Learning about compliance under asymmetric information*

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Abstract

Over time, inspection agencies gather information about firms that cause harmful externalities. This information may allow agencies to differentiate their monitoring strategies in the future, since inspections can be influenced by firms' past performance relative to other competitors in the market. If a firm is less successful than its peers in reducing the externality, it faces the risk of being targeted for increased inspections in the next period. This risk of stricter monitoring might induce high cost firms to mimic low cost firms, while the latter might try to avoid being mimicked. We show that under certain circumstances, mimicking, or even the threat of mimicking, might reduce socially harmful activities and thus be welfare improving.

Keywords: Monitoring and enforcement; externalities; learning; mimicking.

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I. INTRODUCTION

Public administrations design regulations to limit negative externalities arising from production, such as health damages, noise levels, worker safety or environmental pollution. According to Becker's (1968) theory of rational crime, profit-maximizing firms comply with these regulations if the expected penalty of violating exceeds compliance costs. One of the implications of Becker's model is the expectation of widespread non-compliance among firms, if expected sanctions for crimes are small.

For example, expected penalties for environmental crime are generally too lax. Ogus and Abbot (2002) indicate that the UK criminal justice system is in practice characterized by the low number of prosecutions brought by the agency and the failure of the courts to impose fines that reflect the nature of the offence. Although the normal response to major incidents is prosecution, only 23% of such incidents, where the offender was identified, lead to such action being taken and in 17% of the cases no action was taken at all. Also, the average fine for prosecuted businesses and individuals was £6,800 and £1,000 respectively in 1999. Despite these apparently low enforcement levels, substantial compliance among firms is observed in reality. This apparent paradox between low enforcement and high compliance rates has lead to several possible alternative explanations such as self-reporting (Livernois and McKenna, 1999) or regulatory dealing (Heyes and Rickman, 1999). For an overview of this literature we refer to Cohen (1999).

When considering compliance, firms obviously take their control costs, the stringency of the regulations as well as monitoring and enforcement policies, into account. If the inspection agency has a limited available budget (which is normally the case), its monitoring and enforcement strategy might not only be determined by the firms' past compliance status, but also by their performance relative to other competitors in the market. Therefore, if a firm is less successful than its peers in reducing its harmful effect on society, it faces the risk of being targeted for increased inspections in the following period. Knowing that future regulations might be contingent on this relative position of the agents with respect to externality levels, *bad* firms might be tempted to mimic *good* firms, and *good* firms might be tempted to prevent being mimicked by *bad* firms. As a result, actual external costs might be considerably lower than those expected in a static regulatory process.

In this paper we investigate the conditions under which incorporating learning in the regulatory strategy, as well as considering the potential imitation (or threat of imitation) by firms, may result in higher compliance rates than expected and may even be welfare improving for society.

We consider a two-period regulatory model with asymmetric information. The agency is given a budget *per* period to enforce a standard (fixed by law) in an industry composed of high-cost (*bad*) firms and low-cost (*good*) firms. Initially, the agency knows nothing about the type of firms it is dealing with. However, it can collect information on the type of firms by performing audits and measuring externality levels. So, if high-cost firms and low-cost firms choose different externality levels in the first period, this allows the agency to differentiate its monitoring strategy in the second period. The high-cost firms therefore have an incentive to avoid this differentiation by mimicking the behavior of low-cost firms in the first period. However, since imitation is costly for the imitator and since the low-cost firms might try to prevent imitation, such pooling of firms' decisions only takes place under specific circumstances, which depend on the agency's budget, the number of firms in the industry and the monitoring costs.

Note that in this paper we do not deal with an adverse selection model or even a contract setting, because the agency is not offering different contracts to the firms in order to elicit truthful revelation of their types. In fact, the agency cannot offer incentive compatible contracts because it only has one decision variable, namely, the frequency of inspections. Obviously, the lower the inspection probability, the lower firms' expected control costs, and therefore, firms targeted with a larger probability would be tempted to hide their true type. In our setting, the agency simply reacts to firms' compliance decisions and uses all the information that is available by setting its only decision variable – the inspection frequency – as efficiently as possible.

Despite its intuitive appeal, this problem has not been analyzed before in the literature. However, closely related studies are Greenberg (1984), Landsberger and Meilijson (1982) and Harrington (1988), who investigate the relationship between firms' compliance costs and the average level of compliance that can be achieved when both enforcement budgets and the maximum feasible penalty are limited. Enforcement can be made more efficient by dividing firms into groups, contingent on each firms' past performance, and then subject the recent violators to a stricter monitoring and sanctioning policy than the others. However, these papers do not consider the mimicking – avoid mimicking game described above.

Research concerning mimicking behavior can be also found among the economic literature on contract theory. Laffont and Tirole (1988), for instance, studied a simple two-period principal/agent model in which the principal updates the incentive scheme after observing the agent's first-period performance. The agent has superior information about his ability. The principal offers a first period incentive scheme and observes some measure of the agent's first-period performance (cost or profit), which depends on the agent's ability and (unobservable) first-period effort. The relationship is entirely run by short-term contracts. In the second period the principal updates the incentive scheme and the agent is free to accept the new incentive scheme or to quit. The strategies are required to be perfect, and updating of the principal's beliefs about the agent's ability follows Bayes' rule. Laffont and Tirole (1988 and 1990) emphasized a kind of "reverse" incentive constraint: whereas the usual incentive problem consists in preventing *good* types from hiding behind *bad* ones; under spot contracting, *bad* types may pretend they are *good*. In this case, the principal cannot promise to leave a rent to the agent in the future. Informational rents must therefore be concentrated in early periods, and this gives a bad agent an incentive to mimic a good one in the first periods, capture the rent, and then break the relationship. In our setting, however, firms cannot opt out on inspections. The regulation on externality control (e.g. noise limits or emission bounds) is mandatory and any firm can potentially be monitored by the agency.

Mimicking is also related to avoidance activities. Firms have numerous options to avoid apprehension and prosecution (Innes, 2005): they can flee the scene, they can lobby politicians to relax enforcement activities, or they can distance themselves from a violation by using legal means (e.g. exploiting international corporate shells). Malik (1990) shows that one implication of incorporating avoidance behavior is that penalties need not always be set as high as possible. An important strand of literature dealing with avoidance activities is the tax evasion literature. For a recent overview, we refer to Slemrod and Yitzhaki (2002). However, key to the definition of avoidance is the assumption that avoidance is socially detrimental. This is not the case with the mimicking behavior we study here. Since high-cost firms pretend to be low-cost firms, they reduce the level of the negative externality more than they would otherwise do. Under certain circumstances, mimicking, or even the threat of mimicking, might reduce the socially harmful effects of production and be welfare improving.

The remainder of the paper is organized as follows. Section 2 describes the model and the assumptions we make. Section 3 investigates the one-period model, while section 4 deals with the two-period model. In section 5 we study the likelihood of each possible type of equilibrium, depending on the parameters of the model. Section 6 concludes.

II. THE MODEL

We consider an industry composed of N firms that face a negative production externality (such as discharges of hazardous substances, smog precursors or noise). Each firm i can reduce this externality at a cost, which depends on the firms' externality level $e \geq 0$, and also on a parameter θ_i, which defines the firms' type. We assume that the externality control cost function of firm *i* is $c(\theta_i, e)$, with $\theta_i \in \{\theta_H, \theta_L\}$, $\theta_H > \theta_L$, and that it has the usual specification: $c_e(\theta_i, e) < 0$ and $c_{ee}(\theta_i, e) > 0$. We also assume that $c_{\theta_i}(\theta_i, e) > 0$, $c_{\theta_i e}(\theta_i, e) < 0$ and $c_{\theta_i ee}(\theta_i, e) \leq 0$[1]. Therefore, there are two types of firms in the industry: high-cost firms (θ_H) and low-cost firms (θ_L). We assume that the number of high-cost (low-cost) firms is N_H (N_L), such that $N_H + N_L = N$.

We assume that there is a regulation in place, which imposes a uniform externality limit or a standard $\bar{e} > 0$ on the firms. The stringency of the standard and the associated fine,

[1] As we will see later on in equation (1), the relative impact of low-cost firms on the externality is higher than that of high-cost firms if $c_{\theta_i ee}(\theta_i, e) > 0$. This implies that the agency targets known low-cost firms more than known high-cost firms. All our results are then reversed; low-cost firms have incentives to mimic high-cost firms and high-cost firms try to deter imitation. Successful imitation and successful active deterrence of imitation both lead to higher externality levels. Therefore, mimicking and learning might be welfare reducing.

in case a firm is discovered exceeding the standard, are determined by law. For simplicity, the fine F is assumed to be linear:[2]

$$F = f \max\{0; (e - \bar{e})\}, \qquad f > 0.$$

There exists a regulatory agency whose job it is to minimize total external costs caused by the industry, using the regulation in place. This agency has a budget $B > 0$ *per* period (say, *per* year) to spend on monitoring. We assume that the cost *per* inspection is $m > 0$ and that monitoring is perfectly accurate. If firm i is inspected in period j with probability p_{ji} such that $0 \leq p_{ji} \leq 1$, we then have:

$$B \geq m\left(N_H p_{jH} + N_L p_{jL}\right).$$

For simplicity we assume that first a law announces the standard and the fine, and this announcement is followed by two regulatory periods. In each period, the regulatory agency announces the probability of inspection and then each firm reacts by choosing the level of the externality. Once the agency has observed the behavior of the inspected firm, it can update its information about the firms in the next regulatory period. The chronology of decisions is represented in figure 1.

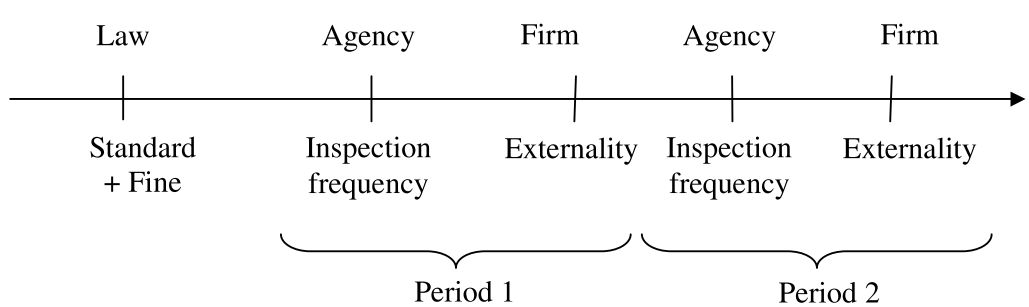

Figure 1: Chronology of decisions

[2] In practice, a linear specification of fines is often encountered for civil fines, since this structure is easy to understand by firms, citizens and administrations. For example, the EPA's Clean Air Act Stationary Source Civil Penalty Policy (1991) describes the civil fines for violating air pollution standards as *"$5000 for each 30% or fraction of 30% increment above the standard"*.

While firms are fully informed about their type (and the types of their partners in the industry), the inspection agency, however, is not. In the first period, the agency is unfamiliar with the regulated firms and it does not have any information on the type of the firms. The agency is not only ignorant of the exact type of each firm; it also has no information about the distribution of the firms' types across the industry. Therefore, the best the agency can do is to inspect all the firms with the same frequency, $p_1 = p_{1H} = p_{1L}$.

The agency can, however, obtain information on a firm's type, because we assume that externality levels are accurately measured throughout the inspections. Hence, if the agency finds that all inspected firms select the same level of the externality in period 1, the inspection agency cannot update its information (that is, it cannot learn). Therefore, in the second period, the agency will continue to use a uniform inspection frequency, $p_2 = p_{2H} = p_{2L}$, where $p_2 = p_1$. However, if the inspected firms in period 1 were found to have chosen different externality levels, the agency can learn whether the inspected firm is of the high or the low type. Moreover, this is not the only piece of information gained by the agency; now it also has a better estimate of the distribution of firm types across the industry. We assume that the proportion of inspected firms that turned out to be of the high type in the first period is an unbiased estimator of the true proportion of high-control cost firms in the industry. Thus, if n_H (n_L) is the number of inspected firms that appear to be of the high (low) type in the first period (such that $n_H + n_L = p_1 N$), we have $\frac{n_H}{n_H + n_L} = \frac{N_H}{N}$.

Now the agency is confronted with three groups of firms, as depicted in figure 2: known high-cost firms, known low-cost firms and firms the agency knows nothing about. The agency can therefore decide to differentiate its inspection strategy and treat each group differently, inspecting with probabilities p_{2H}, p_{2L} and p_{2N}, respectively. We are therefore assuming that the regulator cannot commit himself to not use the information conveyed by the firms' first-period performance in the second period. The simplest way to substantiate this assumption is the changing regulatory framework, such as the fact that the current administration cannot bind future ones.

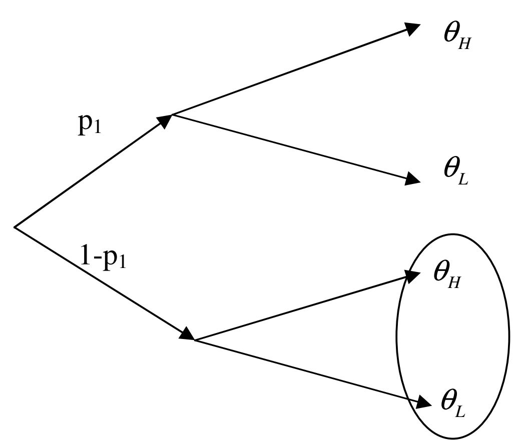

Figure 2

Regarding payoffs, firms choose externality levels that minimize discounted expected costs, composed of control costs and expected fines for non-compliance in each period. These discounted expected costs are the following:

$$C(\theta_i, e_{1i}) + p_1 f \max\{0, e_{1i} - \bar{e}\} +$$
$$\delta\left\{p_1\left[C(\theta_i, e_{2i}) + p_{2i} f \max\{0, e_{2i} - \bar{e}\}\right] + (1 - p_1)\left[C(\theta_i, e_{2i}) + p_{2N} f \max\{0, e_{2i} - \bar{e}\}\right]\right\}$$

where $0 \le \delta \le 1$ is the discount factor.

On the other hand, the agency aims to minimize total external costs in the industry, subject to its financial constraint in each period. The total level of the externality is:

$$E = \left(N_H e_{1H} + N_L e_{1L}\right) + \delta_e \left(n_H e_{2HI} + n_L e_{2LI} + \frac{n_H \left(N - n_H - n_L\right)}{n_H + n_L} e_{2HN} + \frac{n_L \left(N - n_H - n_L\right)}{n_H + n_L} e_{2LN} \right),$$

where subscript HI stands for a firm of type θ_H if it was inspected in the first period, LI for type θ_L if it was inspected in the first period, HN for type θ_H if it was not audited in the first period, and LN for type θ_L if it was not audited in the first period. The parameter $0 \le \delta_e \le 1$ represents the weight the agency gives to future external costs compared to current externalities, and can thus be thought of as an externality discount rate.

The problem needs to be solved backwards to find the sub-game perfect equilibrium. Thus, going back to figure 1, we first have to solve the second period, initially looking at the best response of each firm to a particular probability in that period, and then finding the best monitoring frequencies considering firms' best responses. Afterwards, we have to solve the first period, taking into account that actions in that period affect monitoring strategies in period two.

As a reference case, in the next section we consider the one-period model, under both cases of complete information and asymmetric information.

III. ONE-PERIOD REGULATION

After the standard $\overline{e}$ and the fine *f* are made public knowledge, the agency announces an inspection probability p_i for each firm *i*, which afterwards responds with an externality level e_i.[3] Since we look for a sub-game perfect equilibrium, we first study the optimal behavior of the firms.

3.1 Firms' behavior

Given $\{\overline{e}, f, p_i\}$, firm i solves the following problem:

$$\min_{e_i} \quad c(\theta_i, e_i) + p_i f \max\{0, e_i - \overline{e}\}.$$

The solution of this problem is presented in the following:

Lemma 1. *Given $\{\overline{e}, f, p_i\}$, firm i's optimal externality level, e_i^*, is given by the conditions:*

$$\begin{aligned} & c_e(\theta_i, e_i^*) + p_i f \geq 0, \\ & e_i^* \geq \overline{e}, \\ & \left[c_e(\theta_i, e_i^*) + p_i f\right]\left[e_i^* - \overline{e}\right] = 0. \end{aligned}$$

The intuition of this result is straightforward. Given the policy $\{\overline{e}, f, p_i\}$, the firm can decide to either comply with the standard, or not. The optimal strategy is to comply when the marginal expected penalty for non-compliance is larger than the marginal control costs savings of exceeding the standard; that is, when $p_i f \geq -c_e(\theta_i, \overline{e})$. In that case, the optimal strategy is $e_i^* = \overline{e}$.[4] However, the optimal strategy is to exceed the standard if the marginal expected penalty is below the marginal control cost savings at

[3] Acknowledging a slight abuse of notation, in this section we avoid to use the subscript j (which refers to period), since we are just considering one-period regulation.

[4] In this one-period model, the firm never chooses an externality level strictly below the standard: it just increases control costs, but there are no penalty savings.

the standard. In that case, the firm will choose the externality level such that marginal control cost savings and marginal expected fines are equal. Therefore, we have $e_i^* > \overline{e}$ and $c_e\left(\theta_i, e_i^*\right) + p_i f = 0.$

This expression defines an implicit relationship between the inspection probability and the induced externality level. Using the implicit function theorem, we have:

$$\frac{\partial e_i}{\partial p_i} = -\frac{f}{c_{ee}\left(\theta_i, e_i\right)} < 0, \tag{1}$$

which defines the effect on the externality of a marginal increase in the inspection probability; the larger the probability, the lower the externality level.

For later reference, we define $\mathbb{C}\left(\theta_i, p_i\right) \equiv \min_{e_i}\left[c\left(\theta_i, e_i\right) + p_i f \max\left\{0, e_i - \overline{e}\right\}\right]$. By the envelope theorem, this minimum cost function is increasing in the probability of inspection. That is, $\frac{\partial \mathbb{C}\left(\theta_i, p_i\right)}{\partial p_i} = f \max\left\{0, e_i^* - \overline{e}\right\} \geq 0.$

From lemma 1 we can immediately see that there exists a threshold inspection probability for each type, such that compliance is ensured above that threshold. That minimum probability required is:

$$\overline{p}_i = -\frac{c_e\left(\theta_i, \overline{e}\right)}{f}$$

Obviously, $\overline{p}_H > \overline{p}_L$, since $\theta_H > \theta_L$.

3.2 Agency's behavior

The behavior of the agency depends on the information available about the firms. With perfect information the inspection agency perfectly knows the specific type of each firm, and is thus able to differentiate its monitoring strategy depending on the type. The agency takes into account the optimal response of the firm, presented in Lemma 1 above. Therefore, the agency's optimization problem under complete information is the following:

$$\begin{aligned} \min_{p_H, p_L} \quad & N_H e_H + N_L e_L \\ s.t. \quad & c_e(\theta_i, e_i) + p_i f \geq 0, \qquad i = H, L \\ & e_i \geq \bar{e}, \qquad\qquad\qquad\quad i = H, L \\ & m\left[N_H p_H + N_L p_L\right] \leq B \end{aligned} \tag{2}$$

The first two constraints represent the firms' optimal conditions, as established in Lemma 1, and determine the firms' best responses. The last one is the agency's budgetary constraint.

The following lemma gives us the solution of the agency's (interior) optimal policy in this case.

Lemma 2. *In the one-period game under perfect information, the inspection agency's optimal policy* $\left(p_H^*, p_L^*\right)$ *is the following:*

(i) *If* $m\left[N_H \bar{p}_H + N_L \bar{p}_L\right] \leq B$, *then* $p_H^* \geq \bar{p}_H$ *and* $p_L^* \geq \bar{p}_L$ *such that* $m\left[N_H p_H^* + N_L p_L^*\right] \leq B.$

(ii) *If* $mN\bar{p}_L \leq B \leq m\left[N_H \bar{p}_H + N_L \bar{p}_L\right]$, *then* $p_L^* = \bar{p}_L$ *and* $p_H^* = \dfrac{B - mN_L \bar{p}_L}{mN_H}$.

(iii) *If* $B \leq mN\bar{p}_L$, *then* $\left(p_H^*, p_L^*\right)$ *are such that* $\dfrac{\partial e_H}{\partial p_H} = \dfrac{\partial e_L}{\partial p_L}$ *and* $m\left[N_H p_H^* + N_L p_L^*\right] = B.$

The results of Lemma 2 are very intuitive. Case (i) refers to the situation where the budget available to the inspection agency is sufficient to deter all violations in the industry. Therefore, all firms comply with the regulation[5]: $e_H^* = e_L^* = \overline{e}$. This case is, however, trivial,as well as unrealistic, so in the remainder of the paper we assume that $B \leq m\left[N_H \overline{p}_H + N_L \overline{p}_L\right]$. Case (ii) represents partial compliance: low-cost firms comply with the standard, while the high-cost firms violate it (or comply at the margin): $e_L^* = \overline{e}$ *and* $e_H^* \geq \overline{e}$. Finally, case (iii) is the situation of full non-compliance. In that case, the inspection strategy should be such that the marginal benefits, expressed as reductions in the negative externality, from increasing the inspection probability for the high-cost firms or for the low-cost firms are equal. Put differently, an extra euro spent on monitoring should be used in such a way that the weighted effects on the firms' externality levels are equal for both types. Therefore, this is a cost efficiency condition with respect to how the monitoring budget must be spent.

By contrast, if we assume that the agency knows nothing about the type of firms it is dealing with, the agency only has one possibility: it randomly inspects as many firms as possible within the budget restriction. This is trivially stated in the following lemma.

Lemma 3. *In the one-period game under imperfect information, the inspection agency's optimal policy is* $p = p_H^* = p_L^*$, *where* $p = \dfrac{B}{mN}$.

Note that this result is true even if the agency knew the proportion of types in the industry. Since there is just one-period regulation, and the agency does not know who is

[5] Note that the agency's budgetary constraint is not necessarily binding, and also that we might have multiple equilibria, although they are all equivalent in terms of the total externality induced.

who, it cannot improve using a separating strategy, since there is no rational basis for doing so.

Combining Lemmas 2 and 3, it is easy to see that the uniform inspection frequency is such that $p_L^* \le p \le p_H^*$. This uniform inspection strategy is clearly an inefficient one, since the low-cost firms will be inspected too often and the high-cost firms will be insufficiently monitored. Nevertheless, the agency is not able to improve upon it, since it does not have the necessary information.

IV. TWO-PERIOD REGULATION

We now turn to the case illustrated earlier in figure 1. Since this game must be solved backwards, we first look at the second period decisions. Then, in a later stage, we analyze the strategies in the first period, considering their potential effects on the next period.

4.1 Second period

In the second period the inspection agency chooses the inspection probabilities, and then firms respond as in Lemma 1. Therefore, we study the agency's strategy, taking the firms' responses into account.

On the one hand, if all the inspected firms of the previous period chose the same amount of the externality (pooling equilibrium), the agency cannot learn anything new about them and it cannot differentiate its inspection strategy. This implies that all firms in the second period face a uniform probability of inspection, $p = \frac{B}{mN}$, as established in Lemma 3.

On the other hand, if the previous inspections allow the agency to divide the audited firms into high-cost and low-cost firms (separating equilibrium), then it can differentiate its optimal inspection strategy. In this case, the problem faced by the inspection agency in the second period is:

$$\min_{p_{2H},p_{2L},p_{2N}} E = n_H e_{2HI} + n_L e_{2LI} + \delta_e \left[\frac{n_H}{n_H+n_L}\left[N-n_H-n_L\right]e_{2HN} + \frac{n_L}{n_H+n_L}\left[N-n_H-n_L\right]e_{2LN} \right]$$

$$\begin{aligned} s.t. \quad & m\left[n_H p_{2H} + n_L p_{2L} + \left[N-n_H-n_L\right]p_{2N}\right] \le B \\ & c_e\left(\theta_i, e_{2iI}\right) + p_{2i} f \ge 0, \qquad i=H,L \\ & c_e\left(\theta_i, e_{2iN}\right) + p_{2N} f \ge 0, \qquad i=H,L \\ & e_{2iI} \ge \overline{e}, \quad e_{2iN} \ge \overline{e}, \qquad i=H,L, \end{aligned}$$

where e_{2iI} is the externality level chosen by a firm of type *i* that was inspected in the first period, and e_{2iN} is the level selected in case a firm of type *i* was not inspected. Therefore, the agency now has perfect information about two types of firms (the high-cost and low-costs inspected in the first period), and no information at all about the remaining firms.

The agency's (interior) optimal strategy, when it is possible to separate the types based on previous information, is presented in the following:

Proposition 1. *The inspection agency's optimal policy* $\left(p_{2H}^*, p_{2L}^*, p_{2N}^*\right)$ *in the second period satisfies the following conditions:*

(i) *If* $B \ge mN\overline{p}_L$, *then* $p_{2L}^* = \overline{p}_L$,

$$\frac{\partial e_{2HI}}{\partial p_{2H}} = \frac{n_H}{n_H+n_L}\frac{\partial e_{2HN}}{\partial p_{2N}} + \frac{n_L}{n_H+n_L}\frac{\partial e_{2LN}}{\partial p_{2N}}, \text{ and}$$

$$m\left[n_H p_{2H}^* + n_L p_{2L}^* + \left[N-n_H-n_L\right]p_{2N}^*\right] = B.$$

(ii) *If* $B < mN\overline{p}_L$, *then*

$$\frac{\partial e_{2HI}}{\partial p_{2H}} = \frac{\partial e_{2LI}}{\partial p_{2L}} = \frac{n_H}{n_H + n_L}\frac{\partial e_{2HN}}{\partial p_{2N}} + \frac{n_L}{n_H + n_L}\frac{\partial e_{2LN}}{\partial p_{2N}}, \text{ and}$$

$$m\left[n_H p_{2H}^* + n_L p_{2L}^* + \left[N - n_H - n_L\right] p_{2N}^*\right] = B.$$

Starting with case (ii) of the proposition, this corresponds to the situation where full non-compliance was observed in the first period. The optimality condition determines how funds should be allocated over groups in order to obtain cost efficiency. Note that within the subgroup of firms with a revealed type, the available monitoring budget will be spent such that in equilibrium the marginal benefit of reduced externality levels is equal for high- and low-cost firms (that is, $\frac{\partial e_{2HI}}{\partial p_{2H}} = \frac{\partial e_{2LI}}{\partial p_{2L}}$). This is the result under Lemma 2 (iii). Case (i) of the proposition corresponds to the situation where the known low-cost firms comply with the regulation in the second period $\left(e_{2LI} = \overline{e}\right)$. So, firms of this type are audited with their threshold inspection probability, and the remaining budget is allocated between the other groups such that the cost-efficiency condition is met.

4.2. First period

As we know from lemma 3, in the first period the agency has no information and cannot do better than to randomly inspect firms: $p = \frac{B}{mN}$.

In this period, the relevant issue is to analyze the behavior of the firms, who realize that their actions have an effect on next period's monitoring. Knowing that inspection probabilities in period 2 will be as in Proposition 1, firms can choose between two strategies. On the one hand, high-cost firms can try to mimic low-cost firms or not. If the high-cost firms successfully imitate the low-cost firms (that is, if they pool), the inspection agency will not be able to distinguish between the different firms in the second period and will have to use the uniform inspection probability p, as in Lemma 3. This can be advantageous for the high-cost firms, since they will be inspected with a lower frequency in the second period $\left(p < p_{2H}\right)$, but harmful for the low-cost firms, since they will be inspected more frequently $\left(p > p_{2L}\right)$. For this reason, the low-cost firms might try to deter the high-cost firms from mimicking by reducing the externality levels even more and thus increasing the costs of imitation.

The firms' objective function in the first period is to minimize total expected costs over the complete time horizon, as explained in the model section. High-cost firms will mimic low-cost firms as long as it is cost minimizing for them to do so. Formally, firms will imitate low-cost firms if:

$$\begin{aligned} &\mathbb{C}\left(\theta_H, p\right) + \delta\left\{p\mathbb{C}\left(\theta_H, p_{2H}\right) + \left[1-p\right]\mathbb{C}\left(\theta_H, p_{2N}\right)\right\} \\ &\geq c\left(\theta_H, e_{1L}\right) + pf\max\left\{e_{1L} - \overline{e}; 0\right\} + \delta\mathbb{C}\left(\theta_H, p\right) \end{aligned} \quad (3)$$

where $\mathbb{C}\left(\theta_i, p\right)$ represents the minimum cost for a firm of type i confronted with a probability *p,* and e_{1L} equals the externality level chosen by the low-cost firms. Clearly, costs in the first period increase if this type chooses a strategy different than the static (or one-period) optimal one (e_{1H}^{*}), but costs in the second period can be decreased if type θ_H successfully imitates type θ_L.

This allows us to define a threshold level $\tilde{e}_H$ for which expression (3) holds with equality. This threshold is the minimum externality level that type θ_H could choose to successfully imitate type θ_L.[6] Hence, when the externality level of the low-cost firms is below this threshold $\left(e_{1L} \leq \tilde{e}_H\right)$, it will be too expensive for the high-cost firms to mimic them. However, for externality levels above this threshold $\left(e_{1L} > \tilde{e}_H\right)$, high-cost firms can benefit from imitating the low-cost firms.

Next, we investigate when it is profitable for the low-cost firms to prevent imitation by high-cost firms. Low-cost firms weigh the costs of additional compliance in the first period with the benefit of a less stringent inspection regime in the second period. Formally, this is:

$$\begin{aligned} &c\left(\theta_L, e_{1L}\right) + pf \max\left\{e_{1L} - \bar{e}; 0\right\} + \delta\left\{p\mathbb{C}\left(\theta_L, p_{2L}\right) + \left[1-p\right]\mathbb{C}\left(\theta_L, p_{2N}\right)\right\} \\ &\leq \left[1+\delta\right]\mathbb{C}\left(\theta_L, p\right) \end{aligned} \tag{4}$$

Again, we can calculate a threshold level $\tilde{e}_L$ such that the above expression holds with equality.[7] So, for externality levels below this threshold $\left(e_{1L} \leq \tilde{e}_L\right)$, it is not worthwhile for the low-cost firms to deter mimicking behavior from high-cost firms. The costs of deterrence, i.e. the extra compliance costs, outweigh the associated benefits for this case. However, for externality levels exceeding the threshold $\left(e_{1L} > \tilde{e}_L\right)$, the low-cost firms will lower their levels in order to prevent imitation by high-cost firms.

These thresholds $\left(\tilde{e}_H \textit{ and } \tilde{e}_L\right)$ play a key role in determining the optimal strategies for the firms in the first period, as we present next.

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[6] Clearly, since e_{1H}^* is type θ_H's cost minimizing strategy in a one-period regulation, type θ_H's costs increase with either an increase or a decreases in the externality level. Obviously, we are interested in the case where the externality level is decreased, since this is the only possibility where mimicking can occur.

[7] The same comment as in the previous footnote applies here. We are only interested in the case where type θ_L chooses an externality level lower than its optimal strategy in the one-period regulation (e_{1L}^*) to deter mimicking.

Proposition 2. *The firms' Nash equilibrium strategies in period 1 satisfy the following:*

(i) *If* $\tilde{e}_L > \tilde{e}_H$, *the equilibrium is pooling and the optimal strategy is* $e_{1L} = e_{1H} = e_{1L}^*$.

(ii) *If* $\tilde{e}_L \leq \tilde{e}_H$, *the equilibrium is separating and the optimal strategies are:*

(iia) $e_{1L} = \tilde{e}_L$ *and* $e_{1H} = e_{1H}^*$, *if* $\tilde{e}_H < e_{1L}^*$.

(iib) $e_{1L} = e_{1L}^*$ *and* $e_{1H} = e_{1H}^*$, *if* $\tilde{e}_H \geq e_{1L}^*$.

Therefore, the relative ranking of the two threshold externality levels crucially determines whether the equilibrium is pooling or separating.

In case (i), the decrease in the externality level necessary to avoid mimicking is too expensive for the low-cost firms. It would be necessary to decrease externality levels below $\tilde{e}_H$ $\left(e_{1L} < \tilde{e}_H\right)$ and since this also implies $e_{1L} < \tilde{e}_L$, the low-cost firms will not prevent high-cost from imitating them and they will select e_{1L}^*. Since $e_{1L}^* \geq \tilde{e}_L$, we then have $\tilde{e}_H < e_{1L}^*$. Therefore, it is always profitable for the high-cost firm to imitate the low-cost firms and the low-cost firms will not be able to prevent this. This leads to a pooling equilibrium and the agency's inspections in the first period will not provide the necessary information for differentiating its inspection strategy in the second period.

In case (ii), the low-cost firms can successfully deter the other firms from mimicking them. To this end, the low-cost firms choose $e_{1L} = \min\left\{\tilde{e}_H, e_{1L}^*\right\}$. This level is sufficiently low so as to make it unprofitable for the high-costs firms to imitate them. The high-costs firms will then choose e_{1H}^* in period one. This is a separating equilibrium. The inspection agency is thus able to distinguish between both types of firms after inspecting them.

Table 1 summarizes the sequence of choices depending on the thresholds $\tilde{e}_L$ and $\tilde{e}_H$.

	Period 1		Period 2	
	Agency	Firms	Agency	Firms
Case (i) $\tilde{e}_H < \tilde{e}_L$	Uniform inspections: p	Pooling: $e_{1L} = e_{1H} = e_{1L}^*$	Uniform inspections: p	Separating: $e_{2H}^* = e_{1H}^* > e_{2L}^* = e_{1L}^*$
Case (iia) $\tilde{e}_L \leq \tilde{e}_H < e_{1L}^*$	Uniform inspections: p	Separating: $e_{1H} = e_{1H}^* > e_{1L} = \tilde{e}_L$	Differentiated inspections: p_{2H}, p_{2L}, p_{2N}	Separating: $e_{2HI}^*, e_{2LI}^*, e_{2HN}^*, e_{2LN}^*$
Case (iib) $\tilde{e}_L \leq e_{1L}^* \leq \tilde{e}_H$	Uniform inspections: p	Separating: $e_{1H} = e_{1H}^* > e_{1L} = e_{1L}^*$	Differentiated inspections: p_{2H}, p_{2L}, p_{2N}	Separating: $e_{2HI}^*, e_{2LI}^*, e_{2HN}^*, e_{2LN}^*$

Table 1: Summary of firms' and agency's decisions

4.3. Impact on total externality levels

The strategies chosen by the firms and the agency will influence the total externality levels (and therefore, total external costs), resulting from monitoring and enforcing the standard $\bar{e}$. Depending on the case (see table 1), total external costs will differ. As a reference point, we use the case where the agency cannot learn anything from inspecting firms, even if externality levels differ between firms. So, the reference scenario is analogous to playing the static game with imperfect information (see section III) twice. This implies that total discounted externality levels over the two periods equal:

$$(1+\delta_e)\left(N_H e_{1H}^* + N_L e_{1L}^*\right)$$

From proposition 2 and table 1, we calculate the resulting externality levels in each possible case. This gives:

For case (i): $E = Ne_{1L}^* + \delta_e\left(N_H e_{1H}^* + N_L e_{1L}^*\right)$

For case (iia): $E = \left[N_H e_{1H}^* + N_L \tilde{e}_L\right] + \delta_e\left[E_2\right]$

For case (iib): $E = \left[N_H e_{1H}^* + N_L e_{1L}^*\right] + \delta_e\left[E_2\right]$

with $E_2 \equiv n_H e_{2HI}^* + n_L e_{2LI}^* + \left[N - n_H - n_L\right]\left[\frac{n_H}{n_H + n_L} e_{2HN}^* + \frac{n_L}{n_H + n_L} e_{2LN}^*\right]$, which is the total externality in period 2 for a differentiated inspection policy. Since the agency can always choose a uniform inspection strategy in the second period if that would be better, the level E_2 will never exceed $N_H e_{1H}^* + N_L e_{1L}^*$.

Trivially, this shows that learning is always beneficial. In any case, externality levels in a situation where the agency can acquire information through auditing firms will be lower than in the reference scenario (where such learning was non-implementable).

Comparing cases (iia) and (iib), we also find that the threat of mimicking is a good thing, since the low-cost firms will reduce their externality levels more than in the static case - they might even over-comply with the standard - in order to deter high-costs firms from imitating them. So, when firms choose a separating equilibrium in the first period, resulting total external costs over the two periods are lower if mimicking is actively prevented.

Comparing case (i) with cases (iia) and (iib), we find that successful imitation (that is, the firms are pooled) can reduce total discounted externality levels when the discount rate δ_e is sufficiently low and, for case (iia), also if the number of high-cost firms N_H is sufficiently high. By contrast, for a sufficiently high discount rate, pooling and mimicking by firms might worsen the externality problem, since the second period external costs associated with a uniform inspection probability will exceed those resulting from a differentiated inspection policy (E_2).

V. DISCUSSION

In this section we study the likelihood of each possible type of equilibrium, depending on the parameters of the model. In particular, we are interested in analyzing the influence of the agency's budget, the number of firms and the monitoring costs on the possibility that firms choose the same strategies (i.e. high-cost firms mimic low-cost firms) or different strategies and, therefore, whether the agency can learn the firms' types in the first period and differentiate its strategy in the second regulatory period.

For the purpose of simplicity, let us assume that control costs are quadratic (i.e., the third order derivative is negligible, $c_{eee} \approx 0$). Note that in this case, the marginal externality impacts defined in (1) are constant, that is, $c_{ee}(\theta_i, e)$ is independent of e. Since we are assuming that $c_{ee\theta}(\theta_i, e) < 0$, we have that the marginal externality impact of the high-cost firm is always larger than that of the low-cost firm. Therefore, this means that, for any (p_{2H}, p_{2L}, p_{2N}), we have:

$$\frac{\partial e_{2HI}}{\partial p_{2H}} < \frac{n_H}{n_H + n_L} \frac{\partial e_{2HN}}{\partial p_{2N}} + \frac{n_L}{n_H + n_L} \frac{\partial e_{2LN}}{\partial p_{2N}} < \frac{\partial e_{2LI}}{\partial p_{2L}} < 0$$

Therefore, the best strategy for the agency is a corner solution: it will devote monitoring resources to the type that is more effective in reducing external costs (i.e. the firms that were inspected in the first period and that turned out to be type θ_H); if the agency has enough resources to induce compliance in this group, it will devote the money left to the next group that is more effective in lessening the externality problem (i.e. the non-inspected types in the first period); and then, if there is money left, to the less effective group in reducing the externality (i.e. the inspected firms that turned out to be of type θ_L). The three possible cases are presented next:

<u>Case 1</u>: Available funds are insufficient to enforce $e_{2H} = \overline{e}$ (that is, $B < mn_H \overline{p}_H$).

The agency will devote all its resources to inspecting the group of n_H firms that it knows are high-cost firms. Thus:

$$p_{2H} > p > p_{2N} = p_{2L} = 0$$

Since the agency focuses on a subgroup of firms and inspects only those, we have:

$$p_{2H} = \frac{B}{n_H m} > p = \frac{B}{Nm}.$$

So, high-cost firms that would be inspected in period one would face an increased probability of inspection in period two, and might thus have an incentive to mimic low-cost firms in the first periods. Similarly, low-cost firms have a motivation for trying to prevent imitation by their competitors, since they are not be targeted in the second period if they are not pooled ($p_{2N} = p_{2L} = 0$).

<u>Case 2</u>: Available funds are sufficient to enforce $e_{2H} = \overline{e}$ but not to enforce $e_{2L} = \overline{e}$ (i.e., $mn_H \overline{p}_H \leq B < m\left[N - n_L\right]\overline{p}_H$)

In this case, the agency is able to force known high-cost firms into compliance. Hence:

$$p_{2H} = \overline{p}_H > p \, .$$

The remainder of the budget is then used to monitor the firms that were not inspected in the previous period:

$$p_{2N} = \frac{B - mn_H \overline{p}_H}{\left(N - n_H - n_L\right)m}$$

which is always below $\overline{p}_H$. The different inspection probabilities are ranked as:

$$p_{2H} = \overline{p}_H > p_{2N} > p_{2L} = 0$$

In this case, the likelihood of witnessing pooling is higher than in the first case. After all, the high-cost firms have a greater incentive to mimic the other firms and the low-cost firms have less motivation to deter mimicking, since $p_{2N} > 0$. However, if $p_{2N} < p$, low-cost firms still have a large motivation to deter mimicking. The ranking of p_{2N} with respect to p depends on the availability of funds to deter all high-cost firms in the industry. To see this, note that, since $p = \frac{B}{Nm}$ and $p_{2N} = \frac{B - mn_H \overline{p}_H}{\left(N - n_H - n_L\right)m}$, both the numerator and the denominator of p_{2N} are lower than those of p (there are less available funds to deter unknown firms in the second period, but also a smaller number of unknown firms). Therefore, $p_{2N} \leq p$ if, and only if, the percentage of decrease in the available funds to deter unknown firms is larger than the percentage of decrease in the number of unknown firms. That is, if and only if:

$$\frac{n_H \, m \, \overline{p}_H}{B} \geq \frac{n_H + n_L}{N} \, .$$

Since $\frac{n_H}{n_H + n_L} = \frac{N_H}{N}$, we then have that $p_{2N} \leq p$ if, and only if, $N_H \, m \, \overline{p}_H > B$.

Therefore, since $N_H < N - n_L$, we then have the following ranking of inspection probabilities depending on the agency's budget:

(2a) $p_{2H} = \overline{p}_H > p \geq p_{2N} > p_{2L} = 0$ if and only if $m\, n_H\, \overline{p}_H \leq B \leq m\, N_H\, \overline{p}_H$.

(2b) $p_{2H} = \overline{p}_H > p_{2N} \geq p > p_{2L} = 0$ if and only if $m\, N_H\, \overline{p}_H \leq B < m\left[N - n_L\right]\overline{p}_H$.

Therefore, incentives for low-cost firms to try to prevent imitation (and also for high-cost firms not to mimic) are larger in case (2a).

<u>Case 3</u>: Available funds are sufficient to enforce $e_{2H} = \overline{e}$ and $e_{2N} = \overline{e}$ (i.e. $B \geq m\left[N - n_L\right]\overline{p}_H$).

The inspection agency can now successfully induce all firms, except the low-cost firms that were inspected in the previous period, to comply with the regulation. We have:

$$p_{2H} = p_{2N} = \overline{p}_H > p\,.$$

The rest of the monitoring resources are used to inspect the known low-cost firms and thus:

$$p_{2L} = \frac{B - \left[N - n_L\right]\overline{p}_H}{m n_L}$$

The differentiated inspection frequencies are thus ranked as followed:

$$p_{2H} = p_{2N} = \overline{p}_H > p_{2L} > 0$$

This scenario provides the highest likelihood of finding pooling behavior among firms in the first period.

We are now able to comment on the influence of the available budget, the number of firms in the industry and the level of the monitoring costs on the motivation for mimicking. To begin with an increase in the available budget, *ceteris paribus*, will provide more incentives for imitation. Indeed, a higher budget implies more inspections in any scenario. Thus, the benefits from mimicking increase, since they depend, amongst other things, on the likelihood p of being inspected in the first period. The more likely it is that a high-cost firm can be detected in the first period, the more it stands to gain from hiding among the low-cost firms. For this reason, we also make the following two observations. Firstly, the lower the number of firms affected by the regulation, the higher the incentives for pooling, *ceteris paribus*. Secondly, if inspections become less expensive, *ceteris paribus,* high-cost firms will be induced mimic low-cost firms.

VI. CONCLUSIONS

This paper shows that incorporating learning in regulatory enforcement has implications for the agency and the firms' strategies, as well as for social welfare. We assume that the regulatory agency has the possibility to learn about the true types of the firms it is confronted with through inspection, but only if it finds subgroups of firms performing differently. This type of learning is used afterwards to target known types in the subsequent regulatory period. Since the agency has a fixed enforcement budget (which sometimes can be quite small), we show that it will devote more enforcement resources in the next period to auditing the known high-cost firms (whose reactions to a change in the inspection probability are larger). Only if the agency can successfully induce compliance of the known high-cost firms and it has money left, will it devote effort (resources) to try to improve compliance in the next group, i.e. those firms that where not inspected in the first period. Money left (if any) will then be devoted to the least-efficient group, i.e. the known low-cost firms.

In principle, this targeting strategy can be detrimental for high-cost firms but beneficial for low-cost firms. Therefore, high-cost firms may have an incentive to avoid this situation by trying to mimic the low-cost firms; although low-cost firms may try to avoid being mimicked.

We show that the likelihood of a pooling equilibrium (that is, one in which high-cost firms successfully mimic low-cost firms) depends positively on the agency's budgetary constraint, and negatively on the number of firms in the industry and the monitoring costs. These three factors crucially determine the probability of being inspected in the first period; the larger this probability, the more prone the agency is to target high (low)-cost firms with larger (lower) inspection frequencies in the next period.

To prevent mimicking our results suggest that the agency should not be given a large budget. But interestingly, if the agency significantly discounts future externality reductions and thus focuses on present gains and if the proportion of high-costs firms in the industry is sufficiently large, mimicking, or even the threat of mimicking, might be good for society. These social benefits arise when we compare the externality levels with the separating equilibrium where high-cost firms do not imitate low-cost firms, because it is just not profitable for them to do so (even if low-cost types do not actively avoid being copied). In any case, this avoidance behavior results in a lower amount of global external costs in the first period. On the one hand, the bad types may try to reduce the externality in the first period to avoid future tighter monitoring. On the other hand, the good types may try to reduce the externality as well, in order to differentiate themselves from the bad types and thus prevent being pooled. Sometimes they might even avoid being imitated by doing better than the standard in the first period (i.e. by over-complying).

Several extensions of our model are possible. For example, we have assumed that inspections are accurate, that is, there are no measurement errors. However, in reality, it is sometimes the case that a high-cost firm is wrongly thought to be a low-cost firm (*false positives*). This then implies that the information obtained by the agency from the first period is less valuable than with perfect inspections, and therefore benefits from mimicking are reduced.

It is also interesting to comment on the implications of the model for an infinite time horizon. In that scenario pooling is never optimal. Indeed, there are an infinite number of periods with imitation costs for the high-cost firms, while the benefit of mimicking can only be reached asymptotically (in the 'last' period). This observation also implies that a reversal in strategy (from pooling to separating or from separating to pooling) is never optimal either. Thus, in an infinitely repeated game, the agency is always able to learn the firms' true type through inspections, since high-cost firms never pose as low-cost firms. Finally, the infinitely repeated game converges to a steady-state equilibrium, which involves a cost efficient allocation of the available funds. This implies that the agency's resources are used where they cause the greatest reduction in the level of the externality. This means that in equilibrium it might be optimal for the agency to exclusively target one group of firms and to completely ignore the other firms, depending on the size of the available budget.

Other extensions that incorporate imperfect knowledge of the firms regarding future regulations, commitment issues or imperfect learning, are left for further research.

APPENDIX

Proof of Lemma 1.

The first order conditions of this optimization problem are:[8]

$$c_e(\theta_i, e) + pf - \lambda = 0$$
$$\lambda \geq 0, e - \overline{e} \geq 0,\ \lambda[e - \overline{e}] = 0$$

where $\lambda \geq 0$ is the Kuhn-Tucker multiplier associated to the inequality restriction $e - \overline{e} \geq 0$. Easily combining these conditions, we obtain the desired result.

Proof of Lemma 2.

Deriving the first order conditions to this problem gives:

$$\begin{aligned} &-\gamma_H f + \lambda m N_H = 0 \\ &-\gamma_L f + \lambda m N_L = 0 \\ &N_H - \gamma_H c_{ee}(\theta_H, e_H) - \beta_H = 0 \\ &N_L - \gamma_L c_{ee}(\theta_L, e_L) - \beta_L = 0 \end{aligned} \tag{5}$$

where $\gamma_i \geq 0$ are the Kuhn-Tucker multipliers associated with the firm types' optimal responses ($c_e(\theta_i, e) + pf \geq 0$) and $\beta_i \geq 0$ are the ones associated with $e_i - \overline{e} \geq 0$, where $i = H, L$, and $\lambda \geq 0$ is the one associated to the agency's budgetary constraint. Combining the first two equalities of (5) when $\beta_i = 0$, $i = H, L$ we then have:

$$\frac{\lambda m}{f} = \frac{\gamma_H}{N_H} = \frac{\gamma_L}{N_L} \tag{6}$$

Since $\beta_i = 0$ implies that $e_i - \overline{e} \geq 0$, we then have $c_e(\theta_i, e) + pf = 0$ by Lemma 1, which then implies $\gamma_i \geq 0$. Condition (6) then ensures $\lambda \geq 0$, which then implies that $m[N_H p_H + N_L p_L] = B$. Combining (6) and the two first equations of (5), we then have:

[8] Given the assumptions of our model, these are necessary and sufficient conditions for an optimum. The same applies for the remaining optimization problems in the paper.

$$\theta_H C''(e_H) = \theta_L C''(e_L)$$

which, using (1), is equivalent to:

$$\frac{\partial e_H}{\partial p_H} = \frac{\partial e_L}{\partial p_L},$$

which is case (iii) of the lemma.

Now we consider the case where $\beta_L > 0$ and $\beta_H = 0$ (case ii). Then we have $e_H \geq e_L = \overline{e}$. The budgetary constraint must be such that $mN\overline{p}_L \leq B \leq m\left[N_H \overline{p}_H + N_L \overline{p}_L\right]$. Since we know that $\gamma_L > 0$, the probability of inspecting low-cost firms equals $\overline{p}_L$, and from the budget constraint, we find

$$p_H = \frac{B - mN_L \overline{p}_L}{mN_H}.$$

Finally, case (i) implies that $\beta_i > 0$ for $i = H, L$, which leads to $e_H = e_L = \overline{e}$. Then the budget constraint must satisfy $B \geq m\left[N_H \overline{p}_H + N_L \overline{p}_L\right]$.

Proof of Proposition 1.

The first order conditions of this problem are the following:

$$\begin{aligned}
&\lambda m n_H - \gamma_{HI} f = 0 \\
&\lambda m n_L - \gamma_{LI} f = 0 \\
&\lambda m\left[N - n_H - n_L\right] - \left[\gamma_{HN} + \gamma_{LN}\right] f + = 0 \\
&n_H - \gamma_{HI} c_{ee}\left(\theta_H, e_{2HI}\right) - \beta_{HI} = 0 \\
&n_L - \gamma_{LI} c_{ee}\left(\theta_L, e_{2LI}\right) - \beta_{LI} = 0 \\
&\frac{n_H}{n_H + n_L}\left[N - n_H - n_L\right] - \gamma_{HN} c_{ee}\left(\theta_H, e_{2HN}\right) - \beta_{HN} = 0 \\
&\frac{n_L}{n_H + n_L}\left[N - n_H - n_L\right] - \gamma_{LN} c_{ee}\left(\theta_L, e_{2LN}\right) - \beta_{LN} = 0
\end{aligned} \tag{7}$$

where $\lambda \geq 0$ is the Kuhn-Tuker multiplier associated with the agency's budget constraint, $\gamma_i \geq 0$ are the multipliers associated with firms' best responses such that $i = HI, LI, HN, LN$

Assume that $\beta_i = 0$ for all i. Then we have $e_{2i} \geq \overline{e}$., which then implies $\gamma_i \geq 0$. From the first three equations of (7), we find:

$$\frac{\lambda m}{f} = \frac{\gamma_{HI}}{n_H} = \frac{\gamma_{LI}}{n_L} = \frac{\gamma_{HN} + \gamma_{LN}}{N - n_H - n_L} \geq 0, \tag{8}$$

which implies that the budget is binding.

From fourth and fifth equations in (7), we have:

$$1 - \frac{\gamma_{HI}}{n_H} c_{ee}\left(\theta_H, e_{2HI}\right) = 0,$$
$$1 - \frac{\gamma_{LI}}{n_L} c_{ee}\left(\theta_L, e_{2LI}\right) = 0,$$

which implies:

$$c_{ee}\left(\theta_H, e_{2HI}\right) = c_{ee}\left(\theta_L, e_{2LI}\right), \tag{9}$$

or, equivalently, $\frac{\partial e_{2HI}}{\partial p_{2H}} = \frac{\partial e_{2LI}}{\partial p_{2L}}$.

From sixth and seventh equations in (7), we have:

$$\gamma_{HN} = \frac{n_H}{\left[n_H + n_L\right] c_{ee}\left(\theta_H, e_{2HN}\right)} \left[N - n_H - n_L\right]$$
$$\gamma_{LN} = \frac{n_L}{\left[n_H + n_L\right] c_{ee}\left(\theta_L, e_{2LN}\right)} \left[N - n_H - n_L\right]$$

Thus:

$$\frac{\gamma_{HN} + \gamma_{LN}}{N - n_H - n_L} = \frac{n_H}{n_H + n_L} \frac{1}{c_{ee}\left(\theta_H, e_{2HN}\right)} + \frac{n_L}{n_H + n_L} \frac{1}{c_{ee}\left(\theta_L, e_{2LN}\right)}$$

Using (1), we equivalently have:

$$\frac{n_H}{n_H + n_L}\frac{\partial e_{2HN}}{\partial p_{2N}} + \frac{n_L}{n_H + n_L}\frac{\partial e_{2LN}}{\partial p_{2N}} = \frac{\partial e_{2HI}}{\partial p_{2HI}} = \frac{\partial e_{2LI}}{\partial p_{2LI}}$$

When $\beta_{LI} > 0$, we then have $e_{2LI} = \overline{e}$ and $\gamma_{LI} \geq 0$. In that case, the equilibrium conditions change to $p_{2L} = \overline{p}_L$ and

$$\frac{n_H}{n_H + n_L}\frac{\partial e_{2HN}}{\partial p_{2N}} + \frac{n_L}{n_H + n_L}\frac{\partial e_{2LN}}{\partial p_{2N}} = \frac{\partial e_{2HI}}{\partial p_{2H}}$$

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