

# Nationalization in a Green Market\*

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## Abstract

We analyze whether a government must nationalize a firm in a green market and how many shares he should hold in this firm, so the new public firm takes into account the environmental damage caused by the pollution of industry. Thereby, we used a duopoly model of horizontal product differentiation with price competition. We shows that the government should full nationalize either less or high pollutant firm.

*Keywords:* Nationalization, Mixed oligopoly, Horizontal differentiation, Green market

*JEL classification:* H42; L13; L33; Q58

Some men see things as they are and say, "Why?"  
I dream of things that never were and say, "Why not?"  
—**Robert F. Kennedy, after George Bernard Shaw**  
Quoted by Nalebuff and Ayres (2003).

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# 1 Introduction

In last quarter of past century, we were witness of a larger wave of privatization by governments, which have been attracted by the efficient management of private firm. However, at the beginning of new century, the private firms' image has been deteriorated by the business scandals in EEUU.<sup>1</sup> One of main causes was the difficulty of monitoring private firm's manager by theirs shareholder (for more details read Stiglitz (2003) and Galbraith (2004)). Moreover, at the moment, we are immersed in an international financial crisis, and the European countries have proposed nationalizing banks with financial problems overcome it. Thus, we think about reconsidering a public firm as a government's instrument for improving welfare. Therefore, we arise the question whether a government must nationalize a firm in a green market, given that in this market firms produce pollutant products and nowadays environmental problems become more and more important.

The literature about mixed oligopoly has analyzed when it is beneficial to privatize a public firm. In a seminal paper, De Fraja and Delbono (1989) show that it is socially better for the government to privatize a public firm if the market is competitive enough and the public firm cannot have the move advantage, otherwise the existence of a public enterprise is socially desirable. Matsumura (1998) studies how many shares the government should hold in a privatized firm, and finds that full privatization is not optimal if firms have the same cost and full nationalization is not optimal if a rival firm can enter the market. And recently, Bárcena-Ruiz and Garzón (2006) analyzes government's environmental policy in a mixed oligopoly, in which (private or public) firms produce homogeneous goods. They show that the decision whether to privatize a public firm interacts with the environmental policy of governments.

Contrary to the traditional thought, Porter (1990) and Porter and van der Linde (1995) claim that environmental regulations by governments imply an improvement on welfare because they can open up new investment opportunities, encourage firms to innovate and generate long term gains that can offset the costs of complying with them. In this line, André et al. (in press) shows that firms profit from the existence of a rule penalizing any firm refusing to produce an environmentally friendly good. Other regulatory measurements have been analyzed from a theoretical viewpoint, as the taxation of polluting products when firms cannot evade it (Cremer and Thisse (1999)) and when they can (Macho-Stadler and Pérez-Castrillo (2006) and Macho-Stadler (2008)).

In this paper, we analyze whether a government must nationalize a firm in a green market and how many shares he should hold in this firm, so the new public firm takes into account the environmental damage caused by the pollution of industry. Thereby, we develop a duopoly model of horizontal product differentiation with price competition.<sup>2</sup> Thus, as in Conrad (2005), we consider that each product is characterized by two negatively related properties: the level of environmental quality and the level of power or functional quality. For instance, we can think on the automobile market where a more ecological

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<sup>1</sup>For instance, the scandal that implies the private firm ENRON, one of the most important firm of USA.

<sup>2</sup>There exists others papers that consider models of vertical differentiation in a green market, as Cremer and Thisse (1999), Moraga-González and Padrón-Fumero (2002) and André et al. (in press).

car has a less powered engine.

Our analysis demonstrates that the government should full nationalize either less or high pollutant firm. This result contrast with that obtained by Matsumura (1998), who find that the government should full nationalize a private firm only if it is a monopolist. It is because he considers homogeneous and non pollutant goods. But it is in keeping with Bárcena-Ruiz and Garzón (2006) who find that the public firm should not be privatized when market competition is low enough, although they consider homogeneous goods and they do not consider the possibility of partially state ownership.

The rest of the paper is organized as follows. Section 2 describes the model formally, while Section 3 obtains the results. Finally, Section 4 concludes and suggests some open question for future research.

## 2 The model

We use a duopoly model of horizontal differentiation in which the government decides what firm to (partial or complete) nationalize. The government looks for maximizing social welfare, so the nationalized firm's objective function is weighed sum of their private profit and social welfare according to the government's weight on their capital. There are two firms that produce environmental differentiated products: firm  $l$  that produces less pollutant product and firm  $h$  that produces high pollutant product, and are located at the two endpoints of the unit interval  $[0, 1]$ . With no loss of generality, we consider that firm  $l$  (cleaner) is located at 0 and firm  $h$  (dirtier) is located at 1. Let  $e_i$  be the unit emissions of product  $i \in \{l, h\}$ , so  $e_l < e_h$ . Let  $\Delta^e = e_h - e_l$  be the emission gap between both pollutant products. Given our assumption of horizontal differentiation, each product is characterized by two negatively related properties, the level of pollution and the level of power or functional quality as in Conrad (2005). Thus, the less (high) pollutant product is also less (high) functional one.

A unit mass of consumers is uniformly distributed on the  $[0, 1]$  interval, so those consumers close to 0 prefer the less pollutant product than the high one and those close to 1 prefer high pollutant product than less one. Each consumer is indexed by  $x$ , where  $x \in [0, 1]$ , and can buy at most one unit of the product. Thus, the utility of consumer  $x$  is:

$$U(x) = \begin{cases} u - x^2 - p_l & \text{if he buys the less pollutant product} \\ u - (1 - x)^2 - p_h & \text{if he buys the high pollutant product} \\ 0 & \text{if he does not buy} \end{cases} \quad (1)$$

where  $u$  represents the utility obtained from consuming his ideal product,  $x^2$  ( $(1 - x)^2$ ) represents the disutility from not consuming their ideal product if he buys the less (high) pollutant product and  $p_l$  and  $p_h$  are the price of the less and high pollutant product, respectively. We assume that  $u$  is high enough so all consumer buy at least from a firm, i.e. we assume that the market is fully covered. Let  $x_{lh} = (1 + p_h - p_l)/2$  be the consumer indifferent between buying the less and high pollutant good. Thus, firms' demand functions are as follows (see Figure 1):

$$D_l(p_l, p_h) = x_{lh}; D_h(p_l, p_h) = 1 - x_{lh} \quad (2)$$

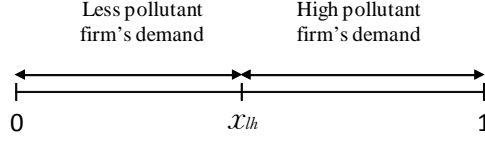


Figure 1: Firms' Demand

We assume that the cost incurred by firms in developing each product is a sunk cost and the production cost is higher by the cleaner firm. Last assumption is because we consider that an ecological product requires a more expensive and sophisticated technology. Thus, the firms' profits are:

$$\pi_l(p_l, p_h) = (p_l - c_l) D_l(p_l, p_h) \quad (3)$$

$$\pi_h(p_l, p_h) = (p_h - c_h) D_h(p_l, p_h) \quad (4)$$

where  $c_l$  and  $c_h$  are the marginal production cost, so that  $c_h < c_l$ . Let  $\Delta^c = c_l - c_h$  be the gap in marginal cost between both firms. We assume the following condition regarding the gap of emission level.

**Condition 1** *The emission gap is bounded by the gap in marginal cost. Formally,*

$$\Delta^c - 1 < \Delta^e < \Delta^c + 1.$$

Condition 1 is needed for ensuring that both firms enter on the market. In particular, the first inequality means that the cleaner product is not too costly so the less pollutant firm enters on the market, and the second inequality means that the high pollutant product is not too pollutant so dirtier firm obtains positive profit.

The government maximizes the social welfare ( $W$ ), which is the sum of consumer surplus (CS) and firms' profit minus the social valuation of environmental damage caused by aggregate pollution as in Moraga-González and Padrón-Fumero (2002) and Bárcena-Ruiz and Garzón (2006).<sup>3</sup> It is given by

$$W(p_l, p_h) = CS(p_l, p_h) + \pi_l(p_l, p_h) + \pi_h(p_l, p_h) - E_T(p_l, p_h), \quad (5)$$

where

$$CS(p_l, p_h) = \int_0^{x_{lh}} (u - z^2 - p_l) dz + \int_{x_{lh}}^1 (u - (1 - z)^2 - p_h) dz, \text{ and} \quad (6)$$

$$E_T(p_l, p_h) = e_l D_l(p_l, p_h) + e_h D_h(p_l, p_h). \quad (7)$$

The government owns  $\alpha \in [0, 1]$  shares in a firm,  $L$  or  $H$ , so the (partial) public firm's objective function is (De Fraja and Delbono (1989) and Matsumura (1998)):

$$\Pi_{0i} = \alpha W(p_l, p_h) + (1 - \alpha) \pi_i(p_l, p_h) = \pi_i(\cdot) + \alpha [CS(\cdot) + \pi_{j \neq i}(\cdot) - E_T(\cdot)] \quad (8)$$

<sup>3</sup>Notice that, unlike these authors, we do not consider the net revenue of the government. It is because we do not let the government to set taxes and subsidies.

where  $i \in \{l, h\}$  and  $0i$  means that the government nationalized firm  $i \in \{l, h\}$ . Notice that public firm maximizes welfare if  $\alpha = 1$ , but it behaves as a private firm if  $\alpha = 0$ , since a higher weight by the government would imply a higher social concern by this firm.

The timing of the game is as follows. First, the government decides what firm to nationalize. Next, it decides his participation (shares) in the nationalized firm. Then, both firms (private and public) set prices. Finally, consumers decide to buy the less or high pollutant product after they have observed firms' prices.

### 3 The Equilibrium

In the next section, we seek to find the subgame perfect equilibrium (SPE) of the game by backward induction. Thus, we now look for the firms' decisions on prices.

#### 3.1 Firms' decisions on prices

We first consider that the government nationalized the less pollutant firm. Thus, the public firm's objective function is:<sup>4</sup>

$$\Pi_{0l} = \pi_l(p_l, p_h) + \alpha\pi_h(p_l, p_h) + \alpha CS(p_l, p_h) - \alpha E_T(p_l, p_h). \quad (9)$$

From maximizing the profit of high pollutant and public firms, (4) and (9) respectively, we find their reaction functions, which are:

$$\begin{aligned} p_l(p_h) &= \frac{1 + c_l - \alpha(1 + c_h + \Delta^e) + p_h}{2 - \alpha} \\ p_h(p_l) &= \frac{1 + c_h + p_l}{2} \end{aligned} \quad (10)$$

Given that  $\alpha \leq 1$ , it is sure that both reaction functions intersect in an unique point. On solving the system of reaction functions (10), we obtain the firms' prices:

$$p_l(\alpha) = \frac{3 + 2c_l + c_h - 2\alpha(1 + \Delta^e + c_h)}{3 - 2\alpha}; p_h(\alpha) = \frac{3 + c_l + 2c_h - \alpha(2 + \Delta^e + 2c_h)}{3 - 2\alpha} \quad (11)$$

By incorporating optimal prices (11) in the indifferent consumer and profit's function (4) and (9), we have:

$$\begin{aligned} x_{lh}(\alpha) &= \frac{3 - \Delta^c - \alpha(2 - \Delta^e)}{6 - 4\alpha}; \pi_h(\alpha) = \frac{(3 + \Delta^c - \alpha(2 + \Delta^e))^2}{2(3 - 2\alpha)^2}; \\ \Pi_{0l}(\alpha) &= \frac{A\alpha^3 + 6B\alpha^2 - 3C\alpha + 6(3 - \Delta^c)^2}{12(3 - 2\alpha)^2} \end{aligned} \quad (12)$$

where  $A = 48u - 48c_h - 28 - 3e_l(4 + e_l) + 6e_h(e_l - 6) - 3e_h^2$ ,  $B = (18 - 24u + 26c_h - c_l(2 - \Delta^e) + e_l(7 + e_l + c_h) - e_h(2e_l + c_h - 17) + e_h^2)$  and  $C = (45 - 36u + c_l^2 + 2c_l(2\Delta^e - c_h - 7) - 4e_h(c_h - 6) + 50c_h + c_h^2 + 4e_l(3 + c_h))$ .

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<sup>4</sup>Notice that the profit functions of each firm is strictly concave with respect to the prices, so necessary conditions for profit maximization are also sufficient.

### 3.2 Government's decisions

Taking into account the previous results, the government chooses the shares in firm  $l$  that maximizes the following social welfare function:

$$W(\alpha) = CS(\alpha) + \pi_l(\alpha) + \pi_h(\alpha) - E_T(\alpha) \quad (13)$$

where

$$\begin{aligned} CS(\alpha) &= \frac{D\alpha^2 - 6F\alpha + 3(36u - 39 + c_l^2 + c_h(c_h - 18) - 2c_l(9 + c_h))}{12(3 - 2\alpha)^2} \\ \pi_l(\alpha) &= \frac{(3 - \Delta^c + \alpha(\Delta^e - 2))(3 - \Delta^c + 2\alpha(\Delta^c - \Delta^e - 1))}{2(3 - 2\alpha)^2} \\ E_T(\alpha) &= e_l D_l(\alpha) + e_h D_h(\alpha) = e_l x_{lh}(\alpha) + e_h(1 - x_{lh}(\alpha)) \\ D &= 48u - 48c_h - 52 + 3e_h^2 - 6e_h(6 + e_l) + 3e_l(12 + e_l) \\ F &= 24u - 26 + c_l(\Delta^e - 6) - 18c_h - e_h(9 + c_h) + e_l(9 + c_h). \end{aligned}$$

We find that the government decides to full nationalize cleaner firm, in spite of there is a private firm, which contrasts with Matsumura (1998) who find that the government should full nationalize a private firm only if a rival firm cannot enter the market. It is because we consider that products are pollutant and horizontally differentiated and consumers have different preferences for pollutant products. This result is summarized in the following proposition.

**Proposition 1** *Under Condition 1, the government full nationalizes the less pollutant firm, so  $\alpha^* = 1$ . Therefore,*

$$\begin{aligned} x_{lh}^* &= \frac{1 + \Delta^e - \Delta^c}{2}; \pi_l^* = \frac{(1 + \Delta^e - \Delta^c)(1 + \Delta^c - 2\Delta^e)}{2}; \pi_h^* = \frac{(1 + \Delta^c - \Delta^e)^2}{2} \\ \Pi_{0l}^* &= W^* = \frac{12u - 1 - 6(e_h + e_l + c_h) + 3(c_l^2 + (\Delta^e + c_h)^2 - 2c_l(1 + \Delta^e + c_h))}{12} \\ CS^* &= \frac{12u - 13 + 18\Delta^e + 6c_h + 3(c_l^2 + (\Delta^e + c_h)^2 - 2c_l(\Delta^e + c_h + 3))}{12}. \end{aligned} \quad (14)$$

Proof: see Appendix.

We now analyze the case in which the government nationalizes the dirtier firm. Thus, the public firm's objective function is:<sup>5</sup>

$$\widehat{\Pi}_{0h} = \widehat{\pi}_h(p_l, p_h) + \alpha \widehat{\pi}_l(p_l, p_h) + \alpha \widehat{CS}(p_l, p_h) - \alpha \widehat{E}_T(p_l, p_h). \quad (15)$$

Following the same method that previous case, we find that the government full nationalizes dirtier firm, as we state in the next proposition.

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<sup>5</sup>Notice that the profit functions of each firm is strictly concave with respect to the prices, so necessary conditions for profit maximization are also sufficient.

**Proposition 2** *Under Condition 1, the government full nationalizes the high pollutant firm, so  $\alpha^* = 1$ . Therefore,*

$$\begin{aligned}
\widehat{x}_{lh}^* &= \frac{1 + \Delta^e - \Delta^c}{2}; \widehat{\pi}_l^* = \frac{(1 + \Delta^e - \Delta^c)^2}{2}; \widehat{\pi}_h^* = \frac{(1 + \Delta^c - \Delta^e)(1 - \Delta^c + 2\Delta^e)}{2} \\
\widehat{\Pi}_{0h}^* &= \widehat{W}^* = \frac{12u - 1 - 6(e_h + e_l + c_h) + 3(c_l^2 + (\Delta^e + c_h)^2 - 2c_l(1 + \Delta^e + c_h))}{12} \\
\widehat{CS}^* &= \frac{12u - 13 - 18\Delta^e - 18c_h + 3(c_l^2 + (\Delta^e + c_h)^2 - 2c_l(\Delta^e + c_h - 1))}{12}.
\end{aligned} \tag{16}$$

Proof: see Appendix.

Notice that demand's firms and public firm's profit coincide in both cases. Moreover, the social welfare also coincides, so the government will be indifferent between nationalizing the less and high pollutant firm. Therefore, we conclude this section with the following corollary.

**Corollary 1** *Under Condition 1, the government full nationalizes the less or high pollutant firm.*

## 4 Conclusions

We analyze whether a government must nationalize a firm in a green market and how many shares he should hold in this firm, so the new public firm takes into account the environmental damage caused by the pollution of industry. To do so the framework used is a duopoly model of horizontal product differentiation with price competition.

Our analysis shows that the government should full nationalize either less or high pollutant firm. This result contrasts with that obtained by Matsumura (1998), who find that the government should full nationalize a private firm only if it is a monopolist. It is because he consider homogeneous and non pollutant goods. But it is in keeping with Bárcena-Ruiz and Garzón (2006) who find that the public firm should not be privatized when market competition is low enough, although they consider homogeneous goods and they do not consider the possibility of partially state ownership.

The main message of this paper is that we must not rule out a public firm as a government's instrument in a green market for improving the welfare of the society. Of course, it is limited by our assumptions, so it would be useful to check the robustness of this result in more complex models. If the result keeps, it would be interesting to analyze the best way of managing a public firm and the essential contents of manager's contract of (private and public) firms in this kind of market for carrying out the actions that damage the environment the less possible. Moreover, in order to find the best government's intervention it is also interesting to compare the social benefits and costs obtained by environmental regulation and nationalization.

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## Appendix

**Proof of Proposition 1.** The government maximizes (13), so the first order condition is:

$$\frac{dW(\alpha)}{d\alpha} = \frac{(1-\alpha)(2\Delta^c - 3\Delta^e)^2}{2(3-2\alpha)^3} = 0.$$

This condition is only satisfied if and only if  $\alpha = 1$ . It is a maximum because the second order condition (17) is non-positive given that  $\alpha \in [0, 1]$ .

$$\frac{d^2W(\alpha)}{d^2\alpha} = -\frac{(4\alpha-3)(2\Delta^c - 3\Delta^e)^2}{2(3-2\alpha)^4} \leq 0. \quad (17)$$

■

**Proof of Proposition 2.** The public firm's objective function is (15). From maximizing the less pollutant and public firms' profits, (3) and (15) respectively, we find their reaction functions, which are:

$$\begin{aligned} p_l(p_h) &= \frac{1 + c_l + p_h}{2} \\ p_h(p_l) &= \frac{1 + c_h - \alpha(1 + c_l - \Delta^e) + p_l}{2 - \alpha} \end{aligned} \quad (18)$$

Given that  $\alpha \leq 1$ , it is sure that both reaction functions intersect in an unique point. On solving the system of reaction functions (18), we obtain the firms' prices:

$$p_l(\alpha) = \frac{3 + 2c_l + c_h - \alpha(2 + 2c_l - \Delta^e)}{3 - 2\alpha}; p_h(\alpha) = \frac{3 + c_l + 2c_h - 2\alpha(1 + c_l - \Delta^e)}{3 - 2\alpha} \quad (19)$$

By incorporating optimal prices (19) in the indifferent consumer and profit's function (3) and (15), we have:

$$\begin{aligned} \hat{x}_{lh}(\alpha) &= \frac{3 - \Delta^c - \alpha(2 - \Delta^e)}{6 - 4\alpha}; \hat{\pi}_l(\alpha) = \frac{(3 - \Delta^c + \alpha(\Delta^e - 2))^2}{2(3 - 2\alpha)^2}; \\ \hat{\Pi}_{0h}(\alpha) &= \frac{A'\alpha^3 + 6B'\alpha^2 - 3C'\alpha + 6(3 + \Delta^c)^2}{12(3 - 2\alpha)^2} \end{aligned} \quad (20)$$

where  $A' = 48u - 48c_l - 28 - 3e_l(12 + e_l) + 6e_h(e_l - 2) - 3e_h^2$ ,  $B' = (e_h^2 + c_l(26 + \Delta^e) + e_l(17 + e_l + c_h) - e_h(2e_l + c_h - 7) - 2(12u - 9 + c_h))$  and  $C' = (45 - 36u + c_l^2 + c_l(50 + 4\Delta^e - 2c_h) - 4e_h(c_h - 3) - 14c_h + c_h^2 + 4e_l(6 + c_h))$ .

Taking into account the previous results, the government chooses the shares in firm  $h$  that maximizes the following social welfare function:

$$\widehat{W}(\alpha) = \widehat{CS}(\alpha) + \hat{\pi}_l(\alpha) + \hat{\pi}_h(\alpha) - \widehat{E}_T(\alpha)$$

where

$$\begin{aligned} CS(\alpha) &= \frac{D'\alpha^2 - 6F'\alpha + 3(36u - 39 + c_l^2 + c_h(c_h - 18) - 2c_l(9 + c_h))}{12(3 - 2\alpha)^2} \\ \hat{\pi}_h(\alpha) &= \frac{(\alpha(\Delta^e + 2) - 3 - \Delta^c)(2\alpha(\Delta^c - \Delta^e + 1) - 3 - \Delta^c)}{2(3 - 2\alpha)^2} \\ \widehat{E}_T(\alpha) &= e_l\hat{x}_{lh}(\alpha) + e_h(1 - \hat{x}_{lh}(\alpha)) \\ D' &= 48u - 48c_l - 52 + 3e_h^2 - 6e_h(e_l - 6) + 3e_l(e_l - 12) \\ F' &= 24u - 26 + c_l(\Delta^e - 18) - 6c_h - \Delta^e(c_h - 9). \end{aligned}$$

The first order condition, which is the same that previous case, is:

$$\frac{d\widehat{W}(\alpha)}{d\alpha} = \frac{(1-\alpha)(2\Delta^c - 3\Delta^e)^2}{2(3-2\alpha)^3} = 0.$$

Thus  $\alpha^* = 1$ , which is a maximum because the sufficient condition is satisfied, as we can see in Proof of Proposition 1. ■