

# Group Theory

## Teaching staff

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## Aims

The purpose of this course is to provide an introduction to some tools in Group Theory. It is assumed that the students are familiar with elementary group theoretical concepts (subgroups, factor groups, isomorphism theorems, Lagrange's theorem, permutation groups, cyclic groups, structure of abelian groups). At the end of the course the students should be able to read advanced books and research papers in group theory. Aside from lectures we will use computer package GAP.

## Syllabus

1. **Examples of groups.** Permutation groups. Linear groups. Further examples. Semidirect product. Wreath product.
2. **Finite groups.** Sylow theorems and their applications. Finite simple groups. Group representations by coset permutations.
3. **Free groups and presentations.** Free groups. Presentations by generators and relations. Finitely presented groups. The word problem. Nielsen-Schreier theorem. Reidemeister-Schreier theorem. Free products. Kurosh theorem.
4. **Nilpotent groups.** Central series. Characterization of finite nilpotent groups. Finitely generated and torsionfree nilpotent groups. Groups of prime power order and their automorphisms.
5. **Solvable groups.** Abelian series and solvable groups. Maximal and minimal conditions in solvable groups. Supersolvable groups. Hall and Carter subgroups. Huppert theorem on supersolvable groups.  $\pi$ -solvable groups. Mal'cev theorem on the solvability of matrix groups.
6. **Introduction to the classification of finite groups.** Highlights on finite simple group classification. Group extensions theory. Extensions, homology and cohomology of groups.

## Practices

The students should perform some computations using computer package GAP (Groups and Algorithms Program).

## Bibliography

- D. Gorenstein. *Finite groups*. Second Edition. Chelsea Publishing Company, 1980.
- M.I. Kargapolov and J.L. Merzljakov. *Fundamentals of the Theory of Groups*. Springer-Verlag, 1979.
- A. Machì. *Introduzione alla Teoria dei Gruppi*. Feltrinelli, 1974.
- D.J.S. Robinson. *A course in the Theory of Groups*. Springer, 1996.
- E. Schenkam. *Group Theory*. Krieger, 1975.

## Methodology

The spin of our course will be given by lectures. There will be some homework assignments which consist of oral presentations of some secondary topics or/and solving problems. Besides this, students will work with GAP.

## Assessment criteria

Students evaluation will be done through participation and oral presentations.