welfare depends only on the consumers' side. In Salop's model all consumers buy a unit of the good, so that welfare losses come only from too much entry (in the case of Salop's model) or too many L-type firms (in our model).

As in Salop, for existence of equilibrium we need to assume that all consumers buy their unit. For this to be true, the expected utility of a consumer must be positive. Therefore, we need that even the consumers that face the highest possible transportation cost have a positive expected utility. Thus, we need:

$$\rho + (1-\rho)\beta - \frac{3}{2n} - c > 0$$

si $\rho = \beta = c = \frac{1}{2} \Rightarrow$

$$\frac{1}{2} + \frac{1}{2}\frac{1}{2} - \frac{3}{2n} - \frac{1}{2} = \frac{1}{4} - \frac{3}{2n} > 0 \Leftrightarrow n > 6$$

3 Free Entry

We examine now the issue of entry in this model. We assume that the firms can decide whether they are high or low type. Let K_L and K_H be, respectively, the entry cost for L-type an H-type firms. We assume $K_H > K_L$, since otherwise all firms would choose to be H-type. We can interpret K as the necessary investment to develop the product. The expected profits must be the same, since otherwise there would only be a type in the market. In Salop's model, the firms' profits were $\frac{1}{n^2} - K$, where n was the number of firms and K the entry cost. In the second period the number of L-type firms in the market is reduced to $(1-\rho)\beta^m n$ and the total number of firms, n' is $[\rho + (1-\rho)\beta^m]\, n$. Hence, we can write the firms' expected profits as follows:

For firms type H :

$$\pi_H = \frac{\delta}{n^2} + \frac{1-\delta}{n^2\,[\rho + (1-\rho)\beta^m]^2} - K_H \tag{4}$$

For firms type L :

$$\pi_L = \frac{\delta}{n^2} + \frac{(1-\delta)\beta^m}{n^2\,[\rho + (1-\rho)\beta^m]^2} - K_L. \tag{5}$$

Note that only for some ranges of the parameters there will be equilibria with both L and H-type firms. For example, in the extreme case where $\delta = 1$, only the first-period profits count, so word of mouth is irrelevant and all firms will choose to be type L. On the other hand, if δ or β are very small, or if K_L is close to $K_{H,}$ all firms will choose to be type H.

At equilibrium we must have $\pi_L = \pi_H = 0$. The first equality implies

$$\frac{1-\delta}{n^2\left[\rho+(1-\rho)\beta^m\right]^2} = \frac{K_H - K_L}{(1-\beta^m)}$$

which combined with the second equality, $\pi_H = 0$ yields (substituting the left hand side above in equation (4))

$$K_L - \beta^m K_H = \frac{\delta\left(1-\beta^m\right)}{n^2}.$$

Substituting n, dividing by K_H and denoting $\frac{K_L}{K_H}$ by γ we have:

$$\gamma - \beta^m = \frac{\delta\left(1-\beta^m\right)}{K_H}\frac{m^2}{A^2}. \tag{6}$$

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Tomemos: $\beta = 0.5, K_L = 0.05, K_H = 0.1,\ \delta = 0.5,\ A = 30;$
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Denote the left hand side above by $g(m)$ and the right hand side by $f(m)$. It can be proved that $g(m)$ is increasing in m and concave, while $f(m)$ is increasing in m and convex.[6] Figure 1 shows $f(m)$ and $g(m)$.

Also, we had: $\pi_H = \frac{\delta}{n^2} + \frac{1-\delta}{n^2[\rho+(1-\rho)\beta^m]^2} = K_H$, so $\delta + \frac{1-\delta}{[\rho+(1-\rho)\beta^m]^2} = n^2 K_H$.

Note that when $m = 0$ we have $f(m) = 0$ and $g(m) < 0$. Therefore, in general, $f(m)$ and $g(m)$ cross twice. However, the first point may imply $m < 1$ which in our model makes little sense. On the other hand, the second crossing may not take place if A is small (m cannot be greater than A and it may happen that $f(A) < g(A)$). It may also happen that they do not cross at all: this happens if $f(m) > g(m)\ \forall m > 1$, that is, if $\delta\left(1-\beta^m\right)\frac{m^2}{A^2} > K_L - \beta^m K_H\ \forall m > 1$.

Consider by the moment the simple case where $\delta = 0$. In this case, the functions $f(m)$ and $g(m)$ cross only once: $f(m)$ is constant and equal to

[6] See proof of Proposition 2.

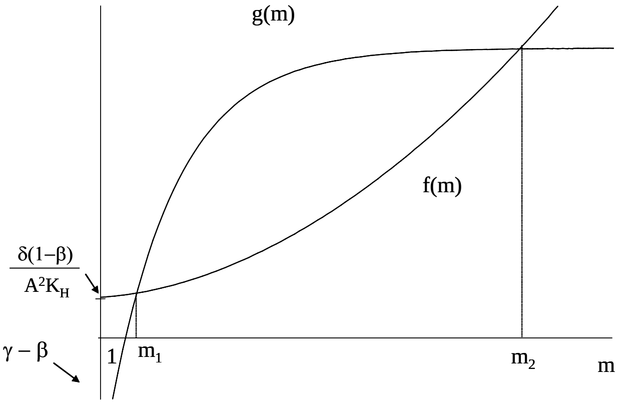

Figure 1: Functions $f(m)$ and $g(m)$ when two equilibria exist.

zero, so the equilibrium is where $g(m)$ crosses the horizontal axis. We have:

$$K_L = K_H \beta^m \tag{7}$$

$$K_H = \frac{m^2}{A^2} \frac{1}{[\rho + (1-\rho)\beta^m]^2} \tag{8}$$

Since, given β, equation (7) above determines m, ρ can be computed from equation (8), and

$$\begin{aligned} K_H &= \frac{m^2}{A^2} \frac{1}{\left[\rho + (1-\rho)\frac{K_L}{K_H}\right]^2}, \\ \rho &= \frac{\frac{m}{A}\sqrt{K_H} - K_L}{K_H - K_L}, \\ \frac{d\rho}{dK_L} &= \frac{-K_H + \frac{m}{A}\sqrt{K_H}}{(K_H - K_L)^2}. \\ \text{And since } K_H &= \frac{m^2}{A^2} \frac{1}{[\rho + (1-\rho)\beta^m]^2} > \frac{m^2}{A^2} \\ \text{we have } \frac{d\rho}{dK_L} &< 0. \end{aligned}$$

Note that, by equation (7) above, if K_L increases, m must decrease and therefore n must increase. Paradoxically, this increase must be mainly fed by the entry of L-type firms, since ρ must decrease. Hence, when K_L increases, the new equilibrium implies more low-quality firms entering the markets, which is somewhat counterintuitive. The reason is as follows: more bad-quality firms in the first period reduce the quality of the signal in the second period, allowing the bad firms to sell more (or, in other words, reducing the probability of a L-firm to be detected). When $\delta = 0$, only this second period matters, but the result holds as well for low values of δ.

Let's consider now the case $\delta \neq 0$. As we saw, there may be two equilibria. Assume we are initially in the one in the left. Suppose now K_L increases. Thus, γ increases and the $g(m)$ function moves upwards. As in the case where δ is zero, the new equilibrium will imply a lower m. Take now the zero profit condition for H-type firms. We had:

$$K_H = \frac{\delta}{n^2} + \frac{1-\delta}{n^2\left[\rho + (1-\rho)\beta^m\right]^2} \tag{9}$$

Since m decreases, n must increase. Both things make the two terms in the right hand side above smaller. The only possibility to compensate this is that ρ decreases, so that we are in a similar situation as when $\delta = 0$. An increase in K_L leads to a new equilibrium where m is lower, n larger, and ρ smaller. If we start in the other equilibrium however, the opposite happens: m increases, so that n decreases and ρ must increase.

Proposition 2 *Given the parameters K_L, K_H, δ, and β, there may be at most two equilibria where L-type and H-type firms coexist, defined by (m_1, n_1, ρ_1) and (m_2, n_2, ρ_2), where $m_1 < m_2$ and the following comparative statics hold:*
a) $\frac{\partial m_1}{\partial K_L} < 0, \frac{\partial n_1}{\partial K_L} > 0, \frac{\partial \rho_1}{\partial K_L} < 0$ *and*
b) $\frac{\partial m_2}{\partial K_L} > 0, \frac{\partial n_2}{\partial K_L} < 0,$ *and* $\frac{\partial \rho_2}{\partial K_L} > 0.$

Proof: See Appendix.

The different possibilities about existence of the two type of equilibria considered in Proposition 2 are illustrated in Figure 1 (both equilibria exist), Figure 2 (only m_1 is an equilibrium), Figure 3 (only m_2 is an equilibrium) and Figure 4 (no equilibrium with both type of firms exists).[7]

It is interesting to note that existence of the first type of equilibrium is ensured by $\frac{K_L}{K_H} < \beta$ and δ small. Intuitively, an equilibrium with low-type active firms is more likely to exist if relative entry costs for those firms

[7]It may also happen that $f(m) > g(m)$ $\forall m$.

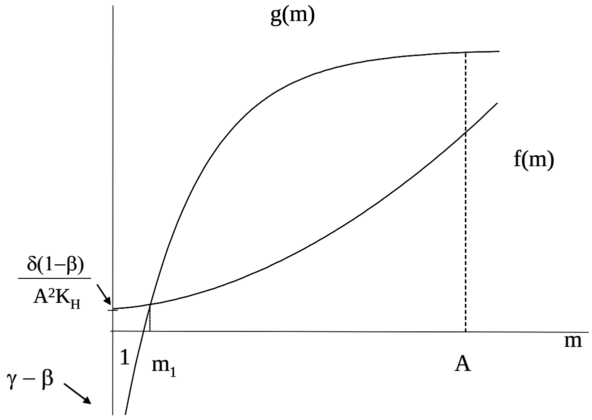

Figure 2: Only equilibrium m_1 exists.

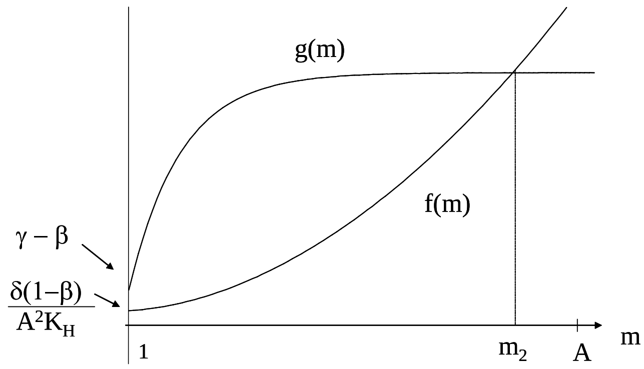

Figure 3: Only equilibrium m_2 exists.

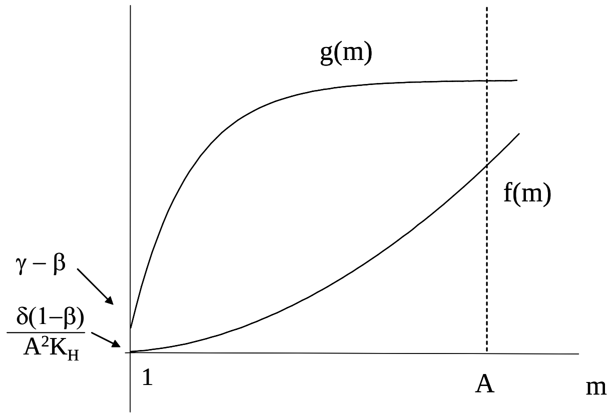

Figure 4: Given the parameters K_L, K_H, δ, and β, no equilibrium exists.

are low, and their probability of producing a good quality product is large. However, a small value of δ makes less likely the existence of the second type of equilibrium, since $f(m)$ is flatter (see Figure 2). The intuition is that if the relative importance of the "learning period" is small then it is more difficult to sustain an equilibrium with well informed consumers (m large) and many active low-type firms.

Our previous result has an interesting implication regarding the evolution of markets as communication about quality improves. According to our results, if the "learning period" in which consumers obtain the information about product's quality is reduced (e.g., because of improvements in information technologies) then it is more likely that the only equilibrium with both types of firms is the one with many competitors but with very noisy signals about quality.

4 Welfare

We can compare the two equilibria obtained above in terms of welfare. Note that production by L-type firms is always bad news: the production cost is higher than the average quality, which is the consumer's valuation, so L-type firms should not produce at all in the social planner's optimum. In our equilibrium above, the only reason why the equilibrium in the left of Figure 1 may be better than the one in the right is that in the later there may be too few firms, and that increases both the prices and the transportation costs that consumers face. The next Proposition states a mild condition on the number of H-type firms (with relation to β) so that the equilibrium with less firms is better than the other one in terms of social welfare.

Let n_H and n_L denote, respectively, the number of H-type and L-type firms. Obviously, $n = n_L + n_H$, and $n_H = \rho n,\ n_L = (1-\rho)\,n$.

Proposition 3 *When two equilibria exist, if $n_H(1-\beta) > \frac{5}{4}$, then the equilibrium with the lowest number of firms yields a higher social welfare than the other one.*

Proof: See Appendix.

We give here a sketch of the proof: we prove that, as long as $n_H(1-\beta) > \frac{5}{4}$, the consumer surplus is decreasing in the number of L-type firms, given the number of H-type firms. This is true for the two periods. On the contrary, given the number of L-type firms, the consumer surplus is increasing in the number of H-type firms (again in both periods). Then we show that n_H is larger in the equilibrium with a smaller n (note that n_H is constant through periods, since no H-type firms exit the market), while n_L, the number of L-type firms, is larger in the equilibrium in the left in both periods. It follows that the consumer surplus is larger in the equilibrium in the right. Since the firms' expected profits are zero in both equilibria, social welfare and consumer surplus are equivalent.

5 Conclusions

One of the most conspicuous changes in the economic environment over the last decades has been the increase in the number of brands and models available to consumers in most goods. We have shown that if consumers' uncertainty about the quality of the goods in the market is significant, a larger number of brands and models may bring about less average information about the quality of the goods. Our main insight, within a framework with an exogenous number of firms is that more competition (reflected in an increase in the number of competing firms) is not necessarily good news from the point of view of consumers. More competition may be associated with noisier signals about the quality of each product, which might outweigh the beneficial effect of competition. With asymmetric information problems, higher abundance of brands and models may bring about a reduction in the average quality of goods. Furthermore, in the long run and assuming free entry, increasing the relative entry cost of low-quality firms might have the paradoxical outcome of increasing the number of active low-quality firms and lowering the proportion of high-quality competitors. We also obtain that technological progress allowing more rapid learning by consumers might also have the paradoxical consequence of reducing the accuracy of the information obtained by those consumers.

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6 Appendix

Proof of Proposition 1:
Denote the numerator in expression (3) by x, and the denominator by y. The derivative of the consumer surplus is $\frac{x'y-xy'}{y^2}$, where
$x' = (\ln\beta)\,\beta^{m+1}(1-\rho) - \frac{5}{4}\frac{1}{A}$, $y' = (\ln\beta)\,\beta^{m}(1-\rho)$,
and its sign depends on the sign of $x'y-xy'$. Simplifying, we have: $sign\frac{dCS}{dm} =$

$$sign\left\{-(\ln\beta)\,\beta^m(1-\rho)\,\rho\,(1-\beta) + \frac{5}{4}\frac{m}{A}\left[(\ln\beta)\,\beta^m(1-\rho) - [\rho+(1-\rho)\beta^m]\right]\right\}.$$

When $m = 1$, we have: $sign\frac{dCS}{dm}\mid_{m=1} =$

$$sign\left\{-(\ln\beta)\,\beta\,(1-\rho)\,\rho\,(1-\beta) + \frac{5}{4}\frac{1}{A}\left[(\ln\beta)\,\beta\,(1-\rho) - [\rho+(1-\rho)\beta]\right]\right\}.$$

Since $(\ln\beta) < 0$ it is clear that the first term is positive, while the second term (negative) goes to zero when A is large. Thus

$$if\ A\to\infty,\ \text{then}\ sign\frac{dCS}{dm}\mid_{m=1} > 0$$

On the other hand, let $m = A$ (m large): $sign\frac{dCS}{dm}\mid_{m=A} =$
$sign\left\{-(\ln\beta)\,\beta^A(1-\rho)\,\rho\,(1-\beta) + \frac{5}{4}\left[(\ln\beta)\,\beta^A(1-\rho) - \left[\rho+(1-\rho)\beta^A\right]\right]\right\}$

$$= sign\left\{\beta^A(1-\rho)\left[(\ln\beta)\left(\frac{5}{4} - \rho\,(1-\beta)\right) - \frac{5}{4}\right] - \frac{5}{4}\rho\right\}$$

which is negative, since all terms above are so.

We know that for low m, CS is increasing on m, and for large m, it is decreasing. Therefore, CS must have a least one maximum. We now show that this maximum is unique: the derivative of the consumer surplus is positive (negative) if and only if $x'y > (<)xy'$, and the consumer surplus has a maximum when $\frac{x'}{y'} = \frac{x}{y}$, that is, when the curve $\frac{x'}{y'}$ cuts $\frac{x}{y}$ (note that $\frac{x}{y}$ is the consumer surplus). But $\frac{x'}{y'} = \beta - \frac{5}{4A(\ln\beta)\beta^m(1-\rho)}$, which is strictly increasing in m. Therefore, $\frac{x'}{y'}$ can only cut $\frac{x}{y}$ once, since that can only happen where $\frac{x}{y}$ has an interior maximum or minimum. Thus, the consumer surplus has a unique maximum.

Proof that m^* is strictly increasing in A, $\frac{\partial m^*}{\partial A} > 0$: It is easy to see that $\frac{d}{dA}(x'y - xy') > 0$. Suppose m_1 maximizes the consumer surplus when

$A = A_1$. If A increases, then $x'y - xy' \mid_{m_1} > 0$, therefore the slope of the CS is strictly positive and the new maximum must be at the right of m_1.

Proof of Proposition 2:
It is clear that both $f(m)$ and $g(m)$ are increasing in m and that $g(m)$ is concave. We show below that $f(m)$ is convex. Therefore the functions $f(m)$ and $g(m)$ can only cross twice.

Note that $f(m) = \frac{\delta(1-\beta^m)}{K_H}\frac{m^2}{A^2} = C * (1-\beta^m)\, m^2$, where $C = \frac{\delta}{K_H A^2}$ is a positive constant. We show that $f''(m) > 0$.

$$\begin{aligned} \frac{f''(m)}{C} &= \frac{d^2}{dm^2}(1-\beta^m)\, m^2 = 2 - 4m(\ln\beta)\,\beta^m - m^2\left(\ln^2\beta\right)\beta^m - 2\beta^m \\ &= \beta^m\left[\frac{2}{\beta^m} - 4m(\ln\beta) - m^2\left(\ln^2\beta\right) - 2\right] \end{aligned}$$

Denote the term in brackets above by $H(m)$; It is clear that $H(0) = 0$. We show that $H'(m) > 0$ so that $H(m) > 0$ $\forall m > 0$.

$$H'(m) = -2\ln\beta\left[2 + m\ln\beta + \frac{1}{\beta^m}\right]$$

Since $\ln\beta < 0$ we only need to show that $2 + m\ln\beta + \frac{1}{\beta^m} > 0$ or $-m\beta^m\ln\beta < 2\beta^m + 1$. Let $q(m) = -m\beta^m\ln\beta$. It is easy to see that $q(m)$ has a maximum at $m = \frac{-1}{(\ln\beta)}$. Thus $q(m) \leq -\frac{-1}{(\ln\beta)}\beta^m\ln\beta = \beta^m$. Hence, $q(m) \leq \beta^m < 2\beta^m + 1$.

The comparative statics regarding m_1 and m_2 comes from equation (6) and Figure 1, by noticing that $g(m)$ moves upwards as $\gamma = \frac{K_L}{K_H}$ increases. Comparative statics about n_1, n_2, $\rho_1$and ρ_2 are obtained from condition $n = \frac{A}{m}$ and equation (9), taking into account that this equation gives an inverse implicit relationship between n and ρ.

Proof of Proposition 3:
Denote by $\pi^x_{t,eq}$ the expected profit of a firm type $x = L,\ H$ at period $t = 1, 2$ in equilibrium eq ($eq = L,\ R$). It is clear that in the first period the firms' expected profit is identical:

a) $\pi^L_{1,eq} = \pi^H_{1,eq}$. Since the number of firms is larger in equilibrium L, it follows that:

b) $\pi^x_{1,L} < \pi^x_{1,R}$. Since $\pi^H_{1,L} + \pi^H_{2,L} = \pi^H_{1,R} + \pi^H_{2,R}$, by b) it must be true that

c) $\pi^H_{2,L} > \pi^H_{2,R}$. Since the expected quality is higher in equilibrium R, the only way the firms' expected profits may be higher in equilibrium L is that the number of firms is lower in this equilibrium (which allows the firms to charge a higher price in spite of the lowest expected quality):

d) $n_{2,L} < n_{2,R}$ or $n'_L < n'_R$, which means $\left[\rho_L + (1-\rho_L)\beta^{m_L}\right] n_L < \left[\rho_R + (1-\rho_R)\beta^{m_R}\right] n_R$. It is clear that $(1-\rho_L)\beta^{m_L} n_L > (1-\rho_R)\beta^{m_R} n_R$ (every term multiplying in the left is larger than its counterpart in the right). It follows that $\rho_L n_L < \rho_R n_R$.

We prove now the signs of the derivatives of the CS with respect to n_L and n_H. We can rewrite equation (2) as

$$CS_1 = \frac{n_H}{n_L + n_H} + \frac{n_L}{n_L + n_H}\beta - \frac{5}{4\,(n_L + n_H)} - c$$

It is easy to see that:

$\frac{dCS_1}{dn_L} = \frac{1}{(n_L+n_H)^2}\left[-n_H(1-\beta) + \frac{5}{4}\right]$; $\frac{dCS_1}{dn_L} < 0 \Leftrightarrow n_H(1-\beta) > \frac{5}{4}$.

$\frac{dCS_1}{dn_H} = \frac{1}{(n_L+n_H)^2}\left[n_L(1-\beta) + \frac{5}{4}\right] > 0.$

On the other hand, in the second period we have:

$$CS_2 = \frac{n_H + n_L\beta^{\frac{A}{n_H+n_L}+1} - \frac{5}{4}}{n_H + n_L\beta^{\frac{A}{n_H+n_L}}} - c$$

Let

$$\begin{aligned} X &= \beta^{\frac{A}{n_H+n_L}} \\ X_{+1} &= \beta^{\frac{A}{n_H+n_L}+1} = \beta X \\ Y &= n_H + n_L\beta^{\frac{A}{n_H+n_L}} = n_H + n_L X \\ Y_{+1} &= n_H + n_L\beta^{\frac{A}{n_H+n_L}+1} = n_H + n_L\beta X \end{aligned}$$

Note that:

$$\begin{aligned} X' &= \frac{-A}{(n_H+n_L)^2}(\ln\beta)\,X \\ (X_{+1})' &= \beta X' \\ Y' &= n_L X' + X \\ (Y_{+1})' &= n_L\,(X_{+1})' + X_{+1} = \beta Y' \end{aligned}$$

We have:

$$\frac{dCS_2}{dn_L} = \frac{(Y_{+1})' Y - \left(Y_{+1} - \frac{5}{4}\right) Y'}{Y^2}$$

$$\begin{aligned}
Sign\frac{dCS_2}{dn_L} &= Sign\left[(Y_{+1})' Y - \left(Y_{+1} - \frac{5}{4}\right) Y'\right] \\
&= Sign\left[\left(\beta Y' Y - Y_{+1} Y' + \frac{5}{4} Y'\right)\right] \\
&= Sign\left[\left(\beta Y - Y_{+1} + \frac{5}{4}\right)\right] \\
&= Sign\left[\beta\left(n_H + n_L X\right) - \left(n_H + n_L \beta X\right) + \frac{5}{4}\right]
\end{aligned}$$

so

$$Sign\frac{dCS_2}{dn_L} = Sign\left[\frac{5}{4} - n_H\left(1 - \beta\right)\right]$$

hence, $\frac{dCS_2}{dn_L} < 0$ iff $n_H\left(1-\beta\right) > \frac{5}{4}$.

Following a similar approach, it is easy to prove that $\frac{dCS_2}{dn_H} > 0$